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An Extreme Value Theory Approach to Calculating Minimum Capital Risk Requirements

by

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Abstract

This paper investigates the frequency of extreme events for three LIFFE futures contracts for the calculation of minimum capital risk requirements (MCRRs). We propose a semi-parametric approach where the tails are modelled by the Generalized Pareto Distribution and smaller risks are captured by the empirical distribution function. We compare the capital requirements form this approach with those calculated from the unconditional density and from a conditional density - a GARCH(1,1) model. Our primary finding is that both in-sample and for a hold-out sample, our extreme value approach yields superior results than either of the other two models which do not explicitly model the tails of the return distribution. Since the use of these internal models will be permitted under the EC-CAD II, they could be widely adopted in the near future for determining capital adequacies. Hence, close scrutiny of competing models is required to avoid a potentially costly misallocation capital resources while at the same time ensuring the safety of the financial system.

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JEL Classifications: C14, C15, G13

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1. Introduction

From a regulatory perspective, the notion that financial institutions should hold risk-adjusted capital as a buffer against potential losses was given international credibility[2] in the BIS Basle Accord of 1988 (see Basle Committee on Banking Supervision, 1988), now widely agreed to be a landmark document in the regulation of financial institutions. While the original Accord focused upon credit risk, regulators have since worked on the treatment of market risk. The calculation of a financial institution's Value at Risk (VaR) is rapidly becoming the standardized approach to the determination of appropriate levels of bank capital. In the EU under the Capital Adequacy Directive II[3] for example, the use of internal risk management models (IRMM), of which J.P. Morgan RiskMetrics™ (1996) is the most widely known, is now permitted as long as the institutions can demonstrate that the model, and the operational procedures relating to the model, are "sound". The IRMMs are used to identify the amount of capital required for each (netted) securities position to cover all but a small proportion of potential losses (typically 5.00%). The sum of these positions is the firm's value at risk relating to its trading exposures.

The standard value at risk methodology[4] assumes that the underlying return generating distribution for the security in question is normally distributed, with moments which can be estimated using past data and do not vary over time. The assumption that the underlying return generating process is normal and stationary over time leads to an under-estimation of both the number and size of extreme events. It is commonly accepted that asset return distributions are fat-tailed. Neftci (1998) argues that it is possible and indeed very likely that extreme events are "structurally" different from the return generating process that operates

[2] Although regulators in the USA and in particular the UK had been operating a risk related system of capital regulation before this date.

[3] See Basle Committee on Banking Supervision (1995).

during less extreme market conditions. Under such circumstances - where liquidity in markets dries up and where routine hedging relationships break down, or become more expensive to execute - the underlying statistical assumption of normality becomes entirely inappropriate. We can think of three such events in the recent past: the "Asian crisis" in September 1997, the "Russian debt crisis" of August 1998, and the "Brazilian crisis" of January 1999. While these crises were not unrelated, each of them was to some degree associated with abnormal trading conditions. For example, after the Russian debt crisis, it was reported that liquidity in the corporate bond market had "dried up" against the background of a "flight to quality" where market participants paid premium prices for US Treasuries and UK gilts.

In this paper we calculate Minimum Capital Risk Requirements (MCRRs) for three of the London International Financial Futures Exchange's (LIFFE) most popular derivatives contracts. We use an unconditional model, a GARCH(1,1) model and a combination of a Generalized Pareto Distribution and the empirical distribution of the returns. Our main finding is that both back-tests and out-of-sample tests of the calculated MCRRs show that the proportion of exceedences produced by the extreme value approach, which concentrates on the tails, are considerably closer to the nominal probability of violations than competing approaches which fit a single model for the whole distribution. The rest of this paper is organized as follows: in Section 2 we present the data sets; in Section 3 we present the extreme value theory; in Section 4 we consider alternative models of conditional volatility; we outline our basic methodology for calculating MCRRs in Section 5; in Section 6 we present our results; and we conclude the paper in Section 7 with suggestions for future research.

[4] For a critical appraisal see Danielsson and DeVries (1997), or Neftci 1998.

2. Data

In this study we calculate MCRRs for three LIFFE futures contracts - the FTSE-100 Index Futures Contract, the Long Gilt Futures Contract and the Short Sterling Interest Rate Futures Contract - based upon their daily settlement prices[5]. The Long Gilt contract trades a notional 10-15 year gilt with a yield to maturity of 7%. The Short Sterling contract is based on a 3-month time deposit with a face value of £500,000. Thus the buyer of such a contract is allocated this amount as a time deposit in an eligible bank on the delivery date, although ut may instead be cash settled at the option of the buyer. Note therefore that the "Long" and "Short" terminology used in the contract titles therefore refers to the contract maturities and not to a long or short position. The data was collected from Datastream International, and spans the period 24/05/1991 to 16/09/1996. Sample observations corresponding to UK public holidays (i.e., when LIFFE was closed) were deleted from the data set to avoid the incorporation of spurious zero returns, leaving 1344 observations, or trading days in the sample. In the empirical work below, we use the daily log return of the original price series.

It is evident from Table 1 that all three returns series show strong evidence of skewness – the FTSE-100 and Short Sterling contract returns are skewed to the right while the returns on the Long Gilt contract are skewed to the left. They are also highly leptokurtic (i.e. fat-tailed). In particular, the Short Sterling series has a coefficient of excess kurtosis of nearly 200. The Jarque-Bera test statistic consequently rejects normality for all three derivative return series. The extreme fat-tailed nature of the three series provides a strong motivation for the estimation methodologies employed in this paper that focus on the tails.

[insert table 1 here]

[5] Because these contracts expire 4 times per year - March, June, September and December - to obtain a continuous time series we use the closest to maturity contract unless the next closest has greater volume, in which case we switch to this contract.

3. Extreme Value Theory

Assuming that $x_1, x_2, \ldots, x_n$ are the realized returns of some data generating process X[6] observed on days $1,2,\ldots,n$, then let Y_n denote the highest daily returns (the maximum)[7] found below a certain level of x. In practice, the distribution of the "parent variable" (X) is not accurately known, therefore the exact distribution of the extremes is also unknown. Thus, most studies focus upon the asymptotic behaviour of the extremes. Extreme value theory is the study of the limiting distribution of the order statistic Y_n,

$$P\{\alpha_n(Y_n - \beta_n) \le x\} \xrightarrow{w} F_Y(y) \qquad [1]$$

where, β_n is the location parameter and α_n (assumed to be positive) is the location parameter. w stands for weak convergence and $F_Y(y)$ is one of the three asymptotic distributions as defined below. If the above equation holds, then it can be said that the distribution function of $x_1, x_2, \ldots, x_n$ belongs to the domain of attraction of $F_Y(y)$. The three distributions, given below, have been justified as the limiting stable distributions of extreme value theory.

The Gumbel distribution (type 1):

$$F_Y(y) = exp\left(-e^{-y}\right) \qquad \text{for } y \in \Re \qquad [2]$$

The Fréchet distribution (type 2):

$$F_Y(y) = \begin{cases} 0 & for\ y \le 0 \\ exp\left(-y^k\right) & for\ y > 0\ (k > 0) \end{cases} \qquad [3]$$

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6 X represents the log price changes.

7 The example given concentrates on the maximum values only. However, an application to minimum values would follow a comparable derivation.

The Weibull distribution (type 3):

$$F_Y(y)=\begin{cases} exp\left(-(-y)^{-k}\right) & for\ \ y<0\ (k<0) \\ 0 & for\ \ y\geq 0 \end{cases} \qquad [4]$$

The shape parameter k reflects the weight of the tail in the distribution of the parent variable X. The lower is k, the fatter is the distribution of X. It also gives the number of finite moments of the distributions, for example, when k is greater than unity the mean of the distribution exists, whereas when it is greater than two the variance is finite and so on. However, k as well as β_n and α_n (known as the "normalizing coefficients") may be different for minima and maxima (see Longin, (1996)).

The tail of the distribution of F_X is either declining exponentially (type 1) or by a power (type 2) or is finite (type 3). According to Gnedenko (1943): the Gumbel distribution can be the limit of bounded and unbounded distributions; only distributions unbounded (to the right) can have a Fréchet distribution as the limit; and only distributions with a finite right end point can have the Weibull distribution as its limit.

The above three distributions can be grouped together by a generalized formula (see Jenkinson, 1955):

$$F_Y(y)=exp\left[-(1-\tau.y)^{1/\tau}\right]\begin{cases} for\ \ y>\tau^{-1} & if\ \ \tau<0 \\ for\ \ y<\tau^{-1} & if\ \ \tau>0 \end{cases} \qquad [5]$$

The tail index, τ, is related to the shape parameter k by $\tau=-1/k$. Thus, the tail index determines the type of distribution. $\tau=0$ corresponds to a Gumbel distribution whereas $\tau<0$

corresponds to a Fréchet distribution and $\tau > 0$ to a Weibull distribution. However, it should be noted that for small values of τ, i.e., large values of k, the Fréchet and Weibull distributions are very close to the Gumbel distribution.

Other fat-tailed distributions, for example, the Student-t and the Pareto distributions among others can be linked to the three extreme value distributions above. Gnedenko (1943) has given necessary and sufficient conditions for a particular distribution to belong to one of the three distributions whereby these conditions can be employed in specific cases to derive the type of asymptotic distribution of extremes. As such, the normal distribution can be seen to lead to the Gumbel distribution; the Student-t obeys the Fréchet distribution with a shape parameter k equal to its degrees of freedom; the stable Paretian law, introduced by Mandelbrot (1963), leads to the Fréchet distribution with a shape parameter k equal to its characteristic exponent.

The distribution adopted in this paper is the generalized Pareto distribution given by the following equation:

$$G(y;\sigma,k) = 1 - \left(1 - k\,\frac{y}{\sigma}\right)^{1/k} \qquad [6]$$

where, k is arbitrary, with the range of y being $0 < y < \infty$ if $k \leq 0$ and $0 < y < \sigma/k$ if $k > 0$. This equation is elaborated below and its interpretation as a limiting distribution is similar to that which motivates equation [5], and thus the idea behind the generalized Pareto distribution is fairly similar to that of the extreme value distributions, collected together in the generalized formula of [5]. Thus the generalized Pareto distribution is employed in this paper forr its intuitive appeal and since it effectively encompasses the three limiting distributions of extreme value theory as special cases.

Let $F(\Delta x_t)$ denote the unknown distribution function of the incremental changes in the log of financial futures prices, the asymptotic theory of extremes is used in approximating the tail areas of $F(\Delta x_t)$. This approach follows Pickands (1975), Smith (1987), Davison and Smith (1990), Embrechts *et al*. (1997) and Neftci (1998).

Closely following Smith (1987) and Neftci (1998), we derive the Generalized Pareto Distribution below. Let *U* and *L* represent the two thresholds of the tails, with *U* representing the 'Upper' threshold and *L* representing the 'Lower' one, such that $\Delta x_t > U > 0$ and $\Delta x_t < D < 0$ lie in the two tails of the distribution $F(\Delta x_t)$. The example derived below is for the upper tail only, however, the replication for the lower tail is similar. The following probability distribution of the random variable Δx_t can be defined as:

$$P(\Delta x_t \leq U) = F(U) \qquad [7]$$

where, $U < x_0$, and $P(\Delta x_t < x_0) = 1$, i.e., Δx_t is bounded by x_0.

Now assuming that e_t, with $e_t \in R^+$, is the exceedance of the threshold *U* at time *t*, then

$$P(\Delta x_t \leq U + e_t) = F(U + e_t) \qquad [8]$$

where, $0 < e_t < x_0 - U$.

$F_U(e_t)$ is given by

$$F_U(e_t) = \frac{F(U + e_t) - F(U)}{1 - F(U)} \qquad [9]$$

with $F_U(e_t)$ representing the conditional distribution of $(\Delta x_t - U)$ given that $\Delta x_t > U$.

Following Pickands (1975), $F_U(e_t)$ can be approximated by the generalized Pareto distribution $G(e_t;\sigma^u,k)$ with

$$G(e_t;\sigma^u,k)=\begin{cases}1-\left(1-\dfrac{ke_t}{\sigma^u}\right)^{1/k} & k\neq 0,\sigma^u>0\\ 1-e^{-e_t/\sigma^u} & k=0,\sigma^u>0\end{cases} \qquad [10]$$

where k is arbitrary, with the range of e_t being $0<e_t<\infty$ if $k\leq 0$ and $0<e_t<\sigma^u/k$ if $k>0$. The case of $k=0$ is interpreted as the limit $k\to 0$, i.e. the exponential distribution with mean σ^u.

Pickands showed that the above equation arises as a limiting distribution for excesses over thresholds if and only if the parent distribution is in the domain of attraction of one of the extreme value distributions. The motivation for the equation is the 'threshold stability' property, i.e., if e_t is generalized Pareto and $U>0$, then the conditional distribution of e_t-U (given $e_t>U$) is also generalized Pareto. Another property is as follows: if n (the number of exceedances) has a Poisson distribution and, conditioning on n, $e_1,\dots,e_n$ are *iid* generalized Pareto random variables, then $max(e_1,\dots,e_n)$ also has a generalized extreme value distribution (see Davison and Smith, 1990, pp. 395).

Going back to Equations [9] and [10], the distance between $G(e_t;\sigma^u,k)$ and $F_U(e_t)$ will converge to zero as $U\to x_0$, i.e. the further we go into the tails:

$$\lim_{U\to x_0}\ \sup_{0<e_t<x_0}\left|F_U(e_t)-G(e_t;\sigma^u,k)\right|=0 \qquad [11]$$

However, further conditions for $F_U(e_t)$ must be satisfied for the above equation to hold, see Pickands (1975) for more details. Moreover, the $G(.)$ is expected to be a 'good' approximation of the $F_U(.)$ as long as the threshold level is high enough. However, an important question would be: 'how high to fix this threshold?' This topic is elaborated in the final part of this section.

The parameters to be estimated from the generalized Pareto distribution are σ^u and k. Methods for estimating the generalized Pareto distribution parameters have been reviewed by Hosking and Wallis (1987). Whereas maximum likelihood estimators exist in large samples provided that $k < 1$, they are asymptotically normal and efficient when $k < 1/2$ (Smith, 1985). Using the same approach as Neftci (1998), the parameters σ^u and k are obtained by maximizing the log likelihood function of $G(e_t ; \sigma^u, k)$.

Assuming that U is high enough so that the generalized Pareto distribution $G(e_t ; \sigma^u, k)$ with $k \neq 0$ is a good approximation for the probability $F_U(e_t)$, then:

$$P(\Delta x_t < e_{t_i}) \cong 1 - \left(1 - \frac{k e_{t_i}}{\sigma^u}\right)^{1/k} \qquad [12]$$

The above equation holds for $k \leq 0$. In the case that $k > 0$, the condition $e_t < \sigma^u k$ must be satisfied for the density to be well defined.

Following the expression [12], the density function of Δx_t can be approximated at an arbitrary observation point e_{t_i}, by the density $G(e_t)$:

$$G\left(e_t\,;\sigma^u,k\right)=\left(\frac{\sigma^u-ke_t}{\sigma^u}\right)^{1/k}\left(\sigma^u-ke_t\right)^{-1} \qquad [13]$$

Finally, by using the density of $G\left(e_t\,;\sigma^u,k\right)$ at each observation point, e_{t_i}, the following log likelihood function is obtained

$$\ell\left(k,\sigma^u\right)=-n\,ln\left(\sigma^u\right)+\sum_{i=1}^{n}\left[ln\left(1-\frac{ke_{t_i}}{\sigma^u}\right)k^{-1}-ln\left(1-\frac{ke_{t_i}}{\sigma^u}\right)\right] \qquad [14]$$

where, n is the number of exceedances in a sample of N observations. In this case, the sample of extremes (n) is obtained by first estimating the standard deviation of the whole sample of the returns and secondly, by selecting all positive and negative increments greater than 1.645 times the standard deviation of the sample in absolute terms to represent the extremes (n).

The results for the estimation of n, σ (the normalizing coefficient) and k (the coefficient determining the fatness of the tail) are given in Table 2(i).

[table 2 here]

The number of extremes (n) for the upper tail is higher than those of the lower tail, except for the Long Gilt contract whereby the number of extremes is 44 in the lower tail compared to 29 in the upper tail. As expected, σ^u is positive for all three contracts, highest for the FTSE-100 index contract, followed by the Long Gilt and then the Short Sterling contracts. The result is quite similar for the lower tail: σ^L is positive for all the contracts, highest for the FTSE-100 index contract, followed by the Short Sterling and then the Long Gilt contracts. Whereas the parameter k is positive in the lower tail for all three contracts (the highest being for the Long

Gilt contract, followed by the Short Sterling and FTSE-100 Index contracts), it is negative for the FTSE-100 Index and Long Gilt contracts in the upper tail.

The next step is to estimate the threshold, T , since it is important to know where the tail starts for the calculation of the MCRRs. Following the definition of U and L,

$$T >> max\left[|U|,|L|\right] \qquad [15]$$

Using the approximation given in expression [9],

$$\frac{F\left(U+e_{t_i}\right)-F\left(U\right)}{1-F\left(U\right)} \cong G\left(e_{t_i}\right) \qquad [16]$$

the following term is obtained by cross-multiplying:

$$1-F\left(U+e_{t_i}\right) \cong 1-\left[F\left(U\right)+G\left(e_{t_i}\right)-G\left(e_{t_i}\right)F\left(U\right)\right] \qquad [17]$$

$F\left(U\right)$ is unknown but since it is the unconditional probability that an observation will exceed the level U , a possible estimate is obtained by using the sample frequency, i.e.,

$$\hat{F}\left(U\right)=\frac{n}{N} \qquad [18]$$

Following Neftci (1998), the estimate of the tail probability is

$$1-F\left(U+e_{t_i}\right) \cong \frac{n}{N}\left(1-\frac{\hat{k}e_{t_i}}{\hat{\sigma}^u}\right)^{1/\hat{k}} \qquad [19]$$

where, $\hat{\sigma}^u$ and $\hat{k}$ are the maximum likelihood estimates of σ^u and k respectively. Denoting this tail probability estimate by α :

$$\alpha=\frac{n}{N}\left(1-\frac{\hat{k}T}{\hat{\sigma}^u}\right)^{1/\hat{k}} \qquad [20]$$

Thus, rearranging [20] we obtain the threshold:

$$T^{\alpha} = \frac{\hat{\sigma}}{\hat{k}}\left[1-\left(\frac{\alpha N}{n}\right)^{\hat{k}}\right] \qquad [21]$$

Again the result for T^{α} (for both the upper and lower tails) is presented in Table 2(ii), with α is set at 0.01 in this paper. For the upper tail, the threshold (i.e. the start of the tail) is set at 0.017 for the FTSE-100 Index contract, at 0.010 for the Long Gilt contract and at 0.003 for the Short Sterling contract. Thus, the threshold is further in the tail for the FTSE-100 Index, followed by the Long Gilt and the Short Sterling contracts. The same result is obtained for the lower tail, with the threshold being 0.018 for the FTSE-100 Index contract, 0.010 for the Long Gilt contract and 0.002 for the Short Sterling contract. The threshold is higher in the lower tail for the FTSE-100 Index contract compared to the upper tail. On the other hand, the threshold is higher in the upper tail for the Short Sterling contract compared with its lower tail.

4. GARCH modelling

In order to provide a benchmark for the evaluation of the results from the extreme value estimation we also calculate MCRRs using a GARCH model. The simple GARCH (1,1) model is given below:

$$x_t = \mu + \omega_t$$

$$h_t = \gamma + \alpha\varepsilon_{t-1}^{2} + \beta h_{t-1} \qquad [22]$$

where, $x_t = Log(P_t / P_{t-1})$, $\omega_t = h_t^{1/2}\varepsilon_t$, $\varepsilon_t \sim$ N(0,1).

Following Brooks *et al.*, (2000), the "best" model of conditional volatility from a large set of candidate models was shown to be the GARCH(1,1) model for all three contracts.

For the purposes of comparison, the probability of an extreme as predicted by the simple GARCH(1,1) model, Extreme Value model and the empirical distribution is estimated. Table 3 shows the probability of the five highest and lowest returns of the three financial futures contracts as predicted by the extreme value procedure, the GARCH(1,1) model together with the values that are predicted by the original empirical distribution function[8].

[table 3 here]

For the GARCH(1,1) model, the conditional volatility is predicted and the probability of an outcome equal to or more extreme than the observed return (conditional on the predicted volatility for each observation) is recorded. In the case of the extreme value procedure, returns are estimated by bootstrapping from the Pareto distribution and the interior of the empirical distribution for common observations. This estimation technique is elaborated in the following section.

As noted, the probability as predicted by the extreme value procedure, and the values that are predicted by the empirical distribution are very similar. On the other hand, it can be seen that the GARCH(1,1) model performs poorly in modelling the tail events compared with the extreme value approach.

5. A methodology for estimating MCRRs

Capital risk requirements are estimated for 1 day, 1 week, 1 month and 3 month investment horizons by simulating the conditional densities of price changes, using Efron's (1982) bootstrapping methodology. For the Generalized Pareto Distribution model, simulation is carried out by bootstrapping from both the fitted tails and the empirical distribution function.

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[8] The distribution function of the log price changes of the contracts.

For the GARCH model, since the standardized residuals ($\hat{\varepsilon}_t / \hat{h}_{t-1}^{1/2}$) from these models are *iid* (according to the BDS test - see Brooks *et al*., 2000) the ω_t are drawn randomly, with replacement, from the standardized residuals and a path of future x_t's can be generated, using the estimates of α, β, γ and μ from the sample and multi-step ahead forecasts of h_t.

In the case of the Generalized Pareto Distribution, the path for future prices is simulated as follows: (1) draw x_t from the empirical distribution with replacement, (2) if $x_t < T(L)$, then draw from the generalized Pareto distribution fitted to the lower tail, (3) however, if $x_t > T(U)$, then draw from the generalized Pareto distribution fitted to the upper tail, (4) on the other hand, if x_t falls in the middle of the empirical distribution, i.e. $T(L) < x_t < T(U)$, then x_t is retained. The number of draws of x_t is equal to the length of the investment horizon. This procedure can be considered as a type of structured Monte Carlo study, where we pay particular attention to the extreme returns in the tails of the distribution. It will be these extreme returns which most strongly influence the value of the MCRR, and hence most influence the likelihood of financial distress.

In practice a securities firm undertaking this procedure would have to simulate the price of the contract when it initially opened the position. To calculate the appropriate capital risk requirement, it would then have to estimate the maximum loss that the position might experience over the proposed holding period[9]. For example, by tracking the daily value of a long futures position and recording its lowest value over the sample period, the firm can report its maximum loss per contract for this particular simulated path of futures prices. Repeating

[9] The current BIS rules state that the MCRR should be the higher of the (i) average MCRR over the previous 60 days or (ii) the previous trading days' MCRR. At the time of writing, it is not clear how CAD II will require the exact calculation to be made.

this procedure for 20,000 simulated paths generates an empirical distribution of the maximum loss. This maximum loss (Q) is given by:

$$Q = (x_0 - x_1) \times Contracts \qquad [23]$$

where x_0 is the price at which the contract is initially bought or sold; and x_1 is the lowest simulated price (for a long position) or the highest simulated price (for a short position) over the holding period. Assuming (without loss of generality) that the number of contracts held is 1, we can write the following:

$$\frac{Q}{x_0} = \left(1 - \frac{x_1}{x_0}\right) \qquad [24]$$

In this case, since x_0 is a constant, the distribution of Q will depend on the distribution of x_1. Hsieh (1993) assumed that prices are lognormally distributed, i.e. that the log of the ratios of the prices, $Ln\left(\frac{x_1}{x_0}\right)$, are normally distributed. However, in this paper, we do not impose this restriction, but instead $Ln\left(\frac{x_1}{x_0}\right)$ is transformed into a standard normal distribution by matching the moments of $Ln\left(\frac{x_1}{x_0}\right)$'s distribution to one of a set of possible distributions known as the Johnson (1949) distribution. Matching moments to the family of Johnson distributions (Normal, Lognormal, Bounded and Unbounded) requires a specification of the transformation from the $Ln\left(\frac{x_1}{x_0}\right)$ distribution to a distribution that has a standard normal distribution. In this case, matching moments means finding a distribution, whose first four moments are known, i.e. one that has the same mean, standard deviation, skewness and kurtosis as the $Ln\left(\frac{x_1}{x_0}\right)$ distribution.

For all the samples of the three contracts, the $Ln\left(\frac{x_1}{x_0}\right)$ distributions were found to match the Unbounded distribution. Therefore, the estimated 5th quantile of the $Ln\left(\frac{x_1}{x_0}\right)$'s distribution is based on the following transformation:

$$Ln\left(\frac{x_1}{x_0}\right)_{1,t} = \sinh\left(\frac{(\pm 1.645 - a)}{b}\right) \times d + c \qquad [25]$$

a, b, c and *d* are parameters whose values are determined by the $Ln\left(\frac{x_1}{x_0}\right)$'s first 4 moments.

From expression 7, it can be seen that the distribution of $\frac{Q}{x_0}$ will depend on the distribution of $\frac{x_1}{x_0}$. Hence, the first step is to find the 5th Quantile of $Ln\left(\frac{x_1}{x_0}\right)$:

$$\frac{Ln\left(\frac{x_1}{x_0}\right) - m}{Sd} = \pm\alpha \qquad [26]$$

where α is the 5th Quantile from the Johnson Distribution, m is the Mean of $Ln\left(\frac{x_1}{x_0}\right)$ and *Sd* is the Standard Deviation of $Ln\left(\frac{x_1}{x_0}\right)$. Cross-multiplying and taking the exponential,

$$\frac{x_1}{x_0} = Exponential\left[(\pm\alpha \times Sd) + m\right] \qquad [27]$$

therefore

$$\therefore \frac{Q}{x_0} = 1 - Exponential\left[(\pm\alpha \times Sd) + m\right] \qquad [28]$$

We also use the unconditional density to calculate MCRRs so that we can make a direct comparison between this and the two other approaches since this much simpler approach ignores both the non-linear dependence in the conditional volatility (which would be captured by the GARCH formulation) and the fat tails of the returns series (which would be accounted for using the extreme value approach). To use the unconditional density, the x_t's are drawn randomly, with replacement, from the in-sample returns.

Confidence intervals for the MCRRs are estimated using the jackknife-after-bootstrap methodology (Efron & Tibshirani, 1993). These confidence intervals are estimated to give an idea of the likely sampling variation in the MCRR point estimates and help determine whether the differences in the MCRRs for the conditional and unconditional models are significantly different.

Assuming that, $Ln\left(\frac{x_1}{x_0}\right)_{(5\%)}$ ~ N(m^*,Sd^*) then, the confidence interval for the $Ln\left(\frac{x_1}{x_0}\right)_{(5\%)}$ is

$$\left\{Ln\left(\frac{x_1}{x_0}\right)_{(5\%)} \pm 1.960 * SE\left[Ln\left(\frac{x_1}{x_0}\right)_{(5\%)}\right]\right\}$$

Therefore, the confidence interval of $\left(\frac{x_1}{x_0}\right)_{(5\%)}$ is $Exp\left\{Ln\left(\frac{x_1}{x_0}\right)_{(5\%)} \pm 1.960 * SE\left[Ln\left(\frac{x_1}{x_0}\right)_{(5\%)}\right]\right\}$

and the confidence interval of $\left(\frac{Q}{x_0}\right)_{(5\%)}$ is given by

$$1 - Exp\left\{Ln\left(\frac{x_1}{x_0}\right)_{(5\%)} \pm 1.960 * SE\left[Ln\left(\frac{x_1}{x_0}\right)_{(5\%)}\right]\right\}$$

The jackknife-after-bootstrap provides a method of estimating the variance of the 5th quantile of $ln(x_1/x_0)$ using only information in the 20,000 bootstrap samples.

To verify the accuracy of this methodology, we compared the actual daily profits and losses of the three futures contracts with their daily MCRR forecasts. In this case, instead of expression (6) we will work with the following:

$$Q = (x_t - x_{t+1}) \times Contracts \quad [29]$$

where x_t is the price of the contract at time t and x_{t+1} is the simulated price at time t+1. This will give us a time series of daily MCRR forecasts. Our measure of model performance is a count of the number of times the MCRR "underpredicts" realized losses over the sample period. This procedure is effectively a back-test of the model's adequacy over the in-sample estimation period.

However, for a fuller evaluation of the results we need to perform an out-of-sample test of the MCRRs based upon the three models, to determine whether the models are likely to be useful in the practical situation where we are determining the capital requirement to cover a period in the future when the parameters of the models are estimated using past data. We therefore calculated MCRRs for a 1 day investment horizon for each contract and for both short and long positions on day t and then checked to see whether this MCRR had been exceeded by price movements in day t+1. We rolled this process forward, recalculating the MCRRs etc., for 500 days, i.e. using the sample period 17th September 1996 to 12thAugust 1998. Out-of-sample tests are not commonly applied in this literature, but are an essential part of the model evaluation process, since it is likely that back-tests will over-state the success of all models, since the data used to assess the adequacy of the MCRR calculations, has also been used to determine the parameters of the models. Moreover, back-tests are likely to be biased towards profligate models which fit to sample-specific features of the data, but are unable to generalize in a genuine out-of-sample forecasting environment.

6. The MCRRs

The MCRRs for the three contracts based upon the unconditional density, the GARCH(1,1) and EVT models are presented in Table 4.

[table 4 here]

Close inspection of the results reveals that the MCRRs are always higher for short compared with long futures positions, particularly as the investment horizon is increased. This is because the distribution of log-price changes is not symmetric: there is a larger probability of a price rise in all three futures contracts than a price fall over the sample period (i.e., the mean returns in Table 1 are all positive), indicating that there is a greater probability that a loss will be sustained on a short relative to a long position. For example, the MCRR for a long Short Sterling position, calculated using the GARCH(1,1) model and held for three months is 3.627%, but is 5.798% for a short position.

The MCRRs based upon the GARCH(1,1) model are always higher than for the unconditional density method of calculation. This result highlights the excess volatility persistence implied in the GARCH(1,1) model (see Hsieh, 1993, for a discussion of this issue). A higher degree of persistence implies that a large innovation in contract returns (of either sign) causes volatility to remain high for a relatively long period, and therefore the amount of capital required to cover this protracted period of higher implied volatility is also higher. The effect of this volatility persistence is considerable – with MCRRs increasing by a factor of two or three in most cases, compared with those generated from the unconditional density. For example, the MCRR GARCH(1,1) estimate for a Short Sterling contract position is 3.627% for a three month investment horizon, whereas the comparable figure for the unconditional density is 1.643%. For the extreme value theory approach, the MCRRs tend to be smaller than the GARCH(1,1) model but greater than the Unconditional Density for the FTSE-100 Index and the Short Sterling contracts, however those for the Long Gilt are smaller than both the conditional and unconditional volatility models. Moreover, capital requirements are highest for the contract which is most volatile, i.e. the FTSE-100 stock index futures contract, while the Short Sterling contract is least volatile of the three and therefore requires less of a capital charge. This holds true for all three alternative methods of estimation.

Approximate 95% confidence intervals for the MCRRs calculated from the unconditional density, the GARCH model and the EVT approach are presented in Table 5.

[table 5 here]

The most important feature of these results is the "tightness" of the intervals around the MCRR point estimates. For example, the 95% confidence interval around the MCRR point estimates of 12.028% for a Long Gilt contract position of three months is 11.787% to 12.509%. Also, in the cases of all three contracts the confidence intervals for the conditional GARCH and unconditional density models as well as the extreme value theory approach never overlap. This indicates that there is a highly statistically significant difference between the MCRRs generated using the conditional GARCH, the EVT and unconditional density.

Table 6 presents the proportion of times that the MCRR is violated during the estimation sample.

[table 6 here]

The back-testing results show that the realized percentages of MCRR violations (for both long and short positions) are in general lower than the nominal 5% coverage. The same holds true for the other two models. Thus, although all the models give rather different sets of MCRRs, the out-of-sample tests show that they are all adequate for the estimation of minimum capital requirements, i.e., the realized percentages of MCRR violations is 5% or less than 5% (with the exception of the EVT model for a long position in the FTSE contract, which is 5.1%). However, if the proportion of exceedences is considerably less than 5%, this implies that the capital charge has been set too high and thus bank capital is tied up in an unnecessary and unprofitable way. In this regard, the extreme value model yields the best results overall, since the proportion of exceedences is much closer to the nominal 5% level while for the others the number of exceedences is too few. In general, the extreme value-generated MCRRs have a proportion of violations which is up to a percentage point higher than those from the unconditional and GARCH models. The most noticeable improvement is for a long position in the Short Sterling contract, where EVT gives 4.9% of exceedences, compared with 4.46 and 4.24 for the unconditional and GARCH models respectively.

The out-of-sample testing results, shown in table 7, are also highly supportive to the extreme value approach compared with its competitors.

[table 7 here]

For example, considering a long position in the Long Gilt contract, the proportion of violations is 2.8% for the unconditional density, and 3.4% for the GARCH model, while it is 4.4% for the MCRR generated using extreme value theory. The superior performance of the extreme value approach indicates that a securities firm who adopted this methodology, could cut is capital requirement by up to one third while still retaining a number of violations which is within acceptable limits.

6. Conclusions

Under CAD II European banks and investment firms will be able to calculate appropriate levels of capital for their trading books using IRMMs[10]. It is expected that these models will be in widespread usage, particularly in London, soon after the necessary legislation has been passed. Given this development in the international regulatory environment, in this paper we investigated certain aspects of this technology by calculating MCRRs for three of the most popular derivatives contracts currently trading on LIFFE.

Our results demonstrate the usefulness of the extreme value approach in providing a superior fit to the data and giving improved back-testing and out-of-sample results. Further research in this area might consider the application of such techniques to other data series or the consideration of alternative fat-tailed distributions. Since the use of these internal models will be permitted under the EC-CAD II, they could be widely adopted in the near future for determining capital adequacies. Hence, close scrutiny of competing models is required to avoid wastage of capital resources whilst at the same time ensuring the safety of the financial system.

[10] This proposal is due to be adopted by the EU's Council of Ministers and the European Parliament under the co-decision procedure.

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Appendix: Tabulated Results

Table 1
Summary Statistics of Derivative Returns

Futures Contracts	FTSE-100	Long Gilt	Short Sterling
Mean	0.00034	0.00013	0.00004
Variance	8.283E-005	2.654E-005	1.680E-006
Skewness	0.29556*	-0.09153*	8.55407*
Kurtosis	2.73215*	3.43428*	199.165*
Normality Test Statistic†	484.2252*	639.9767*	2223267*

Notes: * represents significance at the 5% level (2 tailed-test); † Bera and Jarque test

Table 2

No. of Extremes, Parameters of the Generalized Pareto Distribution & the Threshold Level:

Upper Tail

	FTSE-100 Index	Long Gilt	Short Sterling
n	28	29	19
a	0.02246	0.01243	0.00667
k	-0.02521	-0.12329	0.15124
Threshold (U)	0.01664	0.01003	0.00325

Lower Tail

	FTSE-100 Index	Long Gilt	Short Sterling
n	19	44	15
a	0.05232	0.01324	0.01773
k	0.03680	0.86250	0.54101
Threshold (L)	0.01800	0.00983	0.00189

Table 3

Probability of an Extreme as predicted by the simple GARCH(1,1) model, Extreme Value Model and the Empirical Distribution.

FTSE-100 Index contract

Returns		*Probabilities*	
	GARCH(1,1)	EVT	Empirical
-0.04569	0.00000	0.00070	0.00074
-0.03862	0.00000	0.00075	0.00074
-0.02795	0.00052	0.00165	0.00149
-0.02568	0.00075	0.00170	0.00149
-0.02449	0.00105	0.00220	0.00223
0.053872	0.00000	0.00070	0.00074
0.049636	0.00000	0.00150	0.00149
0.038794	0.00000	0.00166	0.00149
0.035462	0.00020	0.00170	0.00149
0.028351	0.00036	0.00229	0.00223

Long Gilt contract

Returns		*Probabilities*	
	GARCH(1,1)	EVT	Empirical
-0.02284	0.00000	0.00077	0.00074
-0.02123	0.00030	0.00075	0.00074
-0.01941	0.00045	0.00180	0.00149
-0.01873	0.00090	0.00187	0.00149
-0.01860	0.00105	0.00222	0.00149
0.036544	0.00000	0.00079	0.00074
0.019327	0.00015	0.00085	0.00074
0.018795	0.00035	0.00095	0.00074
0.017054	0.00060	0.00157	0.00149
0.016885	0.00086	0.00239	0.00223

Short Sterling contract

Returns		*Probabilities*	
	GARCH(1,1)	EVT	Empirical
-0.00901	0.00000	0.00065	0.00074
-0.00809	0.00000	0.00065	0.00074

-0.00715	0.00003	0.00155	0.00149
-0.00660	0.00005	0.00160	0.00149
-0.00562	0.00025	0.00322	0.00149
0.029236	0.00000	0.00085	0.00074
0.008044	0.00001	0.00090	0.00074
0.007369	0.00006	0.00156	0.00149
0.006933	0.00021	0.00170	0.00149
0.006821	0.00040	0.00249	0.00149

Table 4

Capital Requirement for 95% Coverage Probability as a Percentage of the Initial Value for unconditional density and based on GARCH(1,1) model, and the extreme value theory approach

Horizon	Long Positions			Short Positions		
	Uncond.	GARCH(1,1)	EVT	Uncond.	GARCH(1,1)	EVT
			FTSE-100 Index			
3 months	12.775	25.498	20.391	21.102	32.540	30.820
1 month	7.954	10.417	13.369	10.782	14.567	19.763
1 week	3.272	6.031	5.600	3.845	7.905	5.998
1 day	1.392	4.275	2.340	1.419	5.570	3.161
			Long Gilt			
3 months	7.906	12.028	4.954	10.906	14.070	5.489
1 month	4.855	7.305	3.672	5.623	9.833	4.010
1 week	2.007	4.653	2.506	2.090	5.378	3.005
1 day	0.849	2.932	1.152	0.898	3.276	1.413
			Short Sterling			
3 months	1.643	3.627	2.810	3.061	5.798	4.320
1 month	0.986	2.377	2.001	1.237	4.008	3.010
1 week	0.348	1.423	1.555	0.382	2.799	2.004
1 day	0.127	0.903	0.753	0.130	1.437	0.975

Table 5
Approximate 95% Central Confidence Intervals for the MCRRs given in Table 4.

Horizon	Long Positions			Short Positions		
	Uncond.	GARCH(1,1)	EVT	Uncond.	GARCH(1,1)	EVT
			FTSE-100 Index			
3 months	[12.516, 13.105]	[24.988, 26.517]	[19.575, 20.799]	[20.822, 21.442]	[31.889, 33.842]	[29.587, 31.436]
1 month	[7.815, 8.124]	[10.209, 10.834]	[12.834, 13.636]	[10.581,11.003]	[14.276, 15.149]	[18.972, 20.158]
1 week	[3.181, 3.393]	[5.910, 6.272]	[5.376, 5.712]	[3.759, 3.921]	[7.747, 8.221]	[5.758, 6.118]
1 day	[1.388, 1.403]	[2.486, 2.638]	[2.246, 2.387]	[1.408, 1.431]	[2.941, 3.121]	[3.035, 3.224]
			Long Gilt			
3 months	[7.714, 8.145]	[11.787, 12.509]	[4.756, 5.053]	[10.666, 11.197]	[13.789, 14.633]	[5.269, 5.599]
1 month	[4.764, 4.967]	[7.159, 7.597]	[3.525, 3.745]	[5.556, 5.804]	[9.636, 10.226]	[3.850, 4.090]
1 week	[1.992, 2.049]	[4.560, 4.839]	[2.406, 2.556]	[2.059, 2.141]	[5.270, 5.593]	[2.885, 3.065]
1 day	[0.837, 0.866]	[1.942, 2.061]	[1.106, 1.175]	[0.879, 0.932]	[2.838, 3.012]	[1.356, 1.441]
			Short Sterling			
3 months	[1.552, 1.781]	[3.554, 3.772]	[2.698, 2.866]	[3.034, 3.102]	[5.682, 6.030]	[4.147, 4.406]
1 month	[0.959, 1.017]	[2.329, 2.472]	[1.921, 2.041]	[1.219, 1.265]	[3.928, 4.168]	[2.890, 3.070]
1 week	[0.333, 0.367]	[0.825, 0.876]	[1.493, 1.586]	[0.365, 0.404]	[1.134, 1.203]	[1.924, 2.044]
1 day	[0.118, 0.139]	[0.308, 0.327]	[0.722, 0.768]	[0.119, 0.145]	[0.413, 0.438]	[0.936, 0.995]

Table 6
Backtests: Realized Percentages of MCRR Violations

Contract	Long Position	Short Position
	Panel A: Unconditional Density	
FTSE-100	4.464%	4.390%
Long Gilt	4.464%	3.423%
Short Sterling	4.241%	3.720%
	Panel B: GARCH (1,1)	
FTSE-100	4.241%	3.943%
Long Gilt	4.018%	3.348%
Short Sterling	4.092%	3.274%
	Panel C: EVT	
FTSE-100	5.134%	4.539%
Long Gilt	4.985%	4.539%
Short Sterling	4.911%	4.092%

Note: the nominal probability of MCRR violations was set at 5% (see text for more details).

Table 7
Out-of-Sample tests: Realized Percentages of MCRR Violations

Contract	Long Position	Short Position
	Panel A: Unconditional Density	
FTSE-100	4.400%	3.800%
Long Gilt	2.800%	2.200%
Short Sterling	1.200%	0.200%
	Panel B: GARCH (1,1)	
FTSE-100	5.200%	4.400%
Long Gilt	3.400%	2.200%
Short Sterling	1.000%	0.200%
	Panel C: EVT	
FTSE-100	4.800%	4.600%
Long Gilt	4.400%	3.200%
Short Sterling	1.400%	0.400%

Note: the nominal probability of MCRR violations was set at 5% (see text for more details).