

Sudden stops, productivity and the optimal level of international reserves for small open economies

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Published Version

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Mihailov, A. ORCID: <https://orcid.org/0000-0003-4307-4029> and Nasir, H. (2022) Sudden stops, productivity and the optimal level of international reserves for small open economies. *Open Economies Review*, 33 (5). pp. 825-851. ISSN 1573-708X doi: <https://doi.org/10.1007/s11079-022-09678-2> Available at <https://centaur.reading.ac.uk/106227/>

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To link to this article DOI: <http://dx.doi.org/10.1007/s11079-022-09678-2>

Publisher: Springer

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Sudden Stops, Productivity and the Optimal Level of International Reserves for Small Open Economies

Alexander Mihailov¹ · Harun Nasir²

Accepted: 18 July 2022
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Abstract

This paper contributes to the theory of optimal international reserves by extending the Jeanne-Rancière (Econ J 121:905-930, 2011) endowment small open economy (SOE) model to a SOE with production that accounts for the main sources of economic growth. We, first, derive a richer analytical version of the optimal reserves formula in our set-up, essentially driven by labour-augmenting productivity and the saving rate. Then, under a plausible calibration based on 1975-2020 data averages for typical emerging market countries facing the risk of sudden stops in capital inflows, we find that the optimal reserves-to-output ratio is 7.5%, i.e., the midpoint in the range between that in Jeanne and Rancière (Econ J 121:905-930, 2011), of 9.1%, calibrated to the same sample of 34 middle-income countries, and that in Bianchi et al. (Am Econ Rev 108(9):2629-2670, 2018), of 6.0%, obtained in a different, sovereign debt model without capital and production. We explain the lower optimal reserves-to-output ratio relative to the endowment SOE by the role of capital accumulation as precautionary saving: the accumulated capital stock can potentially be used as a pledge to external creditors in obtaining borrowing, thereby insuring better a SOE against sudden stops. As the countries in our sample appear quite heterogeneous, we also compute the optimal reserves-to-output ratio by region. It turns out that our extended to production insurance SOE model matches well the average reserves-to-output ratio in the data for Latin America, represented by nearly half of our sample, 16 countries, at just above 10%. Yet, for Asia, Africa and Europe our regional model-based ratios understate considerably the respective data averages, suggesting the need to explore alternative modelling approaches.

Keywords Optimal international reserves · Small open economies · Sudden stops · Production technology · Precautionary saving · Insurance contracts

Mathematics Subject Classification E21 · E23 · F32 · F34 · F41 · O40

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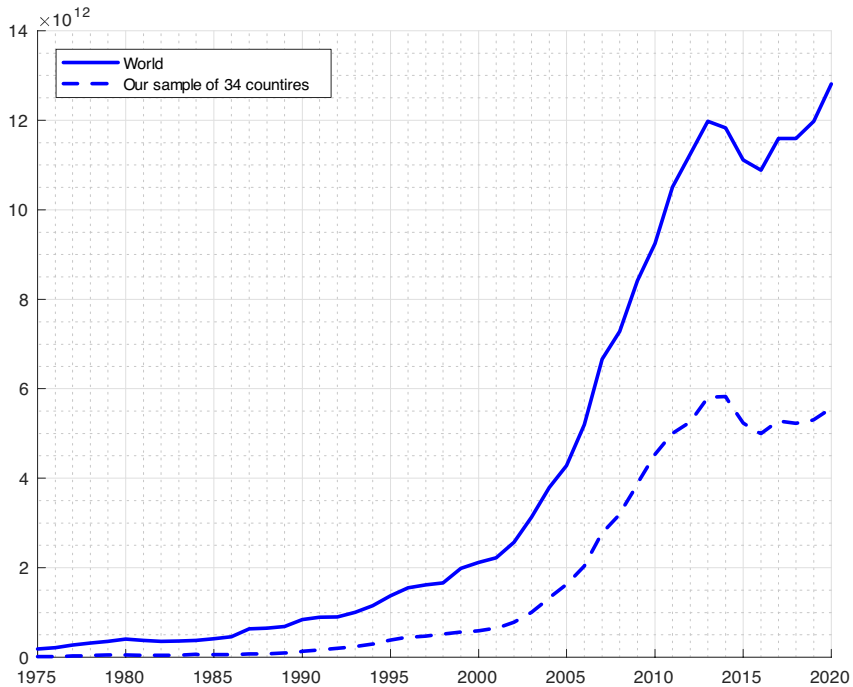


Fig. 1 International Reserves, trn USD. *Source:* IMF's *International Financial Statistics* and World Bank's *World Development Indicators*. Our sample of 34 countries, listed in Table 2 (further below), includes mostly middle-income countries

1 Introduction

International reserves across the globe have increased more than six times since the South-East Asian financial crisis of 1997-98. Figure 1 shows that middle-income countries account for nearly a half of this increase.

Consequently, the accumulation of international reserves in emerging market economies (EMEs) has become one of the most debated issues in open-economy macroeconomics (Chinn et al. 1999; Aizenman and Marion 2003; Dooley et al. 2004; Jeanne and Rancière 2006, 2011; Caballero and Panageas 2007, 2008; Alfaro and Kanczuk 2009; Durdu et al. 2009; Benigno and Fornaro 2012; Calvo et al. 2012; Dominguez et al. 2012; Bianchi et al. 2013, 2018; Arce et al. 2019; Bianchi and Sosa-Padilla 2020). Have many EMEs, in fact, accumulated excessive rather than adequate reserves? And what is an optimal ratio of reserves to output in a small open economy (SOE)? There is no consensus in the recent literature,¹ which offers contradictory explanations on these questions of immediate policy relevance. In particular, there

¹ See our discussion paper version, Mihailov and Nasir (2020), for a detailed literature review.

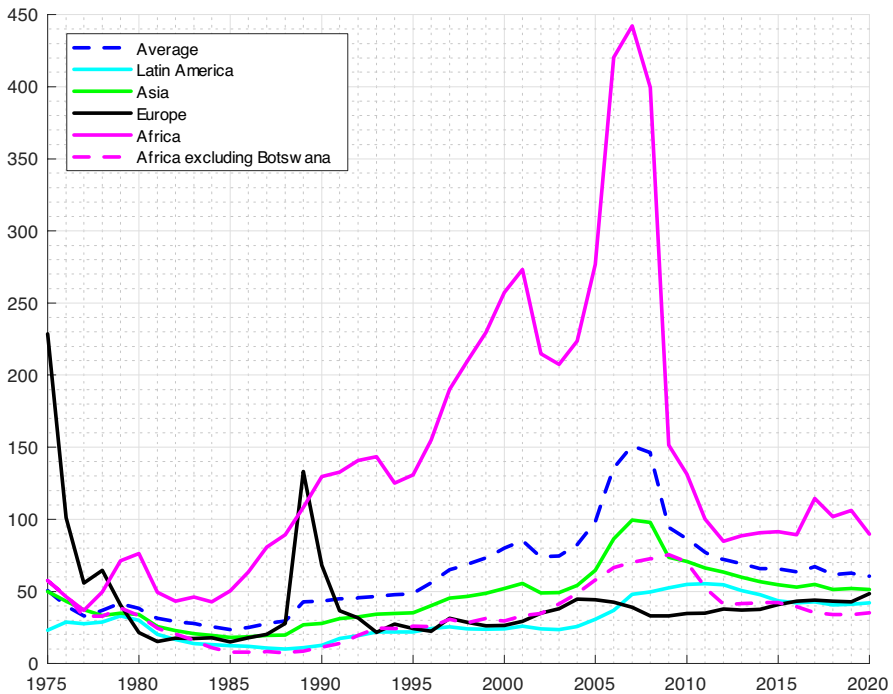


Fig. 2 International Reserves as % of Total External Debt by World Regions in Our Sample. *Source:* Data is for our sample of 34 countries listed in Table 2, as grouped by continent, from IMF's *International Financial Statistics* and World Bank's *World Development Indicators*

is a long-lasting debate whether reserve accumulation is driven by a self-insurance motive against abrupt capital flow reversals; or by the so-called 'new' or 'modern' mercantilism, i.e., hoarding international reserves as part of a development strategy, which facilitates growth by maintaining an undervalued currency and by serving as 'collateral' to encourage foreign direct investment (see, e.g., Aizenman and Lee 2007; Aizenman and Sun 2009).

Figure 2, however, makes it clear that the overall trend of rising international reserves, when expressed in terms of external debt, is largely accounted for by the Asian EMEs, especially after their bitter experience with the South-East Asian financial crisis. This observation points to the hypothesis that international reserves are likely to be accumulated in Asia with different motives and magnitudes compared to the other EMEs, and therefore may need a different modelling approach.

On the other hand, when expressed relative to broad money or GDP, as in Figs. 3 and 4, the increasing trend in international reserves looks more uniform across EMEs, and perhaps may potentially be well explained by a common theoretical framework.

Basically, two main benefits of large reserve holdings have been emphasized: (i) international reserves provide liquidity to smooth consumption (e.g., Jeanne

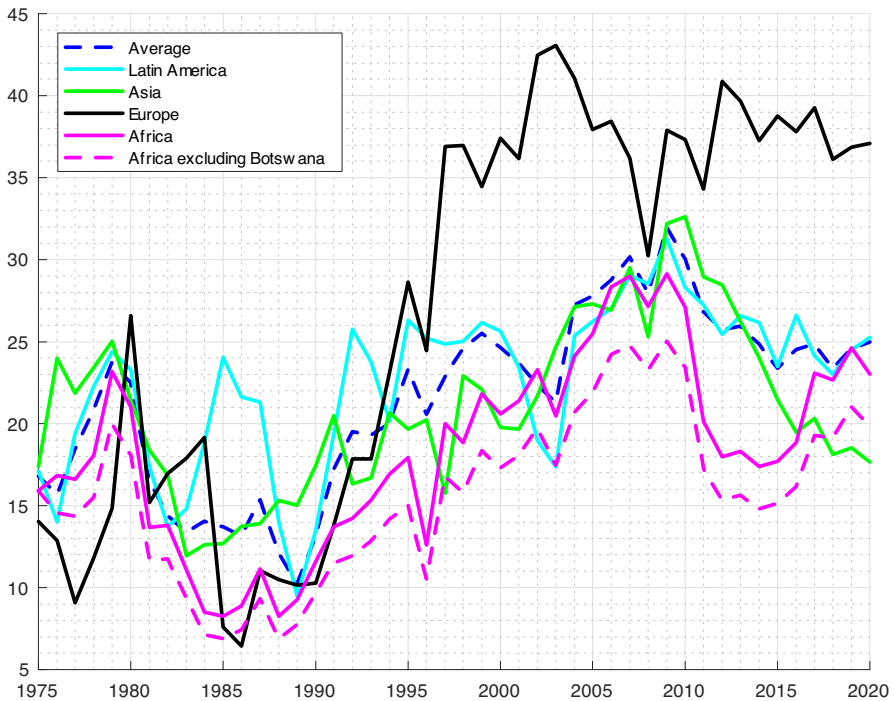


Fig. 3 International Reserves as % of Broad Money by World Regions in Our Sample. *Source:* Data and grouping as in Fig. 2

and Ranci re 2006, 2011); (ii) international reserves give a flexibility to manage sizable capital outflows in periods of crises (e.g., Aizenman et al. 2007). Moreover, it has also been argued that reserve policies can help guard away an economy from a crisis or contribute to a recovery after a crisis (Aizenman and Marion 2004; Dominguez et al. 2012).

The issue of reserve accumulation has been discussed under two main approaches: (i) one of them rationalizes why EMEs hold a high level of reserves as a form of self-insurance against ‘sudden stops’² in capital inflows (Chinn et al. 1999; Greenspan, 1999; Eichengreen and Mathieson 2000; Aizenman and Marion 2003, 2004; Dooley et al. 2004; Aizenman et al. 2007; Dominguez et al. 2012); (ii) the other examines what the determinants of reserve holdings are and, furthermore, what the optimal level of reserves is (Jeanne and Ranci re 2006, 2011; Caballero and Panageas 2007,

² Calvo (1998) seems to have coined and interpreted first this term in a published title. However, Bianchi and Mendoza (2020) clarify (in a footnote) that anecdotal evidence suggests that a comment from the audience in a presentation by Rudiger Dornbusch used the phrase referring to the Mexican crisis of 1994 and quoting Douglas Adams (a British comic writer whose works satirize contemporary life) in the sense that in sharp current-account reversals “it is not the fall that kills you, it is the sudden stop at the end”.

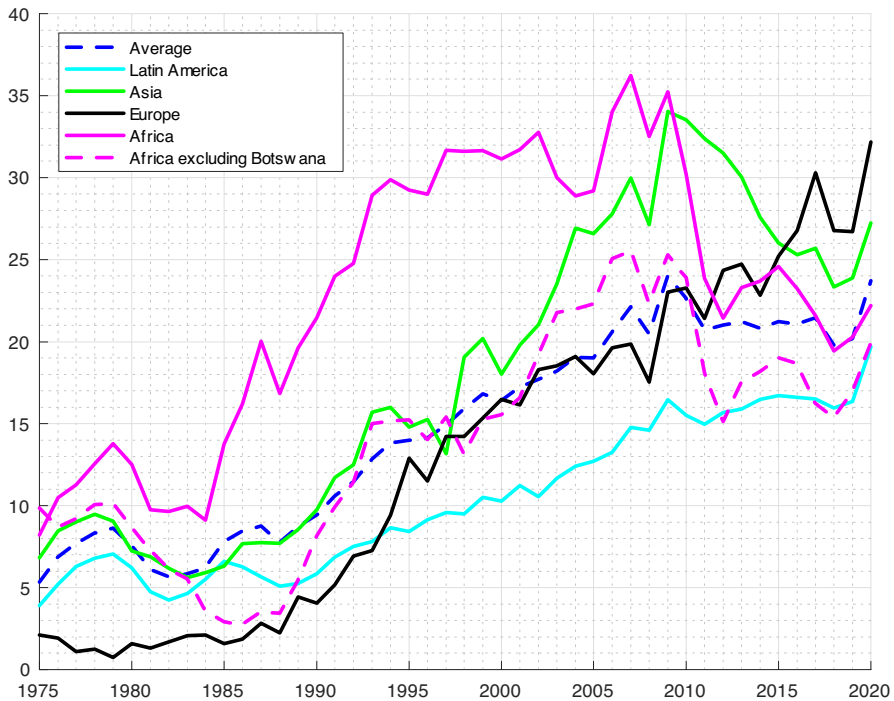


Fig. 4 International Reserves as % of GDP by World Regions in Our Sample. *Source:* Data and grouping as in Fig. 2

2008; Alfaro and Kanczuk 2009; Durdu et al. 2009; Calvo et al. 2012; Bianchi et al. 2013, 2018).

Jeanne and Rancière (2006, 2011), in particular, have considered the role of optimal international reserves as an insurance against sudden stops in capital inflows in an endowment SOE, abstracting from physical capital accumulation through investment. Yet, the literature has not analyzed this important role reserves play in a richer SOE set-up that models production and investment explicitly. Our contribution with the present theoretical paper consists in filling in this gap.

Indeed, most studies on international reserves have focused on other reserve-related issues, such as active reserve management (e.g., Aizenman et al. 2007) or the new type of monetary mercantilism (e.g., Aizenman and Lee 2007). While Jeanne and Rancière (2006, 2011) do consider optimal reserves in a SOE framework, essential features in neoclassical growth theory as well as in reality, such as production technology and production factors, remain outside the scope of their model. In concluding, these authors admit that their analysis is based on a stylized framework and one way to make it more realistic would be to add productive capital and investment. They suggest that the effects of such an extension are *a priori* ambiguous: on one hand, investment offers a new margin to smooth consumption, which would tend to reduce the optimal level of international reserves in terms of output; on the other hand, there will be a new benefit from reserves, namely, to smooth domestic

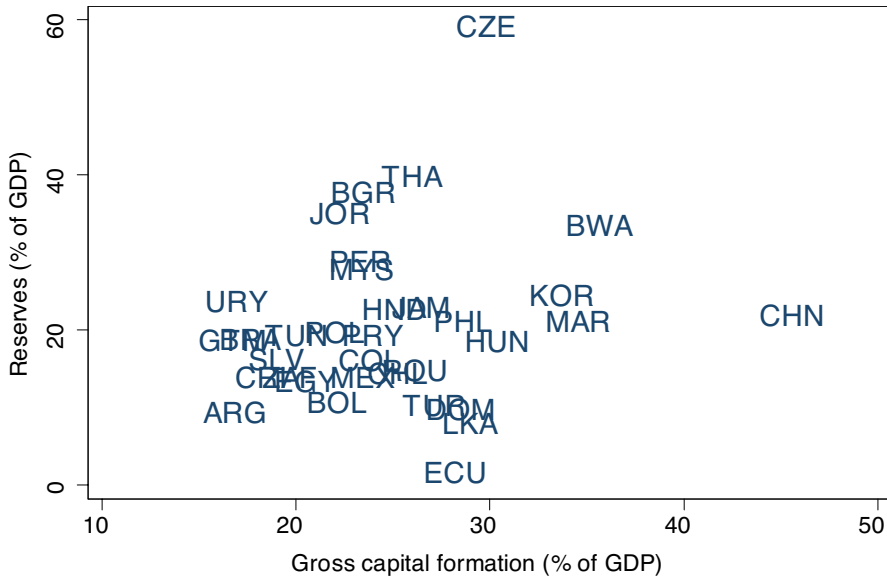


Fig. 5 International Reserves and Investment as % of GDP by Middle-Income Country. *Source:* World Bank, *World Development Indicators*, and authors' calculations. Data on reserves to GDP ratios and gross capital formation are for 2019 (2016 for Jordan only due to lack of data) for the 34 middle income countries in the our/JR sample, as listed in Table 2. The estimated slope coefficient for 2019 (and 15 other years: see footnote 3) is not statistically significant at all conventional levels

investment and output, that may tend to increase the optimal reserves-to-GDP ratio. Which one of these two effects will dominate is not obvious before an explicit careful investigation. This unexplored research question of central interest motivates our present work, also aiming to better understand the key determinants of the optimal level of international reserves in models of production SOEs.

Figure 5 shows a scatter plot of the relationship in 2019 between the ratios of investment and international reserves, respectively, to GDP in the sample of 34 middle-income EMEs used in Jeanne and Ranci ere (2011), referred to henceforth as JR. This figure could serve as another motivation, in particular for the theoretical nature of the study we undertake here. It is insightful to note that the regression coefficient for the scatter plot is not statistically significant at all conventional levels, and this is also the case for 15 other years in the sample.³ Only for 7 years in our sample⁴ such a scatter plot obtains a statistically significant coefficient at the 1% level, a slope which is also minimally positive. In conclusion, there is generally no correlation between investment and reserves in the data, at least for our type of countries. In such a sense, theoretical modelling may help guide such an analysis.

³ 1975, 1976, 1983, 1984, 1985, 1986, 1987, 1994, 1996, 1997, 2000, 2016, 2017, 2018, 2020.

⁴ 1980, 1981, 1982, 2004, 2005, 2009, 2010.

Along these lines of argument, and beyond, our paper proposes an extension to a production economy of the endowment SOE model in JR. More fundamentally, in doing so we bring together two strands of literature that have evolved independently and separately from each other over many years, namely neoclassical growth theory of the 1950s and 1960s and the open-economy theory of capital flows under the risk of sudden stops since the late 1990s. JR have developed an ‘insurance model’ of optimal international reserves where the representative consumer can smooth consumption during sudden stops if the central bank in an endowment SOE holds a stock of international reserves. The authors derive a closed-form expression for the optimal level of reserves relative to the level of output, and quantify it at 9.1%. They agree that this value cannot account for the rising reserve levels in EMEs since the late 1990s, especially in South-East Asia, which can clearly be seen by looking back into Fig. 4.

Our extension of the JR endowment SOE in the present paper aims to incorporate investment, capital, labour and production in this modelling approach. As demonstrated and discussed further down, we first derive a richer analytical version of the optimal reserves formula in our extended set-up, essentially driven by productivity and the saving rate, which the endowment SOE benchmark misses. We further derive novel theoretical results that highlight explicitly the roles of investment, the capital stock, labour and technology on the level of reserves to GDP. We illustrate and analyze the effects of the key determinants of the optimal reserves-to-output ratio, focusing on those that were revealed by the richer production SOE model we worked with, extending the endowment SOE benchmark.

Under a plausible calibration based on the 1975–2020 period for typical emerging market countries facing the risk of sudden stops in capital inflows, we then find that the optimal reserves-to-output ratio in our production SOE model is 7.5%. This is the mid-point in the range between the reserve ratio in Jeanne and Rancière’s (2011) endowment SOE, of 9.1%, calibrated to the same sample of 34 middle-income countries, and that in Bianchi et al. (2018), of 6.0%, obtained in a different, sovereign debt model without capital and production. We explain the lower optimal reserves-to-output ratio relative to the endowment SOE of Jeanne and Rancière (2011) by the role of capital accumulation as precautionary saving in our extension: the accumulated capital stock can potentially be used as a pledge to external creditors in obtaining borrowing. Such a ‘collateral’ function of the capital stock enhanced by flows of saving and investment insures better a SOE against sudden stops, and hence it should optimally hold less international reserves in terms of GDP.

Since the countries in our sample appear quite heterogeneous (e.g., in Figs. 2 through 4), we also compute the optimal reserves-to-output ratio by world region. This regional analysis reveals that the insurance SOE model of reserves we extend to production is validated empirically in terms of matching well the average reserves-to-output ratio in the data for Latin America, at just above 10%. This world region has the highest weight in our sample, and represents nearly half of it, 16 countries, so the empirical validation is important. However, for Asia (with 8 countries in our sample), Africa (with 5 countries) and Europe (with 5 countries too), our regional model-based ratios understate considerably the respective data averages. We conclude that the insurance modelling of reserves we followed is empirically relevant

for Latin American economies, but alternative approaches need to be explored for other world regions – unless this mismatch indicates the extent of reserves hoarding in them.

The rest of the paper is structured as follows. Section 2 presents our extension to production of the JR endowment small open economy. The results of our calibration and quantification exercise, for the whole sample and by regional subsamples, are discussed and illustrated in Sect. 3, and Sect. 4 concludes. Proofs and details on the derivations are provided in the Supplementary Online Appendix.⁵

2 Theory: Optimal Reserves-to-Output Ratio in a Production SOE

In this section, neoclassical growth theory is employed to examine the effects of capital and labour as key production factors on the optimal level of international reserve holdings in the endowment SOE benchmark of Jeanne and Rancière (2006, 2011). Whenever possible, we use the notation of JR to preserve comparability. All assumptions of the JR model are maintained, but now a conventional Cobb-Douglas production function is added. More precisely, we employ a constant returns to scale (CRS) labour-augmenting⁶ Cobb-Douglas production function. As we are seeking a solution for the balanced growth path (BGP) of the SOE model in the long run, we employ this particular production function rather than the alternatives such as Hicks-neutral technology and Solow-neutral technology. Harrod-neutral technology is the only one that is consistent with a solution for the BGP in the long run (see, e.g., Acemoglu (2009) or Jones and Vollrath (2013)).

2.1 Environment

We focus on the optimal level of reserves relative to the level of output that is perceived as insurance, for a production SOE in our case, against losing access to the international credit market. A representative domestic agent, or a private sector, is assumed, as well as a domestic government. There is also an international representative agent, referred to as foreign insurers or the rest of the world (RoW), who provide international reserves to the country. The representative domestic agent in the SOE produces a single (composite) good, which is consumed or invested as physical capital domestically as well as consumed abroad (as SOE exports). The model is set out in discrete time with infinite horizon, using the time subscript $t = 0, 1, 2, \dots$. Apart from the risk of sudden stops in capital inflows, there is no other source of uncertainty. In that sense, the country faces a risk of international liquidity problems in an otherwise deterministic setting, as in JR.

Following JR, the domestic private sector consists of a continuum of atomistic and identical infinitely-lived consumers. Their intertemporal utility U_t is written as

⁵ Our data and code are available upon request as a zip archive.

⁶ Known also as Harrod-neutral technology.

$$U_t = E_t \left[\sum_{i=0, \dots, +\infty} (1+r)^{-i} u(C_{t+i}) \right], \tag{1}$$

where r denotes the constant world interest rate, the period utility function $u(C_{t+i})$ is assumed to be of the constant relative risk aversion (CRRA) type, with CRRA parameter $\sigma \geq 0$ and C being aggregate consumption,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}, \quad \sigma \neq 1, \tag{2}$$

with $u(C_t) = \log(C_t)$ for $\sigma = 1$.

The consumer’s budget constraint now includes investment in physical capital:

$$C_t = Y_t - I_t + L_t - (1+r)L_{t-1} + Z_t, \tag{3}$$

where Y_t is domestic output, I_t is investment in physical capital domestically in order to increase the capital stock and next-period output, L_t is newly-contracted external debt in t with a one-period maturity only and Z_t is a net transfer from the government in t . As in JR, external debt accumulated in $t - 1$ has to be repaid in t at r , captured by $(1+r)L_{t-1}$, and default in paying back external debt as well as foreign lending by the SOE are assumed away. Differently from JR, investment in physical capital provides a third channel of saving in any period t , in addition to the net indebtedness of the SOE to the RoW, $L_t - (1+r)L_{t-1}$, and to the domestic government (or the public sector), entering via the net transfer, Z_t . It is perhaps easier to see the implications of our extension to a production SOE by writing disposable income of the domestic private sector in t , DY_t , compactly as:

$$DY_t \equiv Y_t + L_t - (1+r)L_{t-1} + Z_t.$$

Then, the SOE private-sector budget constraint (3) can be re-written as

$$C_t = DY_t - I_t; \tag{4}$$

and, hence, the SOE private-sector saving in physical capital is defined, as standard, by

$$I_t = S_t \equiv DY_t - C_t. \tag{5}$$

As in neoclassical growth theory, it is common to assume that all firms have an identical production function. Then, the aggregate production function is

$$Y_t = F(K_t, A_N N_t) = K_t^\theta (A_N N_t)^{1-\theta}. \tag{6}$$

K_t denotes the capital stock, N_t is total employment at time t , and A_N is a parameter interpreted in the neoclassical tradition as labour-augmenting technology. $0 < \theta < 1$ measures the capital share in production and assumes CRS. The standard features of this production function are assumed: continuity, twice-differentiability with respect to each argument, positive diminishing returns to each factor and constant returns to scale to both factors – see, e.g., Acemoglu (2009) or Jones and Vollrath (2013).

As in the JR model, there are two states in the economy: the normal – or non-crisis – state (denoted by a superscript n), occurring with probability $1 - \pi$; or a crisis state interpreted as a sudden stop (denoted by superscript s), occurring with probability π . In the non-crisis state, output increases by a fixed rate g_Y and the economy can guarantee a constant portion of the output,

$$F(K_t, A_N N_t)^n = (1 + g_Y)^t F(K_{t-1}, A_N N_{t-1}) \quad (7)$$

$$\alpha_t^n = \alpha. \quad (8)$$

On the other hand, when the economy faces a sudden stop, domestic output decreases by a constant fraction γ below its long-run growth path, and guaranteed output goes down to zero:

$$F(K_t, A_N N_t)^s = (1 - \gamma) F(K_t, A_N N_t)^n \quad (9)$$

$$\alpha_t^s = 0. \quad (10)$$

Due to normalization, the guaranteed output does not drop below a positive level. The sum of the time-varying parameter and the output loss parameter is assumed lower than unity, $\alpha + \gamma < 1$, in order to secure that the domestic private sector does not have difficulty to pay back all the debt during the crisis. The interest rate on external debt repayment is assumed to be higher than the growth rate of the SOE output, $r > g_Y$, to hold the private sector's intertemporal income limited as in JR.

We follow JR in also assuming that after a sudden stop the capital inflow converges to its pre-crisis pattern within a certain number of periods, v . Moreover, the country returns to the normal state, n , in period $t + v + 1$. In reality, a country would gain access to international liquidity as in its pre-crisis level in more than one year, if a sudden stop hits the economy in the current period t . Therefore a 'sudden stop episode' can be defined as the length $[t, t + v]$, as in the JR model. In other words, matching the various times of a crisis stage $s_t = s^0, s^1, \dots, s^v$, in a specific period t the country might be either in the non-crisis state, $s_t = n$, or in some of the crisis states.

The dynamics of output and external credit in a sudden stop episode starting at t are given by:

$$F(K_{t+\tau}, A_N N_{t+\tau})^s = [1 - \gamma(\tau)] F(K_{t+\tau}, A_N N_{t+\tau})^n, \quad (11)$$

$$\alpha_{t+\tau}^s = \alpha(\tau), \quad (12)$$

where $\tau = 0, 1, \dots, v$. In both Eqs. (11) and (12) $\gamma(\tau)$ and $\alpha(\tau)$ are exogenous functions of τ . For $\tau = 0$ in (11) and (12), we see that $\gamma(0) = \gamma$ and $\alpha(0) = 0$, as in the JR model. Furthermore, we similarly assume that the economy returns to its trend path in a monotonic way, in the sense that both are non-negative but $\alpha(\tau)$ is increasing in τ , while $\gamma(\tau)$ is decreasing in τ . When the crisis is over, the private sector can be

financed by international liquidity as in pre-crisis periods, so there will be no restriction to access foreign markets, hence, $\alpha(v) = \alpha$.

Introducing labour, N_t , into the production SOE model requires a description of population growth, assumed to be exogenously given, as in neoclassical growth theory and, more recently, in Gourinchas and Jeanne (2013),

$$\frac{\Delta N_{t+1}}{N_t} = g_N; N_t = (1 + g_N)^t N_0, \tag{13}$$

where N_0 is the population level in a base period and g_N is the constant population growth rate.

Assuming again, as in neoclassical growth theory, that the saving-to-output ratio is constant, s , we can now define (see for more detailed steps Online Appendix B) the rate of growth of capital per capita, $k_t = \frac{K_t}{N_t}$:

$$g_k = \frac{\Delta k_{t+1}}{k_t} = s \frac{y_t}{k_t} - \delta - g_N \tag{14}$$

Then, as in neoclassical growth theory, it turns out that the solution for the BGP in effect imposes a constant capital-output ratio, which is a function of four parameters:

$$k_t = k = \frac{s}{g_K + \delta + g_N} = const \text{ (along BGP)}. \tag{15}$$

Capital accumulation can, further, be written as

$$\Delta K_{t+1} = sY_t - (\delta + g_N)K_t. \tag{16}$$

Investment in the normal state is, then,

$$I_t^n = sY_t = \Delta K_{t+1} + (\delta + g_N)K_t = (g_K + \delta + g_N)K_t. \tag{17}$$

If we replace output by the production function in order to see the effect of its components on the optimal level of reserves, we obtain

$$K_t^\theta (A_N N_t)^{1-\theta} = \frac{g_K + \delta + g_N}{s} K_t,$$

where output is proportional to the capital stock.

The neoclassical BGP concept implies that all key variables grow at the same rate:

$$g_Y = g_K = g_N = g_A. \tag{18}$$

We assume that the capital-labour ratio (or per capita capital) does not grow in sudden stops, equivalent to writing:

$$k_{t+1}^s = k_t^s.$$

In the JR SOE model we here extend to production one of the critical assumptions is related to newly-contracted one-period ahead external debt, L_t . How much can a SOE borrow from foreign lenders? There should be some limit on the amount of output that can be guaranteed by the domestic private sector to foreign creditors. As in the JR model, this restriction is given in what follows by the condition that the external debt must be completely paid back in the next period, which requires:

$$(1 + r)L_t \leq \alpha_t F(K_{t+1}, A_N N_{t+1})^n, \quad (19)$$

where $F(K_{t+1}, A_N N_{t+1})^n$ is trend output in period $t + 1$ (to be defined shortly), α_t is a time-varying parameter used as a proxy for the pledgeability of domestic output to foreign creditors, and the superscript n denotes ‘normal’ times. Assuming, as in JR, that the agents know the value of α_t and $F(K_{t+1}, A_N N_{t+1})^n$ in any current period t , condition (19) states that external debt in period t is default-free as long as (19) is fulfilled. We follow JR in also assuming that the time-varying parameter α_t is an exogenous variable, and it can change with expectations regarding enforcement of creditor rights or penalties on domestic defaulters: because of the possibility of sudden stops, the rigidity of the consumer’s external debt borrowing constraint can fluctuate over time.

In our model extension with physical capital and labour in the CD technology outlined thus far, sudden stops have negative effects on both consumption and investment decisions of domestic consumers, and therefore reduce their welfare. Economic crises reduce trend consumption because consumers’ elasticity of intertemporal substitution in consumption is bounded. Moreover, sudden stop episodes cause a reduction of domestic output which implies a decrease in the consumers’ intertemporal income. Eventually, consumption increases as foreign capital flows return into the economy after the sudden stop. However, our production SOE extension reveals a new feature in the adjustment of the economy, absent in JR: it takes more than one year for investment to recover to its pre-crisis level. Investment continues to decrease after the sudden stop year. Figure 7 later on illustrates this in a 5-year event window: as we shall discuss in due time, there might be many possible explanations for the persistent effect of sudden stops on investment, such as increasing costs of investment, difficulty to find foreign funds for investment, or the preference of external creditors to invest in more stable economies.

The second domestic agent in the SOE is the government – or, equivalently, the monetary-fiscal authority, which plays a critical role in the JR model and in our extension. The task of the government in this set-up is to provide smooth domestic consumption between normal and crisis states. To implement such a policy, the government has as a tool what JR term ‘reserve insurance contracts’. Introducing investment in physical capital and labour-augmenting productivity in our extensions does not affect the government, and we therefore keep all assumptions related to it and its transfers as in JR. Yet, for completeness, we briefly describe the behaviour of the government next.

A reserve insurance contract is a simple contract between the government and foreign insurers. The aim of the government is to protect domestic agents from the case

of a sudden reversal in capital flows; therefore, the government forgoes some funds today in order to gain capital access during the crisis.⁷ In this sense, reserve insurance contracts embody the trade-offs in reserve management, and the mechanism is as follows. Firstly, the government announces a settlement with external creditors in period 0. Then, the external fund providers receive a payment X_t from the monetary authority in period t . This process continues until a crisis occurs. Once the crisis starts at time t , the economy obtains a fund R_t . The monetary authority might sign a new reserve insurance deal with foreign insurers when the sudden stop episode ends.⁸

The government's role can be seen in the budget constraint (3) since it shifts the funds coming from the agreement with foreign investors to the private sector as follows; if the country is in the non-crisis stage,

$$Z_t^n = -X_t; \quad (20)$$

however, if a sudden stop occurs, the government secures a payment in the form of

$$Z_t^s = R_t - X_t. \quad (21)$$

Equation (21) shows the government's gain during the sudden stop of capital inflows. The economy earns R_t from foreign insurers, but should also effect the last payment of the reserve insurance contract, X_t , within the duration of the sudden stop.

There is no change either in foreign insurers' participation condition once we incorporate physical capital and labour. Therefore, all assumptions regarding foreign insurers are kept as in JR. For completeness, we briefly describe their behaviour next.

The role of external creditors is to supply international liquidity to the economy during the sudden stop via the reserve insurance contracts. This definition requires a condition that foreign creditors should agree on the price of the government contracts. This is a critical parameter, which enters the condition for foreign insurers' participation. The marginal utility of funds for the investors at date t is denoted by μ_t . As in JR, it is more expensive in the crisis than in the normal state:

$$\mu_t^s \geq \mu_t^n. \quad (22)$$

The price of insurance depends on the ratio between μ_t^s and μ_t^n . For simplicity, the JR model assumes that the price parity of funds in normal times to funds in the sudden stop episode is fixed and equal or less than one, which we follow:

$$p = \frac{\mu_t^n}{\mu_t^s} \leq 1. \quad (23)$$

⁷ This could be seen as the cost of reserves and JR show that this kind of insurance should be financed by long-term liabilities.

⁸ Since the time of the crisis is unknown, an insurance contract signed in period 0 must be specified as an infinite sequence of conditional payments $(X_t, R_t)_{t=1, \dots, +\infty}$ (see JR).

The JR model considers external investors as being perfectly competitive and as sharing the same time discount rate with the domestic private sector. Under these assumptions foreign insurers supply any ‘reserve insurance contract’ $(X_t, R_t)_{t=1, \dots, +\infty}$ whose present discounted value is non-negative, of the for

$$\sum_{t=1}^{+\infty} \beta^t (1 - \pi)^{t-1} [(1 - \pi)X_t \mu_t^n - \pi(R_t - X_t) \mu_t^s] \geq 0. \tag{24}$$

2.2 Optimal Reserves-to-Output

We continue to assume that the external credit constraint (19) is always binding, which allows for a closed-form solution of this simple insurance problem of the production SOE. The parameter λ , defined as the share of short-term external debt (STED) in GDP, takes the form (for more detailed steps, see Online Appendix B)

$$\lambda = \frac{L_t^n}{K_t^\theta (A_N N_t)^{1-\theta}} = \frac{1 + (1 - \theta)g_N + \theta g_K}{1 + r} \alpha, \tag{25}$$

since the country keeps a constant STED-to-output ratio when (19) is always binding.

Using the CD production function, consumption in the non-crisis state can be written as (see Online Appendix B)

$$C_t^n = \left\{ 1 - s - \lambda \frac{r - [(1 - \theta)g_N + \theta g_K]}{1 + (1 - \theta)g_N + \theta g_K} \right\} K_t^\theta (A_N N_t)^{1-\theta} - X_t. \tag{26}$$

By analogy, consumption in the sudden stop episode can be written as (see Online Appendix B)

$$C_t^s = \left\{ -\gamma - \frac{(\delta + g_N)K_t}{K_t^\theta (A_N N_t)^{1-\theta}} - \lambda \frac{r - [(1 - \theta)g_N + \theta g_K]}{1 + (1 - \theta)g_N + \theta g_K} \right\} K_t^\theta (A_N N_t)^{1-\theta} + R_t - X_t. \tag{27}$$

Since there is no change in the role of monetary(-fiscal) authority, it enters a reserve insurance contract as described above in order to maximize the private sector’s utility subject to the relevant constraints.

The optimal reserves-to-output ratio under the CD production function, ρ_{CD}^* , is then constant, as it was in the JR endowment SOE set-up, but now given by a richer expression, as stated formally in the next proposition.

Proposition 1 (Optimal reserves-to-output ratio in a SOE with labour-augmenting Cobb-Douglas technology) *Assuming the described labour-augmenting Cobb-Douglas production SOE environment with the external credit constraint (19) always binding, the optimal level of the ratio of international reserves to output, $\rho_{CD}^* \equiv \frac{R_t}{K_t^\theta (A_N N_t)^{1-\theta}}$, is constant and given by:*

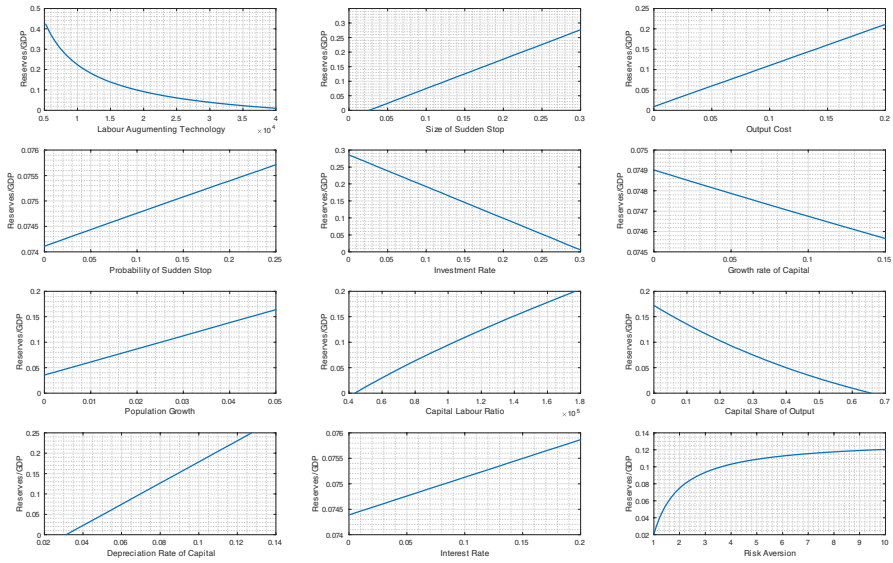


Fig. 6 Optimal Reserves-to-GDP Ratio as a Function of Its Key Determinants. *Source:* Authors’ calculations using data from IMF’s *International Financial Statistics*, Penn World Table 7.0 and World Bank’s *World Development Indicators*

$$\rho_{CD}^* = \frac{\gamma + \lambda - \left(1 - \lambda \frac{r - [(1-\theta)g_N + \theta g_K]}{1 + (1-\theta)g_N + \theta g_K}\right) \left(1 - p^{\frac{1}{\sigma}}\right) + (\delta + g_N) \left(\frac{k}{A_N}\right)^{1-\theta} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)} \left(1 - p^{\frac{1}{\sigma}}\right)} \tag{28}$$

Proof of Proposition 1 See Online Appendix B.

As seen in (28), the optimal level of reserves in terms of GDP with deterministic constant returns to scale labour-augmenting Cobb-Douglas production function in our SOE set-up has many common parameters with the JR endowment benchmark. For example – see, for a visual representation, Fig. 6,⁹ the optimal reserve ratio is a positive function of: (i) the share in output of short-term external debt, λ , as a proxy for the size of the sudden stop; (ii) the output cost of a sudden stop, γ ; (iii) the probability of a sudden stop, π ; (iv) the world interest rate, r ; and (v) the degree of relative risk aversion, σ . Differently from the endowment SOE, the additional determinants in this CD-production version influence the optimal reserves-to-output ratio as follows: (i) labour-augmenting productivity, A_N , negatively; (ii) the investment rate of the economy, s , negatively; (iii) the growth rate of capital, g_K , negatively; (iv) population growth, g_N , positively; (v) the capital-labour ratio, k , positively; (vi)

⁹ The calibration choices underlying the figure are usually sample averages or values used by JR, as in Table 1 and as discussed further below.

the capital share, θ , negatively; and (vii) the depreciation rate of the installed capital stock, δ , positively.

Corollary 1 (Relative optimal reserves-to-output ratio with labour-augmenting Cobb-Douglas technology) *In order to judge about the magnitude of the optimal reserves-to-output ratio derived in Proposition 1, ρ_{CD}^* , relative to the output cost of a sudden stop, γ , and the STED-to-output ratio, λ , we follow JR in re-writing (28) as:*

$$\gamma + \lambda - \rho_{CD}^* = \frac{\left(1 - p^{\frac{1}{\sigma}}\right) \left[1 - \alpha - \gamma + (\gamma + \lambda) \frac{p(1-\pi)}{\pi + p(1-\pi)}\right] - (\delta + g_N) \left(\frac{k}{A_N}\right)^{1-\theta} + p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)} \left(1 - p^{\frac{1}{\sigma}}\right)} \quad (29)$$

Proof of Corollary 1 See Online Appendix B.

It can easily be seen that if the terms $-(\delta + g_N) \left(\frac{k}{A_N}\right)^{1-\theta} + p^{\frac{1}{\sigma}} s$ are ignored, the CD-production SOE optimal reserve formula in (29) would reduce to that in the JR endowment benchmark. Therefore, both models include many similar determinants. However, modelling explicitly production in a SOE, here by labour-augmenting CD technology, avoids the reduction of the special case of $p = 1$, to the so-called Greenspan-Guidotti rule:¹⁰ $\rho_{CD}^* = \gamma + \lambda$. In our richer SOE model with production, even in this case $\rho_{CD}^* = \gamma + \lambda + (\delta + g_N) \left(\frac{k}{A_N}\right)^{1-\theta} - s$, so that population growth, g_N , the steady-state capital-labour ratio, k , and the share of labour in the production process, $1 - \theta$, tend to increase optimal reserves in terms of output, whereas labour-augmenting technology, A_N , tends to decrease it, together with the investment rate, s . Given the latter theoretical result and the related discussion in the Introduction, our production SOE model thus supports the role of saving and investment as increasing the physical capital stock, to be potentially used as a pledge to foreign creditors, and hence insuring a consumption-smoothing role for the SOE residents.

3 Calibration: Quantification and Interpretation of Our Analytical Results

In this section, we analyze some quantitative implications of our production SOE model, comparing it to the respective findings in the JR endowment SOE as well as to other attempts to pin down the ratio of international reserves to output in the recent literature we outlined earlier. To make the comparison as direct and meaningful as possible, we employ data for the same 34 middle-income countries but

¹⁰ See Greenspan (1999) and Guidotti et al. (2004).

extending the original JR sample period, 1975-2003, by 17 years, to 2020.¹¹ In accordance with the JR method, we construct a benchmark calibration based on the average sudden stop in our sample, as updated to 2020. We then discuss to what extent our model of a production SOE is able to explain the recent trend toward a buildup of international reserves in EMEs, in the sample as a whole and by key world regions.

3.1 Results for the Whole Sample

Our calibration of the key parameters determining the optimal reserve-to-output ratio (according to proposition 1 and corollary 1) for the sample as a whole (34 countries throughout 46 years, 1975-2020) are given in Table 1.

We recomputed some of the JR model parameters based on our updated sample, such as the output loss, the size of the sudden stop, and the crisis probability, since they play a modified role in our model extension. We did not change some other JR parameters, such as the risk-free interest rate, the relative price of a non-crisis dollar and the CRRA, as they have no distinct novel role in our model but are necessary for a comparison.

To analyze the behaviour of the model economy, we decompose domestic consumption, C_t , in terms of domestic output, Y_t , less investment, I_t , the financial account, FA_t , income transfers from abroad, IT_t , and reserves decumulation, $-\Delta R_t$,¹²

$$C_t = Y_t - I_t + FA_t + IT_t - \Delta R_t. \tag{30}$$

As in JR, a sudden stop is defined by an unexpected abrupt fall in the financial account. *Ceteris paribus*, it leads to a drop in domestic consumption. This effect can be amplified by a simultaneous drop in output, but can also be mitigated by decumulating reserves.

One can see the correspondence between the national accounting identity (30) and our labour-augmenting Cobb-Douglas production SOE model:

$$\underbrace{C_t^s}_{C_t} = (1 - \gamma) \underbrace{Y_t^n}_{Y_t} - \left\{ \underbrace{(\delta + g_N)K_t}_{I_t} + \underbrace{(-L_{t-1})}_{FA_t} + \underbrace{[-r_t L_{t-q} - (\pi + \omega)R_t]}_{IT_t} - \underbrace{(-R_t)}_{\Delta R_t} \right\}, \tag{31}$$

¹¹ JR applied the World Bank's classification to define their middle-income countries. However, after the publication of their paper this classification has changed: in effect, the sample now includes 7 high-income countries, i.e., Argentina, Chile, Czechia, Hungary, Korea, Poland and Uruguay. Following JR, we also exclude major oil-producing countries from the dataset.

¹² Equation (30) can also be interpreted as decomposing domestic absorption since domestic absorption equals the sum of domestic consumption and investment, $DA_t = C_t + I_t$.

Table 1 Calibration of Parameters

Parameter Interpretations	Parameters	Range						
Panel A: Common Calibration								
Size of a sudden stop	$\lambda = 0.1$	[0, 0.3]						
Output loss	$\gamma = 0.065$	[0, 0.2]						
Risk-free world (net) interest rate	$r = 0.05$	-						
Coefficient of relative risk aversion	$\sigma = 2$	[1, 10]						
Price of a non-crisis dollar	$p = 0.885$	-						
Capital share in output	$\theta = 0.3$	-						
Capital stock depreciation rate	$\delta = 0.06$	[0, 1]						
Panel B: Specific Calibration by World Region								
Probability of a sudden stop	$\pi = 0.1$	[0, 0.25]	Africa (5)	Africa less BWA (4)	Asia (8)	Europe (5)	Latin America (16)	
Labour-augmenting technology	$A_N = 22514$	-	0.115	0.110	0.054	0.087	0.080	
Saving (= investment) rate	$s = 0.226$	[0, /0.48]	25199	24950	18676	32746	19254	
Output per worker	$y = 33779$	-	0.2491	0.2407	0.2800	0.2300	0.1950	
Capital-labour ratio	$k = 87046$	-	35094	34588	29147	51280	28459	
Growth rate of the population	$g_N = 0.015$	[0, 0.035]	76013	74119	82348	146030	70826	
Growth rate of the capital stock	$g_K = 0.062$	[0, 0.015]	0.0225	0.0240	0.0140	0.0021	0.0163	
Growth rate of output	$g_Y = 0.04$	-	0.080	0.077	0.073	0.046	0.056	
			0.044	0.040	0.060	0.026	0.030	

Source: Authors' calculations using data from Penn World Tables 10.0 and World Bank's World Development Indicators

where ω denotes a pure risk premium and might be interpreted as an opportunity cost of holding reserves.¹³ As JR have emphasized, such a decomposition is useful in allowing to infer the magnitude of the shocks hitting the economy in a sudden stop episode, that is, λ and γ , from the empirical behaviour of the terms on the RHS of (31). Furthermore, our extended model to production highlights the dynamics of output as resulting from investment and capital accumulation as well as employment, key macrovariables that are omitted in the JR endowment SOE model. In particular, now – as seen in (31) – investment in sudden stops includes dependence on capital stock depreciation and on labour via population growth, $(\delta + g_N)K_t$.

Following Guidotti et al. (2004) and Jeanne and Rancière (2011), a sudden stop in year t is identified in our sample as a drop in the ratio of capital inflows to GDP exceeding 5% relative to the preceding year. The countries in our sample and the years in which they went through sudden stop episodes are listed in Table 2.

Even though we use the same sample of countries as Jeanne and Rancière (2011), our sudden stop years were defined applying their methodology to our updated dataset, and therefore some minor differences in the sudden stop episodes by country are observed. Moreover, when we calculate capital inflows in our dataset mostly World Bank's *World Development Indicators* (WDI) was used, whereas JR relied on IMF's *International Financial Statistics* (IFS).

Figure 7 illustrates this novel feature of output dynamics in our extension to production, now driven obviously by investment dynamics relative to the JR benchmark. It depicts the average behaviour of consumption and the contribution of the various components on the RHS of (31) in a five-year event window centred around a sudden stop year, where the middle observation '0' labels the latter (output is normalized to 100 in that year of the sudden stop). Although all components of Eq. (30) display a similar pattern with the JR model, investment adds inertia in its own adjustment and, hence, in the adjustment of output. Both investment and output in our production SOE model continue to decrease after the sudden stop period, featuring higher persistence, whereas all other components of Eq. (30) start recovery after period '0', as is the case with output when investment and capital are not modelled in JR. The difficulties in accessing international borrowing facilities after the sudden stop and the capital outflows during the crisis make the private sector vulnerable, and this affects investment decisions. Therefore, a recovery may not be seen in investment and output in the first year after the sudden stop.

The unconditional probability of a crisis, π , is 8% per year for the full sample, 9% per year for the countries which had at least one sudden stop, and 10% for countries which had at least 2 sudden stops in our updated calibration, and thus remains consistent with Jeanne and Rancière (2011), 10%. The STED-GDP ratio, interpreted as the size of a sudden stop, λ , is calibrated at the average level of the ratio of capital inflows to GDP, $\frac{FA_t}{Y_t}$, over our sample of crisis episodes, and is 10%, again consistent

¹³ Because it has no role in affecting productivity and investment, the opportunity cost of holding reserves is not described in our extended set-up. However, in order to enable comparisons between our CD-production SOE model and the JR endowment SOE benchmark, we follow their methodology in expressing $X_t = (\pi + \omega)R_t$.

Table 2 Countries and years of sudden stops

Country	Years of Sudden Stops
Argentina	1989, 2001, 2002, 2018, 2019
Bolivia	1980, 1983, 1994, 2012, 2019
Botswana	1977, 1987, 1991, 1993, 2002, 2009, 2013
Brazil	1983
Bulgaria	1990, 1994, 1996, 1998, 2008, 2009, 2010, 2013, 2017
Chile	1982, 1983, 1985, 1998, 2009, 2020
China	–
Colombia	–
Costa Rica	1984
Czechia	1996, 2003, 2018
Dominican Rep.	1981, 1993, 2003
Ecuador	1983, 1988, 1999, 2000
Egypt	1990, 1993, 2011
El Salvador	1979, 2009, 2020
Guatemala	–
Honduras	1978, 1998, 2005, 2009
Hungary	1994, 1996, 2001, 2006, 2009, 2015
Jamaica	1983, 1985, 1986, 1988, 2002, 2009, 2012, 2015, 2018
Jordan	1976, 1979, 1980, 1984, 1989, 1992, 1993, 1998, 2001, 2008, 2010, 2011, 2014, 2018
Korea	1997, 2008
Malaysia	1984, 1987, 1994, 1999, 2005, 2008
Mexico	1982, 1995
Morocco	1978, 1995
Paraguay	1988, 1989, 2002
Peru	1983, 1984, 1998, 2009, 2011, 2013
Philippines	1983, 1997, 2000
Poland	2017
Romania	1988, 2008, 2009
South Africa	1985, 2020
Sri Lanka	–
Thailand	1982, 1997, 2009, 2011
Tunisia	–
Turkey	1994, 2001
Uruguay	1982, 2002, 2004, 2007, 2009, 2010, 2015

The sample includes countries classified as middle-income by the World Bank, plus 7 high-income countries: Argentina, Chile, Czechia, Hungary, Korea, Poland and Uruguay. A country-year observation is identified as a sudden stop if the ratio of capital inflows to gross domestic product falls by more than 5%. Capital inflows are measured as the current account deficit minus reserves accumulation

Source: Authors' calculations using data from World Bank's *World Development Indicators*

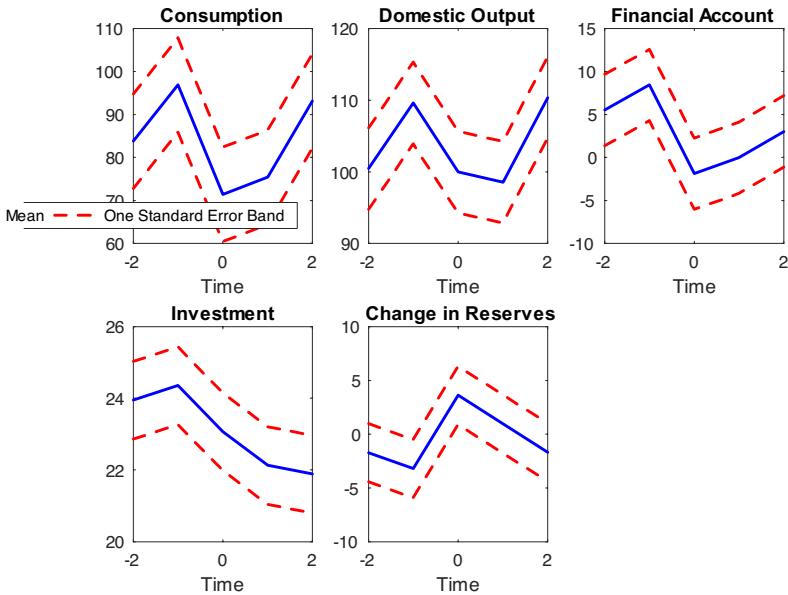


Fig. 7 Average Dynamics of Key Model Variables in Sudden Stops, 1980-2014. Output is normalized to 100 in the year of the sudden stop, i.e., time 0. The blue/solid curve depicts the sample mean and the red/dashed curves around it provide the one-standard-error band. *Source:* Authors' calculations using data from IMF's *International Financial Statistics* and World Bank's *World Development Indicators*

with JR. Output loss, γ , was calibrated at the average difference between the GDP growth rate one period before the crisis and the growth rate in the first year of the capital outflows. We observed in our updated sample almost a 2% decrease in GDP growth rates on average in the first year of capital outflows and a 4.65% decrease when we restrict the sample to countries that suffered an output reduction; however, it shows large variation across countries. JR set this loss to 6.5%, and we use their calibration in order to allow for a more consistent comparison.¹⁴ The risk-free short-term world interest rate, r , the risk aversion parameter, σ , and the price ratio of funds in dollars across states,¹⁵ p , are calibrated as in Jeanne and Rancière (2011) at 5%, 2, and 0.855, respectively.

The role played by the investment rate, s , and the depreciation rate of physical capital, δ , are two additional determinants in extending the optimal reserve formula in JR to a production SOE. We calibrate the investment rate to be equal to 22.6%, which is the sample average of the gross fixed capital formation (% of GDP) in our

¹⁴ JR calculate that output decreases by 4% on average in the first year of sudden stops and by 9% when they only focus attention on subset of the countries in which output fell. Then they take the average of two estimates and set output loss to 6.5%.

¹⁵ Which is based on the calculation of the opportunity cost of reserves in JR.

data from WDI. Following the growth accounting literature, we set the depreciation rate of physical capital to 6% per annum (Caselli 2005; Gourinchas and Jeanne 2013). Our technology parameter, A_N , is calibrated based on a proxy as suggested in Caselli (2005): $A_N = \left(\frac{y}{k^\theta}\right)^{\frac{1}{1-\theta}}$, where y is GDP per worker and k is capital per worker. The average GDP per worker is $\bar{y} = \$33778.65$ for our dataset from WDI and the average capital per worker, \bar{k} , is 87045.54. It is 2.57 times higher than GDP per worker. With the capital share in output taking its standard calibration value of 0.3, e.g., as in Gourinchas and Jeanne (2013), we calculate A_N to be equal to 22514. The average growth rate of the population, g_N , and of the capital stock, g_K , are found to be 1.5% and 6.2% in our updated dataset, respectively.

Based on formula (28), the optimal ratio of reserves to output is quantified at 7.5% in our production SOE model. Notably, this is the mid-point in the range between the analogous ratios in Jeanne and Rancière (2011), of 9.1%, calibrated to the same sample of 34 middle-income countries, and in Bianchi et al. (2018), of 6.0%, obtained in a different, sovereign debt model without capital and production.

3.2 Results by World Regions

Thus far we presented calibration results for the whole of our sample of 34 countries. While they are all emerging market economies, these countries are far from being otherwise very similar, and indeed display a significant degree of heterogeneity, e.g., in Figs. 2–4. This fact has led us to redo our calibrations and calculations specific to each of 4 country groups within our sample, as follows: Latin America, represented by 16 countries and forming roughly half of our sample, as well as Asia (8 countries), Africa (5 countries) and Europe (5 countries). We, finally, calibrated and computed two versions for Africa, with or without Botswana, which is the obvious persistent outlier in terms of maintaining unusually high levels of reserves throughout our sample (see, again, Figs. 2–4).

Figure 8 illustrates the average level of reserves relative to GDP for all our 34 countries (the blue bars) in the sample held during the period 1975–2000. The red line corresponds to 7.5%, i.e., the model-implied optimal level of international reserves to GDP for this group of countries. One could interpret this figure as ‘half-empty vs half-full glass’ of empirical validation of our production SOE model. While we are not too far-off above or below for most countries in our sample, there are Botswana plus 6 other countries, Bulgaria, Czechia, Hungary, Jordan, Malaysia and Thailand, that have maintained a too high average level of reserves in terms of their GDP throughout the 1975–2020 sample period. Yet, Malaysia and Thailand may well have been induced to aim for higher reserves because of the episode of the East Asian financial crisis (1997–1998); whereas the 3 former socialist countries of Eastern Europe have had to build up reserves to maintain economic stability and aim for joining the Euro Area.¹⁶ Botswana has the highest number of sudden stops

¹⁶ In the case of Bulgaria, the introduction of a currency board regime since July 1997 was an additional important incentive to increase quickly and substantially the level of foreign exchange reserves.

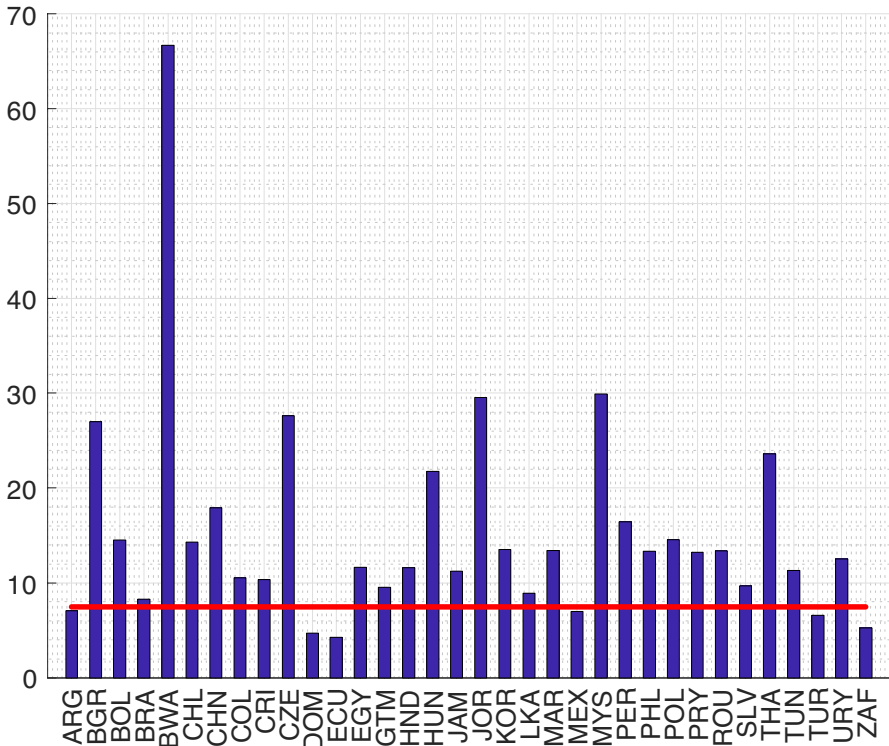


Fig. 8 Average Reserves to GDP in the Data versus Optimal Reserves to GDP in Our Model by Country (in Our Sample), %, 1980-2020. Authors' calculations based on the *World Development Indicator* World Bank Dataset

among the 5 African countries in our sample (see Table 2), and has most likely been increasing the ratio of its reserves-to-GDP over the past decade or two according to the logic of our insurance model here. Finally, Jordan has had the highest number of sudden stops in our sample, 14 years (see, again, Table 2), in addition to experiencing persistently a significant foreign debt; consequently, Jordan seems to have been raising up the level of its foreign exchange reserves gradually to cover it and to surpass this debt level. So, all in all, our results do not appear much surprising, and have a country-specific or regional explanation behind the observed excessive average international reserves held as a ratio to GDP over our sample period, 1975-2020.

Yet, to take account of such heterogeneity, we performed region-specific calibration, under the assumption that heterogeneity will be (much) less across world regions with countries that are more similar geographically, historically and – hence – culturally, as well as in terms of political and economic institutions. Figure 9 highlights our calibration results by regional subsamples.

We interpret this figure in the sense that the insurance SOE model of reserves we extended to production is validated empirically in terms of the average

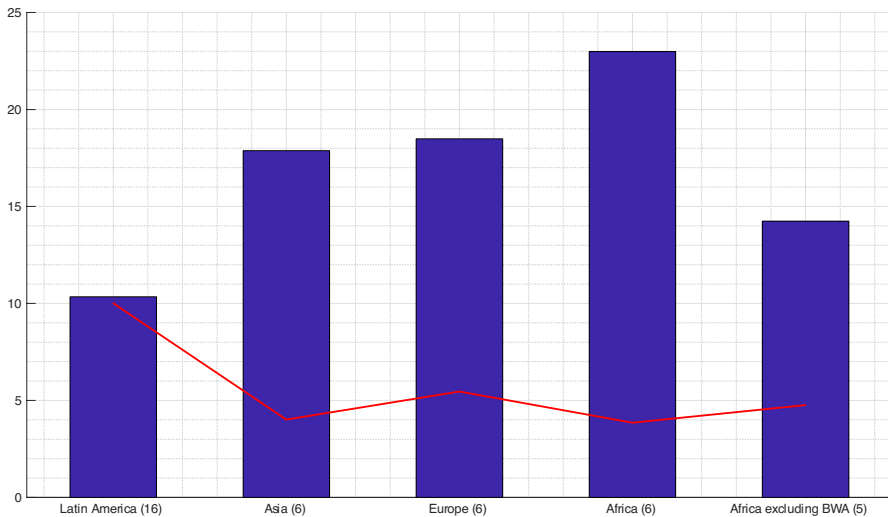


Fig. 9 Average Reserves to GDP in the Data versus Optimal Reserves to GDP in Our Model by World Region (in Our Sample), %, 1980-2020. The blue bars depict data averages by region, while the point value of the red solid line inside each bar corresponds to the quantified outcome of our region-specific calibration. Authors' calculations based on the *World Development Indicator* World Bank Dataset

reserves-to-output ratio in the data for Latin America, at just above 10%. The latter world region has the highest weight in our sample, and represents nearly half of our sample, 16 countries. This is an important finding, new in the literature. Yet, for Asia, Africa and Europe our regional calibration ratios understate considerably the respective data averages, as seen in Fig. 9, which implies that – unless, or to the extent of, indicating reserves hoarding in these regions – alternative modelling approaches should be sought.

Overall, for the sample as a whole and relative to the endowment SOE of Jeanne and Rancière (2011), we explain our lower optimal reserve-to-output ratio, 7.5% vs 9.1%, by the role of capital accumulation as precautionary saving in our extension: the accumulated capital stock can potentially be used as a pledge to external creditors in obtaining borrowing, thereby insuring better a SOE against sudden stops.

4 Concluding Comments

This paper aimed to highlight the role of the neoclassical production factors on the optimal level of international reserve holdings by small open economies facing the risk of sudden stops. To do so, we extended the Jeanne and Rancière (2011) endowment SOE model by adding to it a conventional labour-augmenting Cobb-Douglas production function with constant returns to scale and exogenous population growth, which is consistent with a long-run balanced growth path and the sustained per capita income growth in the data. Our extension to incorporate

investment, capital, labour and production implied a richer analytical version of the optimal reserves formula driven by productivity and the saving rate. We derived novel theoretical results on the role of investment, the capital stock, labour and technology on the optimal ratio of reserves to GDP. We discussed and illustrated the effects of the key determinants of this ratio, focusing on the additional parameters that were revealed by the richer production SOE model.

Under a plausible calibration based on the 1975-2020 period for typical emerging market countries facing the risk of sudden stops in capital inflows, we found that the optimal reserves-to-output ratio is 7.5%. This is the mid-point in the range between the reserve ratio in Jeanne and Rancière (2011), of 9.1%, calibrated to the same sample of 34 middle-income countries, and that in Bianchi et al. (2018), of 6.0%, obtained in a different, sovereign debt model without capital and production. As the countries in our sample appear quite heterogeneous, we also computed the optimal reserves-to-output ratio by region. It turned out that the insurance SOE model of reserves we extended to production is validated empirically in terms of the average reserves-to-output ratio in the data for Latin America, at just above 10%. This world region has the highest weight in our sample, and represents nearly half of it, 16 countries. However, for Asia, Africa and Europe our regional model-based ratios understated considerably the respective data averages, unless we have uncovered a corresponding degree of reserve hoarding.

Thus, we conclude that this type of modelling of reserves is empirically relevant for Latin American economies, but alternative approaches need to be explored for other world regions. We explain the lower optimal reserves-to-output ratio relative to the endowment SOE of Jeanne and Rancière (2011) by the role of capital accumulation as precautionary saving: the accumulated capital stock can potentially be used as a pledge to external creditors in obtaining borrowing; this, in turn, provides a better insurance of a SOE against sudden stops.

One of the drawbacks of our analysis is that the endowment SOE model, which we extended to production, relies on an insurance motive for holding international reserves rather than on the competing mercantilist motive. Alternatively, our model can be refined by introducing explicitly an exchange rate for the SOE and studying its implications for reserves accumulation. Further, instead of the assumption of one single good, the model could be generalized to two goods, with various disaggregation of production structure by sectors, including tradables and nontradables. Finally, the production SOE model of optimal international reserves we derived and quantified by calibration can be subjected to econometric estimation, as in a sequel working paper by Nasir (2020).

Supplementary Information The online version contains supplementary material available at <https://doi.org/10.1007/s11079-022-09678-2>.

Acknowledgements We are grateful to our discussant Anna Lipinska at the 1st Lille-Reading Workshop on International Finance. For constructive feedback, we also thank the anonymous referees, the Editor-in-Chief George Tavlas, Gianluca Benigno, Mark Casson, Miguel Leon-Ledesma, Paul Levine, Kerry Patterson, Cedric Tille, and the audiences at the EEA-ESEM in Geneva, the Applied Economics Meeting

in Sevilla, the Warsaw International Economic Meeting and seminars at several universities. The usual disclaimer applies.

Declarations

Conflicts of interests/Competing interests The authors have no relevant financial or non-financial interests to disclose.

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Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

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