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**Sector, Region or Function?
A MAD reassessment of Real Estate Diversification
in Great Britain**

Peter Byrne* and Stephen Lee**

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Abstract:

This paper re-examines whether it is more advantageous in terms of risk reduction to diversify by sector or region by comparing the performance of the 'conventional' regional classification of the UK with one based on modern socio-economic criteria using a much larger real estate data set than any previous study and the MAD portfolio approach.

The general conclusion of this analysis is that property market sectors still dominate regions, however defined and so should be the first level of analysis when developing a portfolio diversification strategy. This is in line with previous research. When the performance of Functional groups is compared with the 'conventional' administrative regions the results here show that, when functionally based, groupings can in some cases provide greater risk reduction. In addition the underlying characteristics of these functional groups may be much more insightful and acceptable to real estate portfolio managers in considering the assets that a portfolio might contain.

Keywords: *UK, Sectors and Regions, MAD optimisation, functional groups*

***School of Real Estate & Planning,
Henley Business School, University of Reading, RG6 6UD, UK
Phone: +44 (0) 118 378 6340: E-mail: P.J.Byrne@reading.ac.uk**

****Faculty of Finance,
Cass Business School, City University London,
106 Bunhill Row, London, EC1Y 8TZ, UK
Phone: +44 (0) 207 040 5257: E-mail: Stephen.Lee.1@city.ac.uk**

Sector, Region or Function?

A MAD reassessment of Real Estate Diversification in Great Britain

1. Introduction

A variety of papers, mostly written from the 1990's onwards, have examined the sector/regional diversification 'problem' from a number of different perspectives. Even today, when real estate investors attempt to diversify portfolios, it is still basically through a process of naïve sector/region diversification. The use of Modern Portfolio Theory (MPT) has long been advocated however as a more technically rational approach to the construction of real estate portfolios (Lee, 1992).

Using MPT for asset allocation implies a top-down approach. The first decision to be made is how much to allocate to each broad asset category. The second is the optimal allocation within each asset category. The first level decision, 'the place of property' as part of the multi-asset portfolio, was widely scrutinised as long as twenty five years ago (see for example Folger, 1984; Sweeney, 1988; Richard Ellis, 1990; Baring, Houston and Saunders (BHS), 1995; Byrne and Lee 1995). The second level decision; the optimal holding *within* the real estate portfolio, has had increased attention as the quality and quantity of data available have improved. The early research in this area has been reviewed in Byrne and Lee (1997), Lee and Byrne (1998), Veizer (2000) and Katzler (2005).

Out of this comes the (partly academic) question as to whether investors should confine themselves to one region (place or location) and seek diversification by real estate sector within the region, or diversify across regions while remaining within a real estate sector? Another related issue is whether diversification by either real estate sector or region alone produces significantly (better) worse results than full diversification by both sector and region? Additionally and perhaps most importantly, particular interest has been shown as to whether any non-standard classification, especially an economically based approach, can improve portfolio performance.

The basic method for investigating these questions is to inspect the correlations within and between real estate sectors and regions and/or to construct efficient frontiers based on classical mean-variance analysis (Markowitz, 1952, 1959). Both approaches have been criticised. Inspection of correlation coefficients has been of limited value in deciding whether it is better to diversify by sector or region because, in most studies, tests of significant difference between intra and inter correlations have not been undertaken (Ong and Ranasinghe, 2000). Also, correlation matrices provide only one dimension of diversification and in order to investigate the proper benefits the individual risk of the asset must also be considered. This led researchers to construct efficient frontiers based on mean-variance analysis. The use of mean-variance analysis however imposes restrictive assumptions on return distributions and investor utility functions which are not easy to satisfy and should not be ignored. Specifically, either returns must be normally distributed, or investor preferences need to be described by a quadratic utility function. The assumption of normality of returns is generally invalid for most securities including real estate (Young and Graff, 1995, Young *et al.*, 2006, Lizieri and Ward, 2001 and Young, 2008). The quadratic utility function is itself subject to serious limitations, giving it a limited value for describing the actual behaviour of investors (Alexander and Francis, 1986 and Levy and Sarnat, 1994). Finally the use of mean-variance analysis is also prone to practical difficulties when the number of assets exceeds the number of time periods, as the variance-covariance matrix in such situations would be singular or at the very least ill conditioned. Such difficulties have led most studies to use quarterly and biannual data to try and overcome this problem, even though annual data is preferred (see for example Giliberto, 1990, Graff and Cashdan, 1990, Wheaton and

Torto, 1990 and Gyourko and Keim, 1992) because of inconsistencies, lags and seasonality in the higher frequency appraisal based data.

Lee and Byrne (1998) used the Mean Absolute Deviation (MAD) portfolio method of Konno (1988, 1989); an approach that overcomes all these problems. Konno and Yamazaki (1991) demonstrated that this procedure offers several useful properties by comparison with the Markowitz approach (see Byrne and Lee, 1997 and the Appendix). This study revisits sector versus regional diversification within the UK using the Investment Property Databank (IPD) annual data over the 27 years 1981-2007, applying the MAD portfolio method of Konno (1989).

This paper extends the previous work of Lee and Byrne (1998) in several ways. First, there are new data for a much longer period than in the previous paper, where they were significantly affected by the property market crash of the early Nineties. Secondly, the functional classification they used was based on data from the 1981 Census in the UK. Here a more modern classification, with data from the 2001 Census, is used with a different classification of local authorities in the UK, to retest the proposition that such groupings may offer superior diversification benefits.

The remainder of the paper is structured as follows. The next section brings the literature on the topic up to date. Section 3 looks at the data, and considers the way that the data have been partitioned by sector, 'region' and functional grouping. Section 4 discusses the results, and Section 5 concludes.

2. Previous Research

As noted above, the early research in this area has been reviewed extensively elsewhere (see Byrne and Lee, 1997; Lee and Byrne, 1998; Veizer, 2000 and Katzler, 2005) and will not generally be rehearsed here, except when relevant.

Eichholtz *et al.* (1995) tested the benefits of sector/regional diversification in the US and the UK using a set of methods including correlation analysis, principal components, and mean-variance analysis. They concluded that for the US in general, Retail investment should be diversified across regions, while for Office and R&D/Office diversification across real estate types would be preferred. In the case of the UK the opposite result was obtained for Retail and diversification across both real estate sector and region was preferred for Industrial and Offices.

Lee and Byrne (1998) investigated MAD diversification in the UK by comparing a variety of efficient frontiers using annual returns from 392 locations in the IPD *Key Centres* report for the period 1981-1997. In particular they compared the MAD efficient frontiers produced by sector diversification against three types of regional portfolios; those based on the standard administrative regions of the UK, a '3 super-regional' classification, and economically defined regions, based on travel-to-work areas. In line with previous work they found that sector portfolios generally dominated the regional portfolios, however defined, and certain functional groups outperformed the 'standard regional' classification. However, they noted that a 'SuperRest' region, which contained all other 'peripheral' areas beyond London and the Southeast, would have outperformed almost all other diversification opportunities. This confirmed the observation of Eichholtz *et al.* (1995) who noted that diversification benefits increased the further away from London and that a simple 3x3 (Sector/Superregion) classification scheme offered a reasonable investment strategy. Nonetheless they noted that some of the functionally based 'regions' produced results comparable to those of this 'SuperRest' region and might be preferred by fund managers as they provide more insight

into the reasons for a region's performance than that presented by the standard geographical areas.

In a further paper Byrne and Lee (2000) investigated the risk reduction that might be achieved across sectors and regions in the UK and found that the greatest percentage reduction in total risk from naïve diversification, across the three sectors, Retail, office and Industrial and four regions, London, the South East, the South West and the North (i.e. the rest of the regions), occurred within regional portfolios spread across the three sectors. In contrast the sector portfolios, spread across the four regions, showed only minor reductions in risk, with the office sector showing the worst performance. They attributed this to the average correlations within a region being lower than the correlations across the sectors, the lowest average correlation, and so the greatest risk reduction potential, occurring in the regions further away from London. As a consequence, diversification within a region across the sectors was preferable in terms of risk reduction to diversification across regions within a sector.

Fisher and Liang (2000) used the dummy variable methodology of Heston and Rouwenhorst (HR), (1994) to decompose the returns of US real estate into four sectors and four regions. In this approach the returns of real estate are assigned variables that identify sector and regional affiliation. When these dummy variables were regressed on the cross-section of property returns, the estimated coefficients on the dummy variables became the implicit, or "pure", return effects of the different factors. Their results showed that for the NCREIF environment, in the period 1977-1999, sector was more effective than regional diversification.

Using the same HR model as Fisher and Liang, Lee (2001) decomposed total returns from the IPD *Key Centres* series into sector and regional influences. Sector effects accounted for most of the variation in property returns, explaining almost three times the variability of real estate returns than regional factors. Tilting the sector weights away from those of a typical benchmark portfolio lead to greater tracking errors than regional tilts. The results also indicated that two properties in the same sector are likely to be closer substitutes than two properties in the same region. As a consequence, the potential for portfolio risk reduction was greater diversifying across sectors within a region than across regions within a sector.

Andrew *et al.* (2003) using annual data from 1981 to 2002, adopted the HR approach again and found that the sector effect had a greater influence on property returns than regional factors, irrespective the different specifications of sectors and regions.

Using a similar method on monthly data for individual properties over the period 1987 to the end of 2002, Lee and Devaney (2007) examined the time constancy of sector and regional factors on returns. They found that sector dominated regional factors for the majority of time, especially when the market was volatile, but that this difference largely disappeared in calmer stages of the cycle.

The alternative approach to MPT or dummy variables is to use cluster analytic techniques on the returns data of individual property markets (essentially towns or cities) to try identify the extent to which they cluster by sector or region. If the regional dimension plays an important role in return determination property markets will cluster by location. Then as the aggregation of the data continues 'standard' regions should be formed. In contrast, if the returns in the local property market are more determined by property type, sector clusters might be expected.

Cullen (1993), used clustering techniques on 5500 properties from the IPD database, and found Industrial property to be relatively homogenous across the UK, while Retail partitioned

more on ownership and lease terms than on any regional basis. In contrast the office sector displayed a distinct geographical structure, with City Offices showing the greatest difference compared with the rest of the UK.

Hoesli, *et al.* (1997) and Hamelink, *et al.* (2000) using quarterly data from 1977-1995 found similar results in that there appeared to be a geographical dimension to the office and Industrial property sectors, but none for Retail property. The central London office market in particular, and especially the City office market, behaved differently from the Southeast and the rest of the UK, the distinctiveness of London becoming stronger in the second half of their analysis period. A similar conclusion was found for the Industrial sector, which split into a London cluster; the fringe immediately around London and all other 'peripheral' markets. In contrast Retail markets clustered into a single group and failed to show a distinct London dimension. This conclusion was sustained even with a more refined regional classification scheme. They confirmed the findings of Eichholtz *et al.* (1995) that the nine group (3x3) classification offered a useful structure for the real estate portfolio construction process.

Classification of the return data by three property types and 13 administrative regions (the latter constituting the 11 standard regions of the UK with an additional disaggregation of the London property market) resulted in both four and nine cluster solutions. Analysis of the four-cluster solution for the second half of the period also revealed a growing north-south division which was seen to be the result of the continuing decline of the peripherally located traditional manufacturing base of the UK and the corresponding growth of high tech production/service providers located along the transport corridors around London. The period also saw the deregulation of the City of London and the resultant increase in the number of foreign banks entering the London financial market. Drivers of occupier demand in the City of London market therefore became more closely allied to global forces, helping to distinguish this market geographically from the surrounding office markets of Central London and the rest of the UK.

It is their attempts to relate explicitly the results of the cluster derived groupings to the potential underlying economic forces across local markets that add the greatest value. Jackson (2002) noted that by so doing, Hamelink *et al.* (2000) indicated the continuing movement away from regional, administrative groupings, recognising and emphasising the crucial importance of the economic and geographic factors that underpin property markets.

Indeed, a subsequent paper by Jackson and White (2005) provided a further formal test of the efficiency of the more traditional geographic classification systems; namely those of the 11 standard regions and the IPD segments - the latter being a variation of the three super regions of the UK in which London is subdivided into Central London and rest of London. Using cluster analytic techniques and rental growth data from 1981 to 2000 they found that for Retail, whilst there was evidence of a London cluster, membership of the remaining clusters is diverse. By way of contrast the cluster groupings derived from the four IPD segments for both the entire sample period and the respective sub periods revealed a number of similarities. However, they were not in any way absolute, leading them to conclude that on balance neither the standard regions nor the IPD segmentation accurately reflected rental growth patterns in the Retail sector. Clustering of the rental change data for the office sector, however, lead to a different conclusion where they found evidence to support the IPD segmentation.

In summary, these studies showed that for a real estate fund manager the first level of top-down analysis would be the sector allocation to the portfolio because it is this weighting that offers the greatest potential for risk reduction. This apparent superiority of a sector diversification strategy over the conventional regional approach has subsequently led

professionals and academics to ask the question “so - what is a region?” and to wish to characterise regions by economic functional characteristics rather than by physical or administrative geographies. Current thinking therefore is that ‘regions’ defined on their socio-economic characteristics should provide greater risk reduction effect than those based on geography and may be equally, if not more, important than a sector based approach.

Hartzell *et al.* (1987) in the US were the first to test specifically the hypothesis of geographic versus economic regional diversification. They used an eight economic US regional classification system developed by Salomon Brothers. They reported lower correlation coefficients among the eight economic regions by comparison to those of the four arbitrarily defined NCREIF geographic regions of East, Midwest, West and South - the latter groupings being too broad and heterogeneous in nature to provide any significant diversification benefits.

A further test of the efficiency of the purely geographic (four NCREIF regions) and the economic but geographically constrained model (Salomon eight regions) was made by Mueller (1993). This used a wholly economically-based strategy in which nine different employment categories were created based on the US Standard Industrial Classification (SIC) codes. Each of the then 316 Metropolitan Statistical Areas (MSAs) in the US were categorised into one of these categories using a location quotient technique to identify the dominant source of employment within each MSA that was significantly above the US average. The sample period extended from the fourth quarter of 1973 to the fourth quarter of 1990 and therefore included a full property cycle. The data consisted of the return series of a large institutional commingled fund and included properties from within the office, Industrial, Retail and income producing residential sectors. Mueller compared the standard NCREIF four region split, the Hartzell *et al.* (1987) eight region structure and the nine category classification based on SIC's. The study showed a definite improvement using pure economic effects over both the combined geographic/economic strategy and the NCREIF geographic strategy.

Nelson and Nelson (2003) used data on economic performance, business vitality and development capacity at the US State level to form so called ‘capacity clusters’ using the k-means clustering approach. They were able to test the stability of these clusters over an extended period, and found that although there were some anomalies, they could define seven clusters that were remarkably stable. They did not evaluate their performance against sectors, instead they compared the portfolio performance of these clusters with the eight region Salomon Brothers and four region NCREIF classifications and found that, although the time period studied was a significant consideration in shaping relative performance, the capacity clusters were either very or marginally superior in a mean-variance context.

Smith, *et al.* (2004) examined this issue by first asserting that real estate is essentially a local investment – the ‘location primacy’ view once more. They showed that the bulk of institutionally owned real estate lay (lies) in the largest US Metro markets. They used this idea as the basis for collapsing the totality of the market into eight clusters based on economic characteristics, geographic proximity and absolute size. Each segment was lead by one or two size-based anchor locations that drove the nature of rest of the cluster. There were 35 metro areas in seven of the clusters and all the rest in the eighth (so-called Opportunistic) group. 26 non-anchor markets were assigned to the seven main clusters using cluster and principal component analyses as well as an element of subjective assessment to ‘correct’ the placing of certain of the markets within particular clusters. Taken as a whole they call this a size-tiered economic geography of US real estate markets.

In 2005, the paper was updated (Hess and Liang, 2005), to take account of major changes in the definitions of metropolitan areas in the US, which in turn meant that some structural changes occurred in the content of the Metro markets identified in the earlier paper. The overall effect of these major statistical changes was modest in real estate terms however, because of the pre-existing overwhelming concentration of investment in the largest markets. The observed changes in investment patterns did not materially affect the diversification properties of the eight clusters, although it was noted that return performance varied between clusters and that their performance over time was relatively different and in some cases long lasting.

In the most recent paper of this series, Hess and Ruggiero (2009) have examined the extent to which recent changes in market conditions affected their favoured market structure. The conclusion was that their clustering procedure retained its general utility in the long term, but it could be overwhelmed by 'extraordinary events' that produced 'atypical' levels of systematic risk. They also noted that since 2005 there had been a gradual expansion of the geography of investment, especially into the Opportunistic cluster. At the same time however they did also note a slight increase in investment concentration in the anchor markets of each cluster. Across all of these papers perhaps the most telling and sustained conclusion is that investors can use the economic behaviour of a rather small number of 'locations' as "a proxy for the entire real estate investment universe". On this basis it might seem unlikely that significant changes in the structure of portfolio investment would ever be observed, except at the margins.

Heydenreich (2010) used annual office market data over the period from 1981 to 2003 to examine the benefits of economic versus traditional geographic (administrative) diversification in the UK. Using annual employment data from Cambridge Econometrics the UK counties were organised into 11 categories based on the level of specialisation in a number of counties relative to the national average. Then, using mean-variance analysis for holding periods of five and ten years and for a number of sub-periods, the efficient frontiers produced by the 11 economic regions with the 11 Government regions of the UK were compared. The traditional administrative regional approach to diversification yielded inferior results to those produced by the economic regional classification. However, since the analysis was confined to just one sector (Offices) the author was unable to compare the benefits of sector and regional diversification.

There are a rather few departures from this general trend of results. Newell and Keng's (2003) study of quarterly data in Australia for three sectors and three regions over the period 1995-2002, using the HR method, showed that the differences in sector and regional diversification were not as apparent as elsewhere, with regional diversification delivering slightly **greater** benefits than sector diversification. Importantly, they show both sector and region delivering significant diversification benefits. Particularly relevant in this study was the more significant regional contribution to property diversification in Australia, compared to the US and the UK.

Gabrielli and Lee (2009) used four economically defined regions of Italy to test the relative benefits of regional versus sector diversification on an Italian real estate portfolio. Applying the HR method to annual data over the period 1989 to 2007, they calculated constrained cross-sectional regressions to extract the "pure" return effects for the sector and regional factors in 27 Italian cities. They found that sector and regional factors affected real estate returns in almost equal measure and suggested that this was probably a result of using the varied "economic regions" of Italy rather than arbitrary geographical locations as in US and UK studies. In this sense, a diversification strategy based on economically defined regions may be as good as a sector based approach. Nonetheless, their results also show that more recently in Italy the sector factor has started to dominate the regional effect.

3. Data

The real estate data employed in this study are the IPD annual market standing investment total percentage returns over the period 1981-2007 inclusive. At the end of 2007 the data covered 12,234 properties with an aggregate value of £183,769m in 287 funds (IPD, 2008). The compilation used for this study is the *Local Markets Report* data set. These data provide annual returns for Standard Retail, Office and Industrial properties in a large number of locations (essentially towns and cities) in the UK over the period.

The Local Markets (LM) data are presented by real estate sector, and unlike the Key Centres data used previously by Lee and Byrne (1998), they also classify the data into other geographical scales; counties and Standard Government regions.

Although work, reviewed in McNamara and Morrell (1994), has suggested that such large administrative regions may not prove very meaningful in the UK, especially when administratively structured as the Standard Government Regions, an analysis is made here using this highest level 'regional' structure to test whether it has any value in the allocation context.

Based on the numbers above, an average UK institutional investor holds only 43 properties (IPD, 2008), and this effectively limits the number of real estate categories that can be employed. Hoesli *et al.* (1996) argued that for all but the largest investors, a diversification approach based simply on a three sector by three region classification scheme might be a reasonable strategy within the UK. This '3 Super Regions' scheme covers Office, Shop and Industrial properties in London, the rest of the South East and the rest of the UK. Eichholtz *et al.* (1995) also used such a scheme in their analysis of sector versus regional diversification. Therefore, for purposes of comparison, the following analysis also employs this well known '3 Super Regions' classification scheme.

For this analysis the Local Markets were allocated first to the Government Regions in Great Britain (not Northern Ireland). The data were then re-aggregated to form the '3 Super Regions' identified earlier. The South East was subdivided into London and the Rest of the South East, (Eichholtz *et al.*, 1995), basically because London represents the dominant area of institutional real estate investment (Byrne and Lee, 2009, 2010). Hence they are labelled as SuperLondon, SuperSouth and SuperRest.

The third classification returns to the ideas of Green and Owen (1990) used previously by Lee and Byrne (1998). Green and Owen based their analysis on the then UK Department of Employment's Travel-to-Work Areas (TTWAs). Using these data Green and Owen were able to classify 322 areas into a number of clusters based on two different methodological approaches. They were first grouped on a number of selected dimensions of interest, representing urban and regional characteristics. A similar approach was adopted by Champion *et al.* (1987), using a taxonomy based on towns with similar demographic and labour market characteristics which produced the most easily defined clusters of 'similar' towns. A comparable taxonomic approach is used here.

Following the national census in 2001, as part of the UK government's continuing analysis of the economy, the Office of National Statistics (ONS), produced a multivariate classification of local authorities (essentially towns) based on data from that census (ONS, 2003). Taking the Key Statistics from the 2001 national census as its starting point, the ONS selected 42 variables, split into six main dimensions: demography, household composition, housing, socio-economic, employment and industry sector (ONS, 2003). Using a combination of Ward's hierarchical and k-means cluster analysis methods, each Local Authority (LA) was allocated to a group with other LAs to which it was most similar in terms of these 42

variables. This approach grouped the LAs into a number of clusters based on similar characteristics. The hope in such clustering methods is that ‘natural’ groups will emerge - and that they can be labelled sensibly to represent meaningful categorisations. Here, the principal clustering was at the Supergroup level (seven clusters) which then split into a number of groups (13) and then further into still smaller sub-groups (of which there are 24).

The second level groups (the 13 cluster classification) are used in this study. The Northern Ireland Countryside group is excluded. It represents only 1.1% of the UK population and Northern Ireland had a total of only four (Retail) properties in the IPD dataset in 2007. The 12 remaining clusters are shown in Table 1. Here, the Name column is intended to provide a broad descriptor of the cluster’s characteristic features. It will be noted that four of them are associated specifically with London. When the Location column is also considered, there are two more less directly connected; groups 8 and 9; the area immediately around London (Home Counties) and Southern England. It will also be seen that the Thriving London Periphery (group 3) also contains Oxford and Cambridge. Overall the descriptors tend to be ‘up-beat’, ‘Prospering’ and ‘Thriving’ being used several times. This may, amongst other things, be a function of the nature of their local property markets. The remaining columns give the proportion of the total population in the group, the number of LAs in the group and a fairly typical example location from within the group, some of which may not be very obvious.

Table 1: The ONS Area Classification of Great Britain

Group No.	ONS Cluster Name	Location	Pop. %	No. of LAs	Example
1	Regional Centres	Built-up areas throughout E&W	11%	20	Plymouth
2	Centres with Industry	North West and West Midlands	10%	21	Bolton
3	Thriving London Periphery	London Periphery + Oxford and Cambridge	3%	9	Reading
4	London Suburbs	Outer London + Slough and Luton	5%	12	Redbridge
5	London Centre	Inner London	2%	8	Islington
6	London Cosmopolitan	Inner London, Except Brent	3%	7	Haringey
7	Prospering Smaller Towns	Throughout the E&W	22%	113	Stroud
8	New and Growing Towns	Southern England	5%	24	Dartford
9	Prospering Southern England	Home Counties	9%	44	Horsham
10	Coastal and Countryside	Coastal E&W + some inland areas	10%	52	Christchurch
11	Industrial Hinterlands	South Wales and Northern England	12%	31	Sunderland
12	Manufacturing Towns	Southern Yorkshire + isolated locations	9%	34	Ellesmere Port

Source: ONS (2003)

It should be noted that Table 1 gives the number of LAs in each group. In contrast, in Table 2, the *Local Markets* are shown assigned to the Regions, Super Regions, Functional groups and Sectors. Because a Local Market may only exist for one sector or for no sectors at all, it is not possible to relate the LA numbers in Table 1 directly to Table 2. In this table therefore Panel A shows the proportions of the Local Markets for which any Sector time series is available. The complete data set is substantially larger, geographically and as a time series, than that used previously by Lee and Byrne (1998). Full time series (27 years) - necessary for the optimisation analysis - are not available however for all of these locations. Hence, Panel B of the table shows the reduced set for which full data were available and this set was used to generate the principal results. It can be observed that for the most part

the proportions in the cells change only marginally, although there are some more substantial differences especially in the Office sector.

Table 2: Number of Local Markets in Sector/Regions 2007
Panel A: All Markets – Panel B: Markets with full time series

	PANEL A			PANEL B		
Government Regions	Retail	Office	Industrial	Retail	Office	Industrial
London	16%	19%	11%	17%	25%	15%
South East	16%	32%	23%	16%	32%	28%
South West	10%	8%	8%	11%	7%	6%
East of England	10%	16%	13%	9%	10%	18%
East Midlands	6%	4%	8%	5%	4%	5%
West Midlands	10%	5%	10%	9%	7%	9%
North West	10%	6%	9%	10%	5%	6%
Yorks. and Humberside	8%	4%	6%	9%	4%	5%
North East	3%	1%	1%	3%	1%	1%
Scotland	7%	4%	5%	7%	4%	5%
Wales	4%	2%	5%	4%	1%	1%
Super Regions						
SuperLondon	16%	19%	10%	17%	25%	15%
SuperSouth	15%	32%	22%	16%	32%	28%
SuperRest	68%	49%	67%	67%	44%	57%
Functional Groups						
Regional Centres	15%	13%	9%	16%	18%	13%
Centres with Industry	8%	8%	8%	9%	8%	11%
Thriving London Periphery	6%	9%	3%	6%	11%	5%
London Suburbs	6%	6%	5%	7%	8%	7%
London Centre	5%	7%	1%	5%	10%	3%
London Cosmopolitan	2%	3%	3%	2%	3%	4%
Prospering Smaller Towns	19%	14%	21%	19%	8%	15%
New and Growing Towns	9%	10%	10%	8%	11%	15%
Prospering Southern England	7%	27%	16%	7%	21%	19%
Coastal and Countryside	8%	2%	1%	7%	3%	2%
Industrial Hinterlands	8%	2%	13%	9%		2%
Manufacturing Towns	5%		10%	4%		5%
Total by Sector	100%	100%	100%	100%	100%	100%

It will be seen in Table 2 that the data are fairly evenly spread across sectors but not regions. Looking at Panel A (containing all the location data) it is clear that the proportion of office property is highest in the two Southern Super regions, but that this is much less so for Retail (more generally ubiquitous) and for Industrial, where the SuperRest dominate with 68 and 66% respectively. In Panel B where only locations with full times series are shown, approximately half of the locations in Panel B have been in two regions, London and the South East, reflecting a continuous institutional bias towards the South of England particularly for Offices where the proportion rises from 19 to 25% for SuperLondon, implying the strong and sustained investment in this 'region'. The data also reflect a preference for service-rich areas such as Regional Centres and, for Offices especially, for the clusters described as thriving or prospering in the South of England. A further inference from these patterns is that the data may not therefore reflect the complete performance of real estate in regions other than the 'South of England'. As a consequence, although the relative performance of other regions in explaining real estate returns may not be thought to be significant because of the overwhelming influence by size and value of individual properties in the metropolitan areas, this might not be the case.

4. Results

Figure 1 shows the overall pattern of total returns for the IPD annual series over the study period (note the sharp downturn seen in the 2006 to 2007 period which continued into 2008). This is presented to demonstrate the overall structure of returns and performance in

the early study period - 1981-1995 and the additional period - 1996-2007. The pattern is clearly cyclical in some systematic way, although the two periods are no means the same. The related statistics for the three periods, 1981-2007, 1981-1995 and 1996-2007 are shown at the bottom of Table 3. This shows in particular that the period since the earlier study, was characterised by higher returns and lower volatility. Table 3 also shows the equally weighted average returns and risk measures for the 11 Government Regions; three Super regions; 12 functional groups and the three real estate sectors, using the Panel A and Panel B datasets from Table 2.

Table 3 shows in general that the better returns were outside London and the South East for the Government and Super regions and that the further away from the South, the lower was the risk. The pattern of return and risk for the functional clusters is, in comparison, much more diverse, but the London-based groups do show rather higher returns, albeit with higher risks. Finally in terms of sectors, Industrials offer the highest returns but at higher risk, while Retail shows the lowest risk, at an average return more than 100 basis points higher than for Offices over the period.

Although individual risk and return characteristics are important, the attractiveness, or otherwise, of an asset class as a diversifier in a portfolio is influenced more by its correlation with the other asset classes, than its individual features. It is through the less than perfect positive correlation between assets that diversification is achieved. An analysis of the correlation coefficients between assets can give some clues as to the assets that will have a positive allocation in a mixed-asset portfolio. The simple inspection of a correlation matrix, however, is unlikely to provide a clear indication of the assets that will offer the efficient investment combination in such a multivariate problem. As an alternative Meric and Meric (1989) suggest that calculating what they call a *Dependency Index* can indicate the relative attractiveness of an asset, where the index is an average of the correlation coefficient between a particular asset's returns and the returns of the other assets. These dependency indices are shown in the final columns of Panels A and B of Table 3. In this case these values have all been calculated with the overall market effect removed, so that they represent the relative performance of each category over the period.

Table 3: Summary Statistics: Mean Returns and Risks: 1981 -2007
Regions, Super Regions, Functional Groups and Sectors
Panel A: All Markets – Panel B: Markets with full time series

	PANEL A			Depend. Index	PANEL B			
	Return %	Risk (SD)	Risk (MAD)		Return %	Risk (SD)	Risk (MAD)	Depend. Index
Standard Regions								
London	11.71	10.06	7.41	0.038	12.03	11.00	8.27	0.166
South East	10.43	8.88	6.91	0.394	10.92	10.42	7.78	0.507
South West	10.97	8.72	6.48	0.385	11.57	9.67	7.41	0.539
East of England	10.67	9.42	7.09	0.390	11.35	10.68	7.87	0.511
East Midlands	11.97	9.67	6.84	0.353	12.24	10.95	7.79	0.476
West Midlands	11.07	8.60	6.09	0.446	11.55	10.63	7.50	0.539
North West	11.31	8.22	6.11	0.428	11.37	9.41	7.03	0.599
Yorks. and Humberside	11.31	9.28	6.85	0.431	11.82	9.86	7.27	0.573
North East	11.87	8.38	6.33	0.389	12.42	9.20	6.93	0.485
Scotland	10.76	7.26	5.51	0.415	11.05	8.16	6.16	0.509
Wales	11.44	9.16	6.81	0.374	11.50	9.96	7.51	0.440
Super Regions								
SuperLondon	11.71	10.06	7.41	0.038	12.03	11.00	8.27	0.166
SuperSouth	10.43	8.88	6.91	0.394	10.92	10.42	7.78	0.409
SuperRest	11.26	8.74	6.45	0.476	11.56	10.01	7.36	0.610
Functional Groups								
Regional Centres	11.23	9.23	6.96	0.498	11.18	9.21	6.93	0.578
Centres with Industry	11.90	10.06	7.27	0.424	11.82	10.09	7.24	0.563
Thriving London Periphery	10.35	9.92	7.57	0.338	10.36	9.88	7.58	0.420
London Suburbs	11.64	10.74	8.09	0.311	11.47	10.71	8.07	0.443
London Centre	12.48	11.96	8.88	-0.570	12.48	11.96	8.88	-0.538
London Cosmopolitan	13.93	11.02	8.57	-0.078	13.34	10.82	8.38	0.082
Prospering Smaller Towns	11.69	10.77	7.80	0.461	11.87	10.94	7.94	0.581
New and Growing Towns	11.38	10.57	7.78	0.458	11.41	10.41	7.66	0.493
Prospering Southern England	10.86	10.45	7.90	0.362	11.02	10.38	7.83	0.468
Coastal and Countryside	11.19	9.73	7.62	0.456	11.44	9.91	7.76	0.406
Industrial Hinterlands	11.68	8.73	6.55	0.432	11.33	9.03	6.70	0.466
Manufacturing Towns	11.79	9.65	6.77	0.432	12.17	10.31	7.12	0.420
Sectors								
Offices	10.08	10.35	7.90	-0.525	10.06	10.63	7.87	0.340
Retail	11.24	9.52	7.27	0.422	11.29	9.50	7.25	0.322
Industrial	12.55	10.62	7.70	0.098	13.31	10.55	7.88	0.392
IPD Annual All Property								
1981-2007	10.61	8.71	6.38					
1981-1995	9.68	10.40	7.80					
1996-2007	11.77	6.29	4.57					

In Panel B of Table 3, the dependency indices show that there have been differences in average performance between these areas and classes over the study period. While there is some consistency in the average correlations (moderate positive) it is clear that there is a diversity in relative performance that might be usefully deployed in the portfolio context. On the other hand, it might be felt that for the most part the average intercorrelations are positive and rather too high for really effective diversification across the 'asset classes'. There are some notable exceptions, the London classes being the most obvious, and with the sectors showing some interesting variations on the overall pattern. It is also worth noting that the average index for functional groups is much lower than that for the Government regions, implying that there might be more opportunity to use these clusters for meaningful risk reduction.

Using the MAD portfolio optimisation method, efficient frontiers were generated for the three real estate sectors and for the three categories of 'region'; Standard, 'Super' and 'Functional'. The portfolio asset weights were unconstrained.

Figure 2 shows a comparison between the sector performances in the period up to 1998 and the new longer time series. As would be expected given the sector performance figures in Table 3, the sectors have pushed out towards the top left of the chart, reflecting the different weight which the later period gave to the overall risk/return values, but the relative positions of the three sectors is in effect unchanged.

Some comparative results are shown in summary in Figures 3(a), 3(b) and 4 and 5¹. These enable a view to be formed as to the relative advantages of diversifying by sector, or 'region', however defined.

Sector versus Super Regions (Figure 3a)

SuperSouth and SuperLondon have continued to be affected by the relatively poor performance of the Office sector, a fact noted in earlier work, and sustained though to the present. They and the Office sector are thus somewhat dominated by the Retail, Industrial and SuperRest frontiers. In respect of these last three, SuperRest generally dominates. Given Table 3, showing the higher returns and lower risks of the Government Regions outside London, this result occurs simply because of the large number of diverse combinations of locations and real estate types within SuperRest.

Sector versus Functional groups (Figure 3b)

In Figure 3(b) the Sectors are compared with selected functional groups and these groups dominate sectors in a number of cases. Group 1 is the Regional Centres and Group 6 - London Cosmopolitan is the London boroughs immediately around the core of London. Here the total mix of real estate is relevant, since both groups are essentially those with the greatest overall diversity of investment, but Retail and Industrial holdings are again significant, and this is reflected in the overlap of their frontiers with the Retail and Industrial sector frontiers. The Office frontier is dominated by six functional groups.

Super Regions versus Functional groups (Figure 4)

Figure 4 compares the efficient frontiers of the 3 Super Regions with the 'best' performing Functional groups. In this case there is no clear dominance, with many frontiers intersecting, but SuperRest and SuperLondon are relatively dominant. On the other hand, functional clusters mostly from the south of England, ie groups 4,5,6,and 9, do not perform much less well than the superregions, and may offer a richer (and finer) description of the 'region's shape' than the superregions can do. It can still be argued however that the 3x3 categorisation of Superregions and Sectors might offer a generally acceptable portfolio structure model, but that functional systems should not be dismissed.

Standard Regions versus Functional groups (Figure 5)

When functional groupings (economic/non-contiguous) are compared with the 'best' of the administratively defined and contiguous Government regions, the results in Figure 5 show that only the London Region performs better than any of the Functional groups, but at a higher risk. Otherwise Regional Centres dominates Scotland and the next best performing functional groups, which are again broadly similar to those in Figure 4. Clearly again these functionally formed clusters have more value in diversification terms than most of the 'conventional' regional structures.

¹ In all cases full results are available from the authors.

5. Conclusion

If a top-down strategy is to be used in the construction of real estate portfolios, the first issue is whether it is more advantageous in terms of risk reduction to diversify by sector or region. Eichholtz *et al.* (2000) examined this question for the UK, using hypothetical real estate data, but for technical reasons only considered three real estate types and three 'Super Regions'. This study utilises a much larger UK actual real estate data set than any previous study and by applying the MAD portfolio approach is able to compare the performance of the 'conventional' regional classification with one based on modern socio-economic criteria.

Given the analysis above, and in line with most previous research, the first level of investigation is still the sector. Next, the real estate portfolio manager needs to consider the perennial 'What is a region?' question. The results presented here for Great Britain show that when functionally based groups are compared with Government Regions, greater risk reduction benefits accrue from the functional structures. As was noted earlier, the performance of the clusters used here is in no significant way inferior to that of the most efficient Super Region.

In particular functional groups may be much more insightful and acceptable to real estate portfolio managers. For example the SuperRest efficient frontier is composed of locations which are found within the variety of the functional clusters, the investment characteristics of which are likely to be better understood and more easily monitored by real estate professionals.

The general conclusion to be drawn from this is that diversification across the SuperRest region would have outperformed almost all other diversification strategies. However, in comparing Functional grouping with this Super Regional approach, this economically based classification produced results that were almost equally as good. It remains clear that the principal issue to be resolved is the development of a set of widely acceptable functional groupings, since the evidence indicates that such groupings do offer generally superior risk/return performance than the static Government Regional classification still widely used in the UK. Furthermore, in the development of a real estate portfolio diversification strategy, portfolio managers might well prefer the richer descriptions that such definitions of functionality allow.

There is though a further fundamental question that is associated with this work. This is whether the top-down, optimising, approach of MPT and its related methodologies can ever actually equate to the reality of portfolio construction as practised, where the starting point is more likely to be what is observed at the bottom - the individual property and its characteristics - that drive the investment decision process, and thus (un)intentionally limit the extent to which a portfolio can be optimal in terms of risk and return. This is an area where major research is now required to fully, and perhaps finally, settle the Sector/Region 'problem'.

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Figure 1:

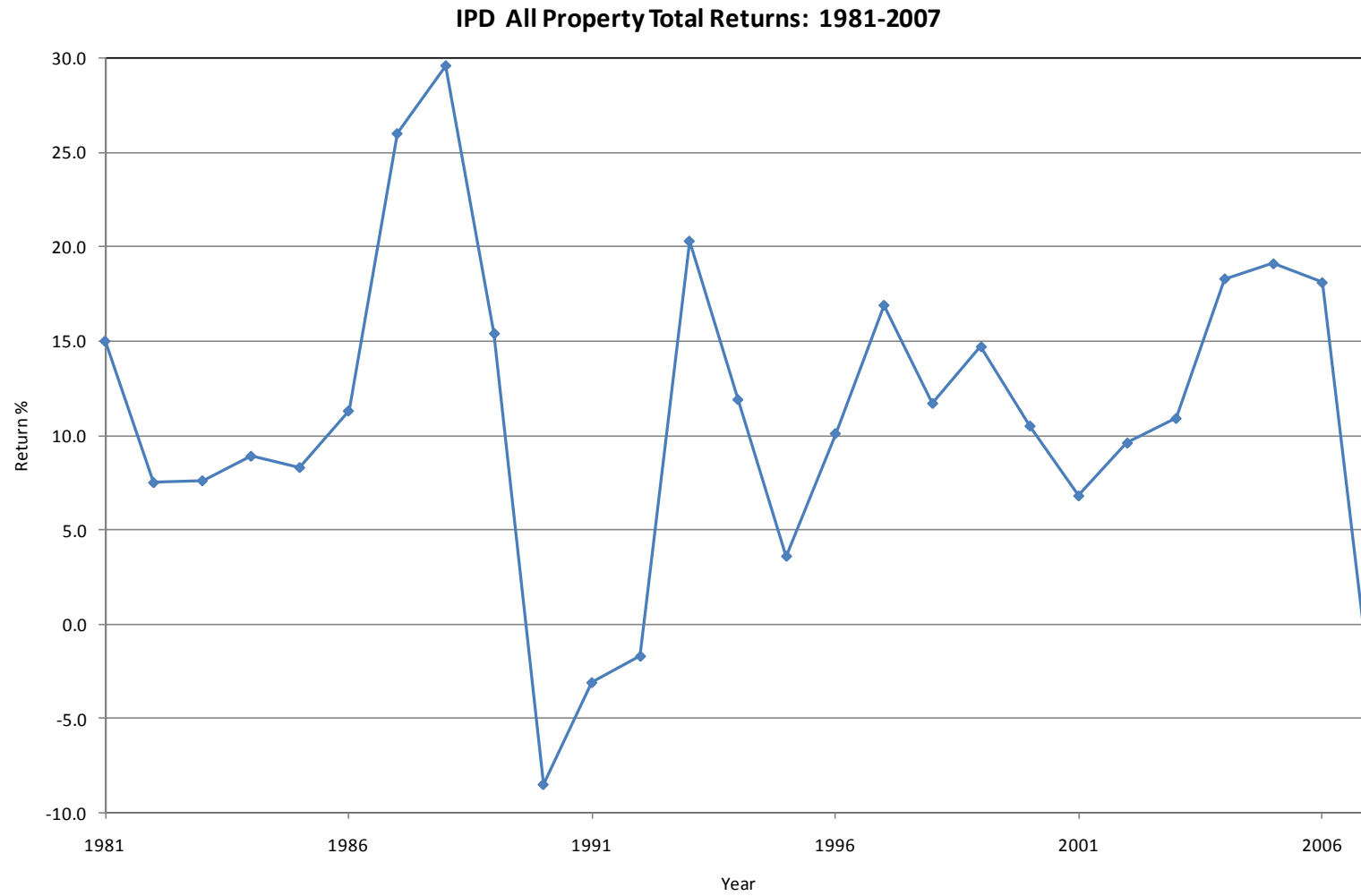


Figure 2:

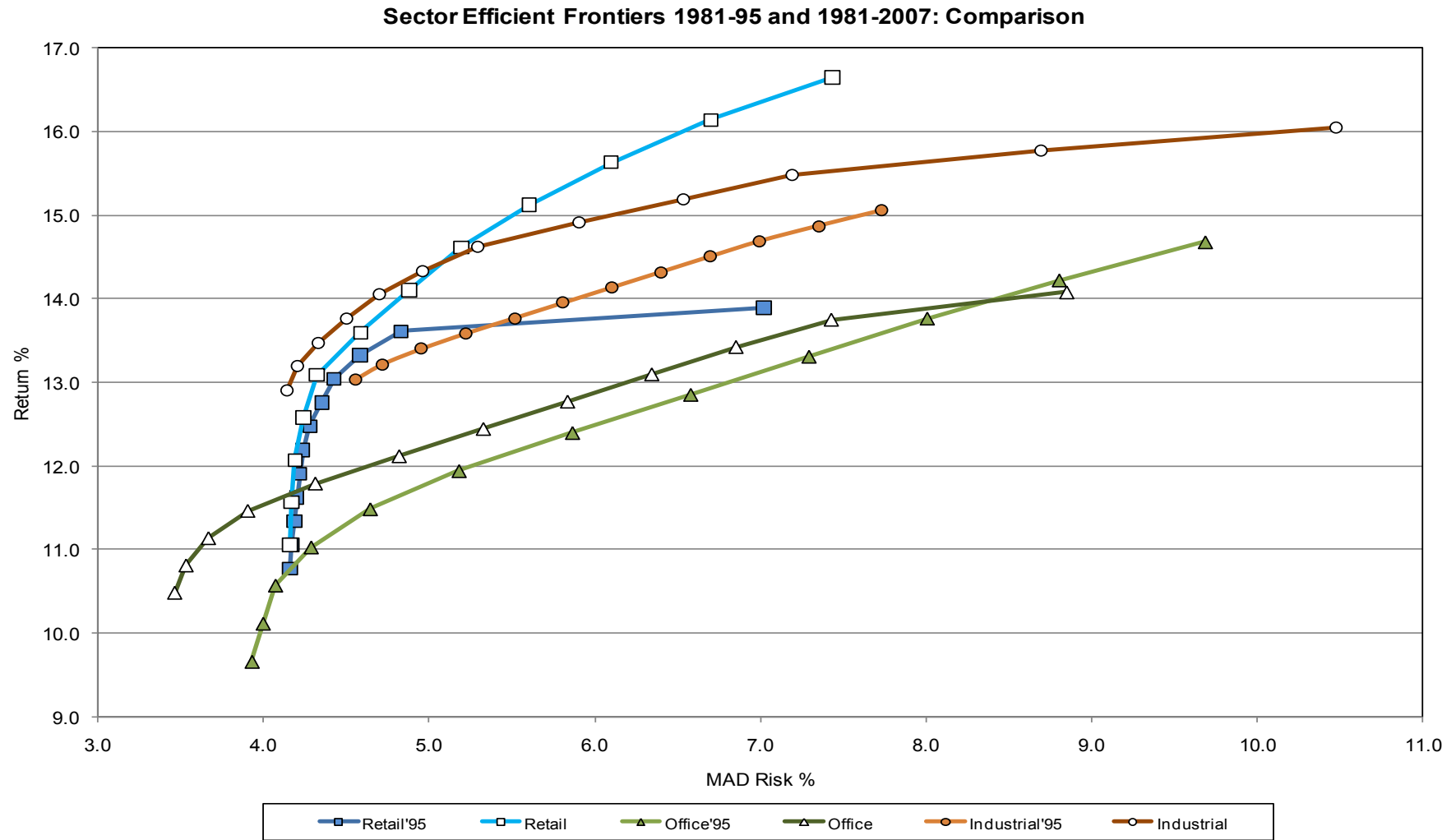


Figure 3(a):

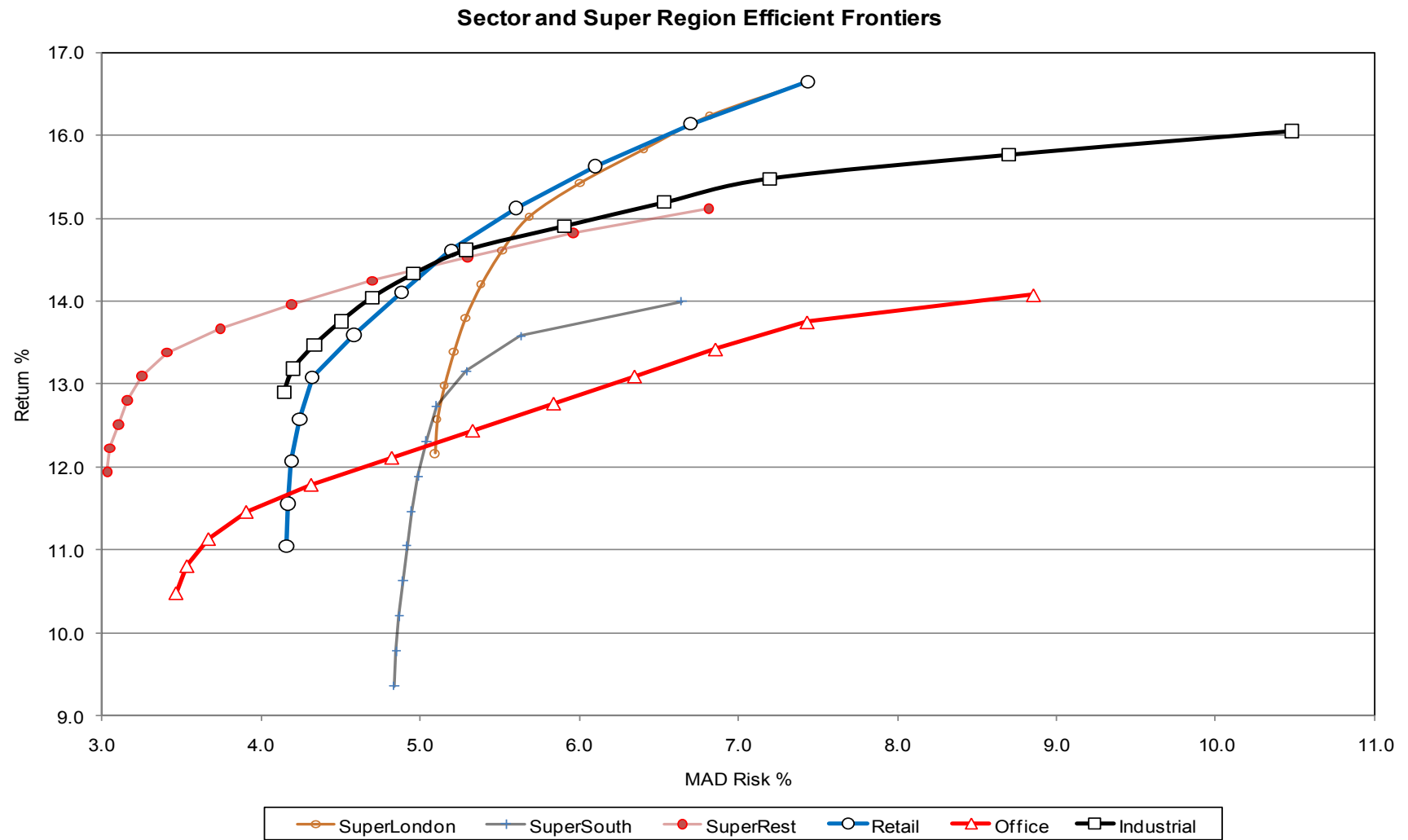


Figure 3(b):

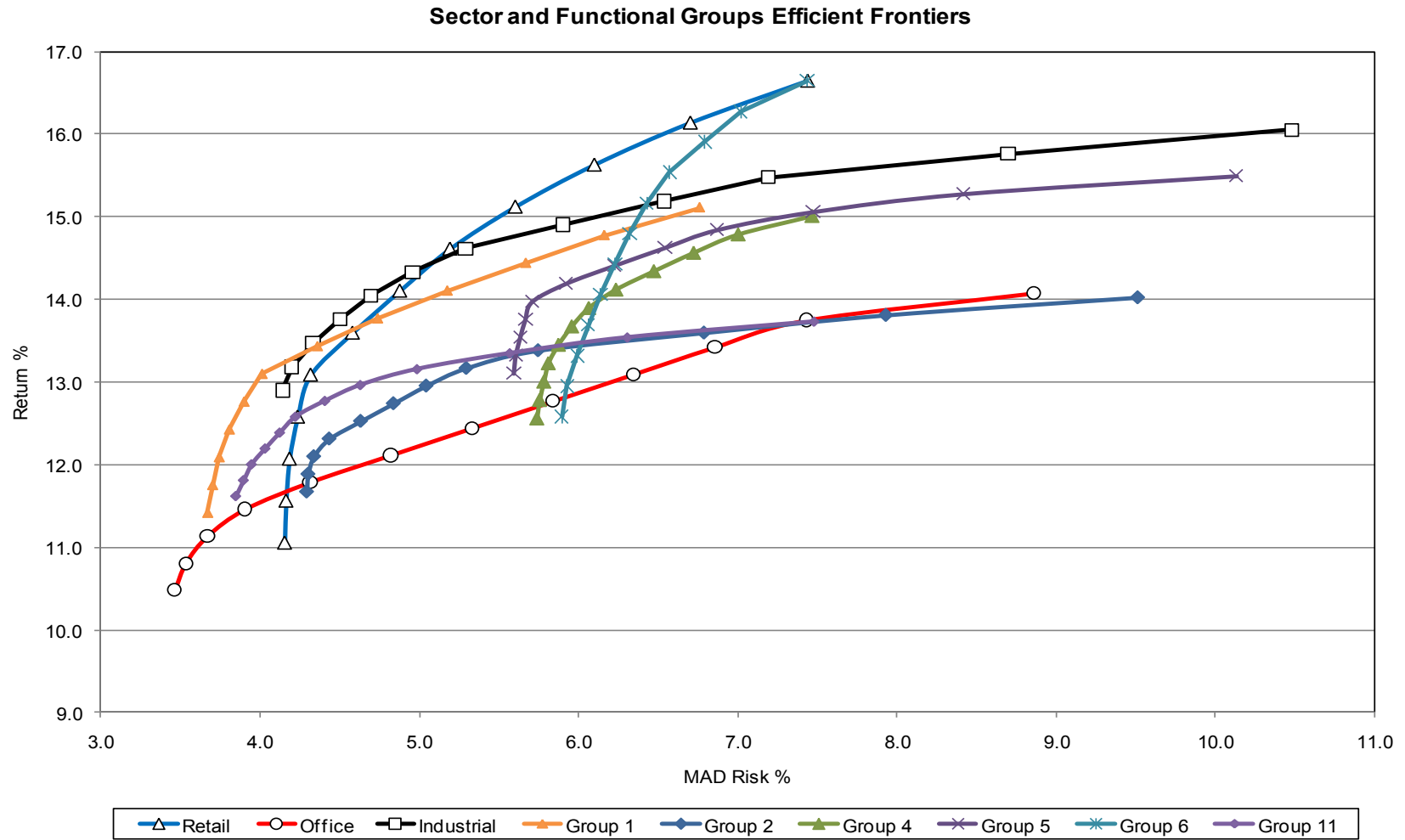


Figure 4:

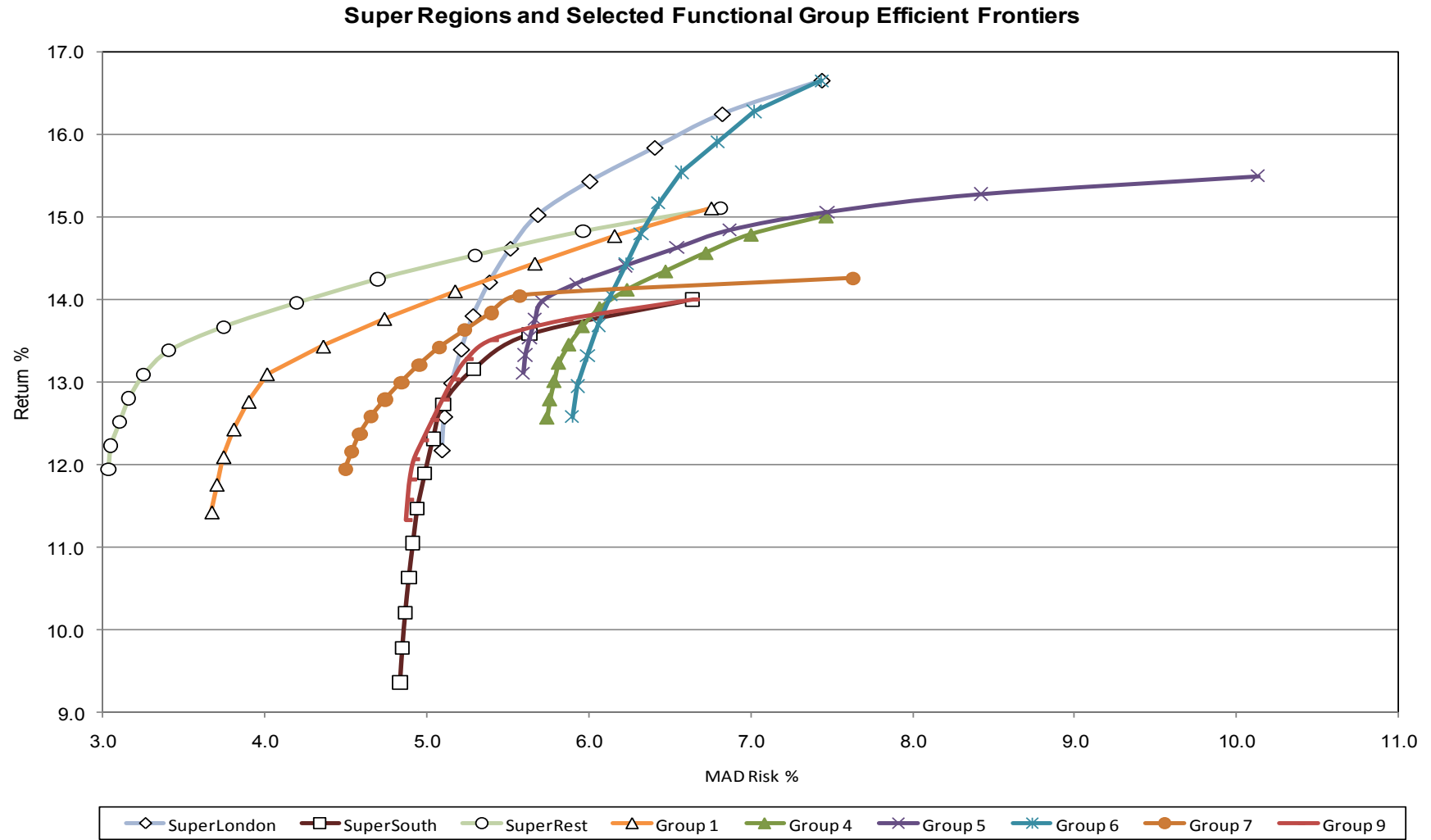
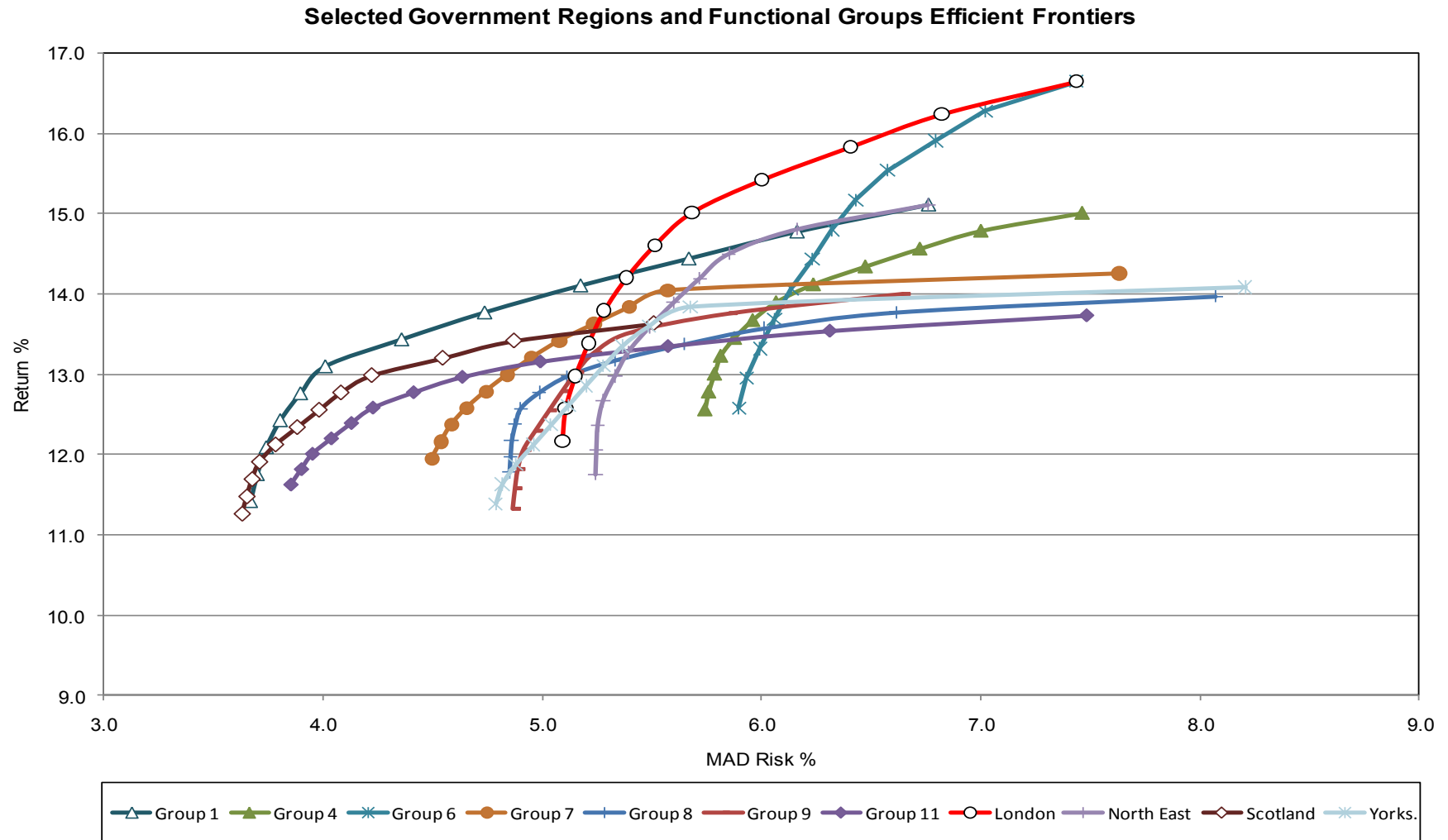


Figure 5:



Appendix

The MAD Portfolio Approach

The Markowitz method for portfolio selection is formulated as a parametric quadratic programming problem where the objective is to minimise portfolio risk for various levels of return, that is:

$$\min \quad \sigma_{\text{port}} \quad (1)$$

subject to:

$$E(R_{\text{port}}) = R^* \quad (2)$$

$$\sum_{i=1}^m w_i = 1 \quad (3)$$

$$0 \leq w_i \leq 1 \quad (4)$$

where:

w_i = weight associated with asset class i

$E(R_{\text{port}})$ = Expected Return of the portfolio

m = number of assets.

R^* is the required rate of return of the investor.

The weights w_i must be positive (no short sales allowed) and the fractions (asset proportions) of the total portfolio must sum to 1.

The method proposed by Konno (1988) uses the MAD of returns as the measure for risk.

For this, let $r_{i,t}$ be the realised return of asset i during each time period t ($t = 1, \dots, T$), available from historical data. The expected value of the return of asset i can be approximated by the mean derived from these data where:

$$E(R_i) = \sum_{t=1}^T r_{i,t} / T \quad (5)$$

Then the MAD_{port} can be approximated as follows

$$MAD_{\text{port}} = \frac{1}{T} \sum_{t=1}^T \left| \sum_{i=1}^m (r_{i,t} - \sum_{t=1}^T r_{i,t} / T) w_i \right| \quad (6)$$

The Mean Absolute Deviation optimisation problem is then:

$$\min \quad \sum_{t=1}^T \left| \sum_{i=1}^m a_{i,t} w_i \right| / T \quad (7)$$

subject to:

$$E(R_{\text{port}}) = R^* \quad (8)$$

$$\sum_{i=1}^m w_i = 1 \quad (9)$$

$$0 \leq w_i \leq 1 \quad (10)$$

and where:

$$a_{i,t} = (r_{i,t} - \sum_{t=1}^T r_{i,t} / T), \quad i=1, \dots, m; \quad t=1, \dots, T \quad (11)$$

is the per period deviation of the realised return r for asset i , in time period t , from the mean return of asset i over time T .

This is actually equivalent to the linear programme:

$$\min \sum_{t=1}^T y_t / T \quad (12)$$

subject to:

$$y_t + \sum_{i=1}^m a_{i,t} w_i \geq 0, \quad t=1, \dots, T \quad (13)$$

$$y_t - \sum_{i=1}^m a_{i,t} w_i \geq 0, \quad t=1, \dots, T \quad (14)$$

$$E(R_{\text{port}}) = R^* \quad (15)$$

$$\sum_{i=1}^m w_i = 1 \quad (16)$$

$$0 \leq w_i \leq 1 \quad (17)$$