Depth to mate and the 50-move rule

NOTE

DEPTH TO MATE AND THE 50-MOVE RULE

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1. INTRODUCTION

The first author's DTM_{50} 'EGT' endgame tables (Huntington, 2013; Haworth, 2014a/b) provide 'DTM' Depth to Mate information as moderated by the FIDE (2014) '50mr' 50-move rule and the ply-count *pc*. This note puts that achievement in the context of earlier DTM computations (Nalimov et al., 2000/2001; Wu and Beal, 2001a/b; Bleicher, 2015) and data from previous studies of 50mr-impact (Tamplin and Haworth, 2004; Bourzutschky et al., 2005; Tamplin, 2015). It compares some DTM_{50} statistics with the intrinsic, unmoderated DTM and DTZ_{50} data.[3] Datasets supporting these results are available (Huntington and Haworth, 2015) and include a pgn file, its annotation, and the fuller statistics which cannot be accommodated here.

When considering 50mr impact, the ply-count *pc* or rather the ply-remaining count $pr \equiv 100\text{-}pc$ must be borne in mind. A position's $dtm_{50,pc}$ may increase as *pc* increases until the win becomes a 'frustrated win', a '50mr-draw'. Even a mate in two ply will be frustrated if 99 ply have been expended, e.g., in KNNKP. These 'EM_{50}' DTM_{50} EGTs are the first to provide depths for any value of *pc*. Clearly, $\text{maxDTM}_{50,pc\geq 0} \geq \text{maxDTM}_{50,pc=0}$.

The FIDE 50-move rule is not an intrinsic rule of the game but a 'rule of play' introduced by Ruy López (1561) for the convenience of professional coffee house players. The results here show that it has major impact on KBBKN and KNNKP, and therefore on the upstream KBBKNN, KNNKNP and KNNKPP. For s6m, s7m and s8m chess, maxDTZ is greater than 80, 240 and 510 moves respectively (Haworth, 2014b). The 50mr, now backed by the 75-move-rule, will apparently frustrate ever more subtle wins as the number of men increases.

The perspective here is that these extreme cases of extended wins, rather than being denied by rules of play, should become part of the culture, experience, record and history of chess, at least when an EGT-armed computer-engine demonstrates infallible play.

2. COMPUTING DTM: ALGORITHM, LANGUAGE AND PROGRAM

When generating any EGT, the fundamental principles are that (a) successor endgames' EGTs are computed first, and (b) a position can be given a depth:

- temporarily when one of its immediate successors has been given a depth, but
- definitively only after enough of its successors have been given their definitive depth.

Any computation begins by identifying 'mated' (in 0 ply) positions. Positions which can be assigned a depth may be found by repeated, linear sweeps of the whole endgame and the first sweeps efficiently net many positions. Shortly though, the more selective and efficient method is to 'unmove' from the 'frontier' of positions which have just been given a depth in the last cycle of the algorithm.

The best known DTM computations are those of Nalimov (2000/2001) which created EGTs for the whole of sub-7-man (s7m) chess (Bleicher, 2015). Nalimov employed linear sweeps rather than the more retrograde 'unmove' algorithm, giving a position a depth at the earliest opportunity but lowering it later as required. As evidenced by the results here, this occurred many times in generating the KNNKP EGT. In contrast, Wu (2001a/b) worked exclusively in unmove mode. His key idea was to defer identifying 'mates in *m*+1' until all 'mates in *m*' had been identified. As each *dtm* is associated with one cycle of the algorithm, it only required two bits per position rather than one- or two-bytes, economising on memory by a factor of four or eight.

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[3] DTZ ≡ minimaxed depth to the zeroing of the ply-count, i.e., to a pawn-push, capture and/or mate which end the current phase of play.

Huntington reverts to the principle of assigning depths as soon as possible. Each algorithm cycle corresponds to a specific number, *pr*, of ply remaining: *pr* effectively increases from 0 to 100 during the computation. Lower values of DTM_{50} become possible when more ply are available. Table 3 shows that the highest variability of DTM_{50} seen so far occurs in KNNKP with 25 different mate-depths. By noting on what cycle a DTM_{50} value is set, the various ply-ranges corresponding to a specific dtm_{50} can be identified for each chess position.

EGT generation, as a computation task, is challenging because the results are not self-evidently correct: the subtlety and toxicity of errors is well known (Hurd and Haworth, 2010). Where clarity of coding and correctness are particularly at risk, as has been argued in classic texts elsewhere (Hughes, 1990; Bird, 2014), 'FPL' functional programming languages, without side effects, may be preferred to imperative languages. The declarative style of FPLs, using higher levels of abstraction, creates programs which are more readable and understandable. Research in FPLs created an embarrassment of riches and ideas, and the best of these were brought together in the language HASKELL, 'standardised' in HASKELL 98 (Peyton-Jones, 2002) and then in HASKELL 2010 (Marlow, 2010). HASKELL is most liked for its elegance as a language, for its type system, and for the confidence it creates that compiled code is likely to be correct code (MacIver, 2015) and was the choice of the first author here.

FPLs once had a reputation for inefficiency but modern compilers have largely offset this and performance is competitive today. For example, FPLs allow new methods of whole-program optimisation made possible by the promise of purity. Also, code can be written much more compactly due to the expressive power of treating functions as ordinary values that can be built at runtime. Certainly, there are new kinds of pitfalls which programmers must contend with: the challenge of balancing use of space and time remains. Small changes can have a large effect on performance by, e.g., causing an optimization to become inapplicable. Worse, unexpected memory leaks are common, and measures must be taken to prevent overzealous time optimisation that causes excess space usage. Such snags plagued earlier versions of the DTM_{50} EGT generator, and in general avoiding them is an active area of compiler research. The EGT itself consists internally of an array where each cell value stored the possible mate lengths for various ranges of PC, and an 'overflow' hash table to store values that do not fit in an array cell. The cell's size has to be fixed in advance and chosen with care. Too small, and the overflow structure becomes heavily used, which is much less space/time efficient than an array; too large, and much memory is wasted on unused array space.

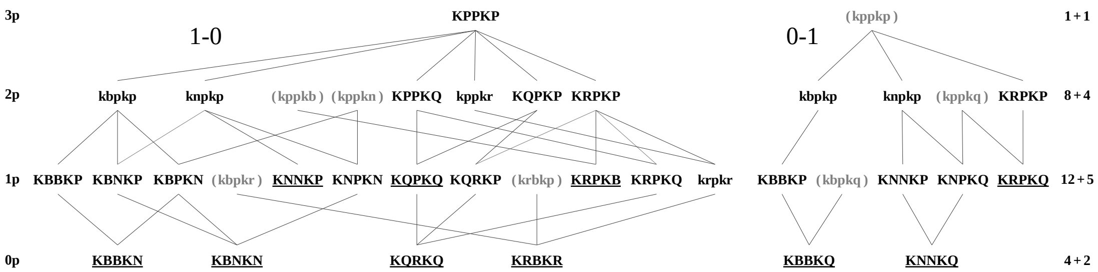

Figure 1. 5-man endgame wins, 1-0 and 0-1, at first apparently DTZ-susceptible to the 50-move rule.

3. THE RESULTS

DTM endgame tables presume a ply-count of $pc = 0$ and therefore the $DTM_{50,pc}$ results with $pc = 0$ provide a natural first focus. The later consideration of a free-ranging *pc* places emphasis on the cost of $pc > 0$ and the variability of $dtm_{50,pc}$.

3.1 $DTM_{50,pc}$ with zero ply-count *pc*

For sub-5-man chess, no endgames are affected as maxDTM < 100 ply. Figure 1 shows those 5-man endgames, ranked by number of pawns, which appear at first to be susceptible to the 50mr, 1-0 and/or 0-1 wins[4] perhaps being converted to 50mr-draws or lengthened in some sense. Tamplin and Haworth (2004) identified the actual 50mr impact in DTZ terms and this is annotated under four headings in Figure 1 and Table 1.

[4] In fact, the 50mr only affects both 1-0 and 0-1 wins in the endgames KBBKP, KNNKP, KRPKQ and KRPKP of Figure 1.

#	#P	Endgame	Res.	maxDTM, ply wtm #	maxDTM wtm dtm	maxDTM btm #	maxDTM btm dtm	EZ: ≡ EZ_{50} ?	EZ % wins frustrated wtm	EZ % wins frustrated btm	EZ % wins dtz_{50} > dtz wtm	EZ % wins dtz_{50} > dtz btm		$maxDTM_{50}$, ply pc = 0 wtm #	pc = 0 wtm dtm	pc = 0 btm #	pc = 0 btm dtm		any pc wtm #	any pc wtm dtm	any pc btm #	any pc btm dtm
01	0	KBBKN	1-0	32	155	43	156	Δ	21.05	48.20	0	0	↓	275	131	319	132	↓≡	275	131	319	132
02	1	KBBKP	1-0	1	147	15	146	Δ	ε	ε	0.01	ε	↓	1	131	4	132	↓↑	1	137	16	136
03	0	KBNKN	1-0	2	213	1	212	Δ	0.52	1.93	0	0	↓	11,204	159	5,140	160	↓≡	11,204	159	5,140	160
04	1	KBNKP	1-0	9	207	9	208	Δ	ε	ε	ε	ε	↓	5	161	10	162	↓↑	8	175	2	174
05	1	KBPKN	1-0	1	199	1	192	Δ	ε	ε	ε	ε	↓	6	175	4	174	↓↑	1	177	1	176
06	2	kbpkp	1-0	92	133	52	134	≡	0	0	0	0	≡	92	133	52	134	↑↑	3	145	3	146
07	1	(kbpkr)	1-0	3	89	3	88	≡	0	0	0	0	≡	3	89	3	88	↑↑	5	89	5	90
08	1	KNNKP	1-0	2	229	4	228	Δ	26.35	46.87	42.16	30.80	↓	7	223	12	222	↑↑	28	255	3	256
09	1	KNPKN	1-0	5	194	12	193	Δ	ε	ε	ε	ε	↓	5	161	13	162	↓↑	5	163	13	162
10	2	knpkp	1-0	10	113	10	114	≡	0	0	0	0	≡	10	113	10	114	↑↑	1	129	2	130
11	2	(kppkb)	1-0	1	85	1	86	≡	0	0	0	0	≡	1	85	1	86	≡≡	1	85	1	86
12	2	(kppkn)	1-0	2	99	1	100	≡	0	0	0	0	≡	2	99	1	100	==	1	99	1	100
13	3	KPPKP	1-0	6	253	7	254	Δ	ε	ε	ε	ε	↑	4	281	4	282	↑=	5	281	4	282
14	2	KPPKQ	1-0	7	247	2	200	Δ	0.01	0.01	0	0	↑	17	275	1	200	↑≡	17	275	1	200
15	1	kppkr	1-0	1	107	1	106	≡	0	0	0	0	≡	1	107	1	106	≡≡	1	107	1	106
16	2	KQPKP	1-0	1	209	6	244	Δ	ε	ε	ε	ε	~	4	191	9	274	↑↑	4	233	9	274
17	1	KQPKQ	1-0	5	247	13	246	Δ	0.02	0.08	0.03	0.10	↑	36	273	30	274	↑↑	3	275	30	274
18	1	KQRKP	1-0	1	79	3	134	Δ	0	ε	0	0	↓	1	79	3	122	~↑	5	107	3	122
19	0	KQRKQ	1-0	3	133	31	134	Δ	ε	ε	0	0	↓	3	121	31	122	↓≡	3	121	31	122
20	1	(krbkp)	1-0	1	55	4	72	≡	0	0	0	0	≡	1	55	4	72	↑↑	1	73	1	74
21	0	KRBKR	1-0	28	129	19	128	Δ	0.01	0.02	0	0	↓	939	111	162	112	↓≡	939	111	162	112
22	1	KRPKB	1-0	3	145	13	146	Δ	ε	ε	ε	ε	≡	3	145	13	146	↑↑	5	149	15	150
23	2	KRPKP	1-0	3	111	3	136	Δ	0	ε	ε	ε	↓	3	111	23	118	~↑	1	121	9	120
24	1	KRPKQ	1-0	45	135	1	108	Δ	ε	ε	ε	0	~	22	123	1	114	~=	48	123	1	114
25	1	krpkr	1-0	33	147	4	148	≡	0	0	0	0	≡	33	147	4	148	↑↑	2	161	1	162
26	1	KBBKP	0-1	54	164	82	165	Δ	5.85	8.47	0.07	0.02	↓	3	134	7	135	↓↑	4	136	2	137
27	0	KBBKQ	0-1	74	162	15	161	Δ	8.49	1.46	0	0	↓	3,116	124	1,030	123	↓≡	3,116	124	1,030	123
28	2	kbpkp	0-1	2	100	3	101	≡	0	0	0	0	≡	2	100	3	101	≡≡	2	100	3	101
29	1	(kbpkq)	0-1	3	100	2	99	≡	0	0	0	0	≡	3	100	2	99	↑↑	1	104	2	103
30	1	KNNKP	0-1	11	146	9	147	Δ	0.14	0.06	0.10	ε	↓	12	130	13	131	↓=	16	130	13	131
31	0	KNNKQ	0-1	10	144	2	143	Δ	0.05	0.01	0	0	↓	162	124	104	123	↓≡	162	124	104	123
32	2	knpkp	0-1	3	114	3	115	≡	0	0	0	0	≡	3	114	3	115	≡≡	3	114	3	115
33	1	KNPKQ	0-1	1	124	2	109	Δ	ε	0	0	0	↓	1	114	2	109	↑↑	1	146	1	145
34	3	(kppkp)	0-1	3	84	3	85	≡	0	0	0	0	≡	3	84	3	85	≡≡	3	84	3	85
35	2	(kppkq)	0-1	12	82	6	81	≡	0	0	0	0	≡	12	82	6	81	↑↑	3	86	1	87
36	2	KRPKP	0-1	5	200	6	205	Δ	0.14	0.05	0.10	ε	↓	3	188	2	193	↓≡	3	188	2	193
37	1	KRPKQ	0-1	3	206	1	207	Δ	0.06	0.02	0.02	ε	↓	21	194	1	195	↑↑	7	214	3	213
38	2	(kppkn)	0-1	12	32	31	33	≡	0	0	0	0	≡	12	32	31	33	↑↑	2	36	10	35
39	0	krbkq	0-1	3	140	4	139	≡	0	0	0	0	≡	3	140	4	139	↑↑	3	144	4	143
40	0	KBBKNN	1-0	11	211	1	212	Δ	50.15	70.98	1.75	2.78	↓	1	179	1	178	↓↑	2	181	1	180
41	1	KNNKNP	1-0	198	275	30	274	Δ	?	?	?	?	↓	160	223	11	222	↓≡	160	223	11	222
42	2	KNNKPP	1-0	2	259	3	258	Δ	?	?	?	?	↓	1	257	2	256	↑↑	2	269	2	268

Table 1. Data for some endgame wins, the first 37 being those of Figure 1.[5]

The four-way DTZ-based taxonomy is indicated as follows:

- obviously unaffected wins (7) (bracketed 'lower case'), e.g., KBPKR (1-0), KPPK(Q/P) (0-1):
  maxDTM ≤ 100 ply ⇒ EM_{50} ≡ EM ⇒ there need be no 50mr-effect here or 'downstream',
- perhaps affected wins (6) (unbracketed lower case), e.g., K(B/N)PKP (1-0 and 0-1):
  the DTZ_{50} and DTZ EGTs, i.e., EZ_{50} ≡ EZ, are identical but perhaps $dtm_{50} > dtm$ somewhere,
- affected wins (14) (upper case), e.g., KBBKP and KRPKP (both, 1-0 and 0-1), KPPK(Q/P) (1-0):
  maxDTZ ≤ 100 ply but there are 50mr draws, i.e., EM_{50} ≢ EM,
- obviously affected wins (10) (underlined, upper case), e.g., KNNKP (1-0) and KRPKQ (0-1):
  maxDTZ > 100 ply ⇒ 50mr-draws ⇒ EM_{50} ≢ EM.

Beyond the scope of Figure 1, Table 1 provides DTM_{50} data for a number of other endgames which are affected. A second DTZ/Z_{50} review (Bourzutschky et al., 2005) provides further context for Pawnless 6-man endgames.

[5] $maxDTM_{50,0}$ is compared with maxDTM. $maxDTM_{50,pc}$ is compared with maxDTM and then $maxDTM_{50,0}$. '↓', '≡', '↑' and '~' mean, respectively, 'if anything, less than', 'identical', 'if anything, more than' and 'considering wtm/btm, both less than and more than'. '=' rather than '≡' indicate that the number of maximal positions has changed.

With increasing likelihood, $maxDTM_{50,pc=0}$ is greater than, less than or equal to maxDTM as indicated in Table 1:

$maxDTM_{50,\,pc=0}$ > maxDTM:

(1-0) KPPKP, KPPKQ, KQPKP btm, KQPKQ, KRPKQ btm

$maxDTM_{50,\,pc=0}$ < maxDTM:

(1-0) KB(B/N)K(N/P), KBPKN, KNNKP, KNPKN, KQPKP btm, KQRK(P/Q), KRBKR, KRPKP, KRPKQ wtm (and 6-man) KBBKNN, KNNKPP;

(0-1) KBBK(P/Q), KNNK(P/Q), KNPKQ, KRPK(P/Q)

			Value		depths in ply						
id	Endgame	FEN	1-0?	5-way	dtc	dtm	dtm50	dtz	dtz50	dtz50′	Notes: annotations to the DTZ_{50}' metric
01	KBNKN	8/8/1N6/8/6B1/1K3n2/8/k7 b - - 0 1	1-0	-2	100	158	160	100	100	100	$maxDTM_{50}$ s6m_P-less pos for $pc \geq 0$
02	KPPKP	8/8/8/1p3K2/3P4/3P4/7k/8 b - - 0 1	1-0	-2	8	248	282	2	2	2	$maxDTM_{50}$ s6m pos for $pc \geq 0$
03	KRBKQ	k4B2/8/8/8/6q1/8/K3R3/8 w - - 0 1	0-1	-2	82	140	140	82	82	82	$maxDTM_{50}$ P-less 2-man win for $pc = 0$
04	KRBKQ	k4B2/8/8/8/6q1/8/K3R3/8 w - - 11 1	0-1	-2	82	140	144	82	82	82	$maxDTM_{50}$ P-less 2-man win for $pc \geq 0$
05	KRPKQ	4q3/1R6/8/8/k7/1PK5/8/8 b - - 0 1	0-1	2	137	193	195	77	77	77	$maxDTM_{50}$ 2-man win for $pc = 0$
06	KRPKQ	2k5/8/8/R7/8/3K4/1P4q1/8 w - - 33 1	0-1	-2	126	182	214	64	64	64	$maxDTM_{50}$ 2-man win for $pc \geq 0$
07	KNNKP	6k1/p7/8/8/7N/7K/2N5/8 w - - 0 1	1-0	2	178	181	223	30	56	56	$maxDTM_{50}$ KNNKP pos for $pc = 0$
08	KNNKP	8/8/5N2/p7/8/k1K5/8/1N6 b - - 4 1	1-0	-2	167	190	256	43	93	93	$maxDTM_{50}$ KNNKP pos for $pc \geq 0$
09	KQPKQ	3Q4/8/8/5K2/8/3P4/7k/1q6 b - - 0 1	1-0	-2	220	240	274	100	100	100	$maxDTM_{50}$ KQPKQ pos for $pc = 0$
10	KQPKQ	3Q4/8/8/5K2/8/8/3P3k/1q6 w - - 88 1	1-0	2	141	163	275	1	1	1	$maxDTM_{50}$ KQPKQ pos for $pc \geq 0$
11	KBBKNN	7k/7B/8/2B5/3K4/2n5/8/5n2 w - - 0 1	1-0	2	9	133	179	9	55	55	$maxDTM_{50}$ KBBKNN pos for $pc = 0$
12	KBBKNN	2n5/8/3B4/8/3K4/1B6/6n1/2k5 w - - 43 1	1-0	2	21	139	181	21	55	55	$maxDTM_{50}$ KBBKNN pos for $pc \geq 0$
13	KBBKN	8/8/8/7B/4k3/4B3/3K4/1n6 w - - 0 1	'1-0'	1	119	143	—	119	—	119	
14	KBBKP	8/8/8/7B/4k3/4B3/1p1K4/8 b - - 0 1	'1-0'	-1	6	144	—	6	—	1	1. … b1=N+′′′′, 50mr-draw
15	KBNKN	8/8/3K4/8/8/3B4/k7/1n1N4 w - - 0 1	'1-0'	1	139	199	—	139	—	139	
16	KBNKP	8/8/3K4/8/8/3B4/kp6/3N4 b - - 0 1	'1-0'	-1	9	200	—	9	—	1	1. … b1=N′′′′, 50mr-draw
17	KBPKN	1n6/3P4/8/8/1K6/7B/8/k7 w - - 0 1	'1-0'	1	1	199	—	1	—	1	1. d8=N′′′′, 50mr-draw
18	KNNKP	K1k5/3N1N2/8/8/4p3/8/8/8 w - - 0 1	'1-0'	1	169	169	—	164	—	164	
19	KNPKN	kn6/3P4/1K6/8/8/8/3N4/8 w - - 0 1	'1-0'	1	1	191	—	1	—	1	1. d8=B′′′′, 50mr-draw
20	KPPKP	8/4P3/8/8/8/4P3/kp1K4/8 b - - 0 1	'1-0'	-1	2	244	—	2	—	1	1. … b1=Q′′′′, 50mr-draw
21	KPPKQ	8/4P3/8/8/8/4P3/k2K4/1q6 w - - 0 1	'1-0'	1	1	243	—	1	—	1	1. e8=Q′′′′, 50mr-draw (dtz = 102p)
22	KQPKP	8/4Q3/8/8/8/K7/6Pp/5k2 w - - 0 1	'1-0'	1	5	191	—	1	—	1	1. g4′′′′, 50mr-draw
23	KQPKQ	4Q3/8/8/8/8/4P3/k2K4/1q6 b - - 0 1	'1-0'	-1	222	242	—	102	—	102	
24	KQRKP	Q7/2k5/8/8/8/8/R2p4/K7 b - - 0 1	'1-0'	-1	2	134	—	2	—	1	1. … d1=Q′′′′, 50mr-draw
25	KQRKQ	Q7/2k5/8/8/8/8/R7/K2q4 w - - 0 1	'1-0'	1	119	133	—	119	—	119	
26	KRBKR	8/3B4/8/1R6/5r2/8/3K4/5k2 w - - 0 1	'1-0'	1	117	129	—	117	—	117	
27	KRPKB	K1R5/8/3k4/3P4/8/8/1b6/8 w - - 0 1	'1-0'	1	113	131	—	105	—	105	
28	KRPKP	6R1/P6K/1k6/8/8/8/3p4/8 b - - 0 1	'1-0'	-1	2	136	—	2	—	1	1. … d1=Q′′′′, 50mr-draw
29	KRPKQ	6R1/P7/2q5/2k5/8/8/8/6K1 b - - 0 1	'1-0'	-1	2	118	—	2	—	2	1. … Kb6′′′′ 2. a8=Q′′′′, 50mr-draw
30	KBBKNN	8/6B1/8/8/2B1n3/6K1/3k3n/8 w - - 0 1	'1-0'	1	1	147	—	1	—	1	1. Kxh2′′′′, 50mr-draw
31	KBBKP	8/8/6B1/3K4/5B2/8/p7/3k4 b - - 0 1	'0-1'	1	1	157	—	1	—	1	1. … a1=Q′′′′, 50mr-draw
32	KBBKQ	8/8/6B1/3K4/5B2/8/8/q2k4 w - - 0 1	'0-1'	-1	136	156	—	136	—	136	
33	KNNKP	3k3N/3N4/3K4/8/8/8/7p/8 b - - 0 1	'0-1'	1	1	145	—	1	—	1	1. … h1=Q′′′′, 50mr-draw
34	KNNKQ	3k3N/3N4/3K4/8/8/8/8/7q w - - 0 1	'0-1'	-1	126	144	—	126	—	126	q.v., KNNKP '0-1'
35	KNPKQ	1k1K4/4P1N1/8/8/8/6q1/8/8 w - - 0 1	'0-1'	-1	6	124	—	6	—	1	1. e8=N′′′′, 50mr-draw
36	KRPKP	8/8/8/5PR1/8/2K5/5p2/k7 w - - 0 1	'0-1'	-1	2	188	—	2	—	2	1. Kd4′′′′, f1=Q′′′′, 50mr-draw
37	KRPKQ	8/7R/6K1/8/5P2/8/8/k6q b - - 0 1	'0-1'	1	116	165	—	3	—	2	1. … Qe4+′′ 2. f5′′′′, 50mr-draw

Table 2. Positions p01-p37: example $maxDTM_{50}$ wins and 50mr-draws.[6]

3.2 DTM_{pc} with free-ranging ply-count

With pc being allowed to range over all values 0-99, each endgame will have:

- a $maxDTM_{50,\,pc\geq0}$ and a set of positions and a set of pc-values for which $dtm_{50,pc} = maxDTM_{50,pc\geq0}$,
- 'maximum penalty' positions for which ply-count pc produces max $dtm_{50,pc} - dtm$,
- 'maximum variety' positions for which varying pc gives most values of $dtm_{50,pc}$.

Five 5m endgames have $maxDTM_{50,\,pc=0}$ > maxDTM and sixteen more have $maxDTM_{50,pc\geq0}$ > maxDTM:

- within Figure 1: (1-0) KBPK(P/R), KNNKP, KNPKP, KQPKP, KQRKP wtm, KRPKB, KRPKP wtm, KRPKQ btm, KRPKR and (0-1) K(B/N/P)PKQ, KRPKQ,
- beyond Figure 1: (0-1) KPPKN, KRBKQ and (6 man) KNNKPP.

[6] ′′′′ ≡ absolutely unique value-preserving move; ′′ ≡ unique metric-optimal move; ′ ≡ metric-equi-optimal move

Tables 2 and 3 provide a list of positions[7,8] illustrating 50mr-impact:

- p01-p06: zonal $maxDTM_{50,\,pc=0}$ and $maxDTM_{50,\,pc\geq0}$ positions,
- p07-p12: endgame-specific $maxDTM_{50,\,pc=0}$ and $maxDTM_{50,\,pc\geq0}$ positions,
- p13-p37: '50mr-draw' frustrated wins or saved losses,
- p38-p46: DTM-lengthened wins or losses with $pc = 0$,
- p47-p50: zonal 'maximum *pc*/DTM-penalty' positions,
- p51-p56: endgame-specific 'maximum *pc*/DTM-penalty' positions,
- p57-p60: zonal maximum of the number of *pc*-determined mate-depths $DTM_{50,pc}$,
- p61-p63: endgame-specific maximum number of mate-depths, *nmd*.

			Value		depths in ply						
id	Endgame	FEN	1-0?	5-way	dtc	dtm	dtm50	dtz	dtz50	dtz50′	Notes: annotations to the DTZ_{50}' metric
38	KNNKP	6k1/p7/8/8/7N/7K/2N5/8 w - - 0 1	1-0	2	180	181	223	32	58	58	$dtm_{50} = dtm$ + 42p; $maxDTM_{50,\,pc=0}$ pos.
39	KPPKP	8/8/8/2K4p/4P3/4P3/k7/8 b - - 0 1	1-0	-2	8	248	282	2	2	2	$dtm_{50} = dtm$ + 34p; $maxDTM_{50,\,pc=0}$ pos.
40	KPPKQ	8/4P3/8/8/1q6/3KP3/k7/8 w - - 0 1	1-0	2	1	241	275	1	1	1	$dtm_{50} = dtm$ + 34p; $maxDTM_{50,\,pc=0}$ pos.
41	KQPKQ	8/K7/8/8/7q/4PQ2/8/k7 b - - 0 1	1-0	-2	220	240	274	100	100	100	$dtm_{50} = dtm$ + 34p; $maxDTM_{50,\,pc=0}$ pos.
42	KRPKQ	1K4R1/P7/8/8/8/8/1k6/q7 b - - 0 1	1-0	-2	4	112	114	4	4	4	$dtm_{50} = dtm$ + 2p; $maxDTM_{50,\,pc=0}$ btm pos.
43	KBBKNN	8/8/6n1/8/k3BB2/8/n1K5/8 w - - 0 1	1-0	2	1	133	149	1	55	55	$dtm_{50} = dtm$ + 16p
44	KBBKP	8/8/8/1k6/8/8/p4BB1/3K4 b - - 0 1	0-1	2	1	123	125	1	13	13	$dtm_{50} = dtm$ + 2p
45	KRPKP	8/8/5K2/8/2R2P2/8/6p1/k7 b - - 0 1	0-1	2	1	159	163	1	11	11	$dtm_{50} = dtm$ + 4p
46	KRPKQ	8/4q2R/k5K1/8/5P2/8/8/8 b - - 0 1	0-1	2	113	163	167	3	41	41	$dtm_{50} = dtm$ + 4p
47	KPPKP	8/4P3/8/8/8/2K1P3/k3p3/8 w - - 99 1	1-0	2	1	17	275	1	1	1	max 5m_win *pc*/DTM-cost = 258p
48	KRRKN	k7/3R4/8/8/8/K2R4/8/4n3 w - - 97 1	1-0	2	4	5	81	4	4	4	max 5m_P-less_win pc/DTM-cost = 76p
49	KNNKR	5r2/5N2/8/8/2N5/K1k5/8/8 b - - 98 1	0-1	2	1	3	73	1	1	1	max 2-3m_P-less_win pc/DTM-cost = 70p
50	KRPKQ	8/8/8/8/2K5/4k3/RP6/4q3 b - - 88 1	0-1	2	13	29	191	7	7	7	max 2-3m_win pc/DTM-cost = 162p
51	KBBKN	k7/8/B2B4/8/3K4/8/8/6n1 w - - 90 1	1-0	2	9	11	45	9	9	9	max KBBKN pc/DTM-cost = 34p
52	KNNKP	7k/2K3Np/3N4/8/8/8/8/8 w - - 91 1	1-0	2	17	17	207	8	8	8	max KNNKP pc/DTM-cost = 190p
53	KQPKQ	8/1K6/8/8/q7/5Q2/4P3/k7 w - - 96 1	1-0	2	7	33	275	1	1	1	max KQPKQ pc/DTM-cost = 242p
54	KRBKR	8/8/B7/1R6/8/1K6/6r1/k7 w - - 90 1	1-0	2	9	11	37	9	9	9	max KRBKR (1-0) pc/DTM-cost = 26p
55	KRBKR	5r2/B7/8/8/2R5/1k6/8/K7 b - - 98 1	0-1	2	1	3	57	1	1	1	max KRBKR (0-1) pc/DTM-cost = 54p
56	KBNKNN	8/4B3/4N3/7n/k1K4n/8/8/8 w - - 98 1	1-0	2	1	3	161	1	1	1	max KBNKNN pc/DTM-cost = 158p
57	KNNKP	8/p7/2N3K1/8/8/8/2N1k3/8 w - - *pc* 1	1-0	2	120	121	→223	42	44	44	max sub-6-man *nmd* = 25
58	KQNKR	7N/6rk/8/8/K7/8/8/6Q1 w - - *pc* 1	1-0	2	11	29	→61	11	11	11	max sub-6-man_P-less *nmd* = 12
59	KRNKQ	7N/8/6R1/k7/8/8/K7/2q5 w - - *pc* 1	0-1	-2	8	42	→66	8	8	8	max 2-3m_P-less win *nmd* = 11
60	KRPKQ	8/8/4R3/3K2q1/2P5/8/7k/8 w - - *pc* 1	0-1	-2	58	104	→164	31	31	31	max 2-3m win max *nmd* = 14
61	KBBKN	5n2/1BB5/8/8/8/2K5/8/3k4 w - - *pc* 1	1-0	2	17	37	→51	17	17	17	KBBKN max *nmd* = 8
62	KQPKQ	8/8/8/3P4/6k1/3K2q1/3Q4/8 w - - *pc* 1	1-0	2	99	115	→159	29	29	29	KQPKQ max *nmd* = 13
63	KRBKR	6B1/8/5r2/8/1K6/7R/8/1k6 w - - *pc* 1	1-0	2	23	33	→41	23	23	23	KRBKR max nmd = 5

Table 3. Positions p38-p63: example elongated wins, and maximum penalty/variety positions.

4 SUMMARY AND VIEW FORWARD

The impact of the 50-move rule on sub-6-man chess, including the effect of the ply count *pc*, has been identified. Given a shortage of remaining ply, the winning strategy is more adaptable in some endgames than in others. The 50mr impact increases as the number of men increases, and has been observed in key 6- and 7-man endgames of interest. This impact should be measured as the 50mr, now backed up by a mandatory 75-move rule, truncates chess as experienced over the board, whether played by man or machine. The evolving data files associated with this note (Huntington and Haworth, 2014) provide:

- extended versions of Tables 1-3,
- (annotated) pgn files illustrating various positions and lines mentioned here, and
- a set of spreadsheets of the first author's complete data.

The $maxDTZ_{50}$ figures inferred from these DTM_{50} EGTs confirm previous figures (Tamplin and Haworth, 2004). Some $maxDTM_{50,\,pc}$ positions and lines have been published in an evolving collection of chess records (Haworth, 2014b) which facilitates the comparison of DTM, DTM_{50}, DTZ and DTZ_{50} lines.

[7] The 5-way value-scale is ±2 for unconditional '50mr-wins/losses', ±1 for '50mr-draws' and 0 for unconditional draws.

[8] The list includes a frustrated win but not necessarily a DTM-elongated win for every affected endgame.

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References

Bird, R. (2014). Thinking Functionally with Haskell. CUP. ISBN 978-1107452640.

Bleicher, E. (2015). Nalimov DTM EGT query service. http://www.k4it.de/index.php?topic=egtb&lang=en.

Bourzutschky, M. S., Tamplin, J. A. and Haworth, G. M^cC. (2005). Chess endgames: 6-man data and strategy. *Theoretical Computer Science*, Vol. 349, No. 2, pp. 140-157. ISSN 0304-3975. doi: 10.1016/j.tcs.2005.09.043. http://centaur.reading.ac.uk/4524/.

FIDE (2014). Laws of chess, post 2014-07-01. http://www.fide.com/fide/handbook.html?id=171&view=article.

Haworth, G.M^cC. (2014a). Chess Endgame News. *ICGA Journal*, Vol. 37, No. 1, pp. 25-9, with supporting pgn dataset at http://centaur.reading.ac.uk/37158/.

Haworth, G.M^cC. (2014b). Chess Endgame Records. Dataset at http://centaur.reading.ac.uk/34268/.

Hughes, J. (1990). Why Functional Programming Matters. From *Research Topics in Functional Programming* (ed., Turner, D.A.), pp. 17-42. Addison Wesley, Longman. ISBN 978-0201172362 (1990).

Huntington, G. (2013). Endgame tablebases with the fifty-move rule. http://galen.metapath.org/egtb50/.

Huntington, G. and Haworth, G.M^cC. (2015). Depth to Mate and the 50-Move Rule. *ICGA Journal*, Vol. 38, No. 1, pp. 50-55. This note plus supporting pgn dataset and annotation at http://centaur.reading.ac.uk/39103/.

Hurd, J. and Haworth, G. M^cC. (2010). Data assurance in opaque computations. *Advances in Computer Games 12* (eds., H. J. van den Herik and P. Spronck), LNCS Vol. 6048, pp. 221-231. ISSN 0302-9743, ISBN 978-3-642-12992-6, doi 10.1007/978-3-642-12993-3_20.

'Kronsteen' (2012) http://preview.tinyurl.com/of9gk49. DTM_{50}: a new metric for EGTs.

López de Segura, R. (1561). *Libro de la invencion liberal y arte del juego del axedrez*. Book of the liberal invention and art of the game of chess.

MacIver, D.R. (2015). http://hammerprinciple.com/therighttool/items/haskell/c-2 on the 'Hammer Principle' website: a comparison of HASKELL and C++.

Marlow, S. (ed.) (2010). https://www.haskell.org/onlinereport/haskell2010/. HASKELL 2010, Language Report.

Nalimov, E.V., Haworth, G.M^cC. and Heinz, E.A. (2000). Space-Efficient Indexing of Endgame Tables. *ICGA Journal*, Vol. 23, No. 3, pp. 148-162. ISSN 1389-6911. http://centaur.reading.ac.uk/4562/.

Nalimov, E.V., Haworth, G.M^cC. and Heinz, E.A. (2001). Space-Efficient Indexing of Endgame Databases for Chess. *Advances in Computer Games 9*, (eds. H.J. van den Herik and B. Monien), pp. 93-113. Institute for Knowledge and Agent Technology (IKAT), Maastricht, The Netherlands. ISBN 9-0621-6576-1. http://centaur.reading.ac.uk/4563/.

Peyton-Jones. S. (ed.) (2002). https://www.haskell.org/onlinereport/. HASKELL 98, The Revised Report: language and libraries.

Tamplin, J.A. (2015). 'Nalimov format' DTC/Z/Z_{50} EGT query service. http://chess.jaet.org/endings/.

Tamplin, J.A. and Haworth, G.M^cC. (2004). Chess endgames: data and strategy. In: *Advances in Computer Games: Many Games, Many Challenges* (eds., van den Herik, H. J., Iida, H. and Heinz, E. A.), pp. 81-96. Proceedings of Advances in Computer Games (10). Kluwer Academic Publishers, Norwell MA. http://centaur.reading.ac.uk/4547/.

Wu, R. and Beal, D.F. (2001). Computer Analysis of some Chinese Chess Endgames. *Advances in Computer Games 9*, (eds. H. J. van den Herik and B. Monien), pp. 261-273. Institute for Knowledge and Agent Technology (IKAT), Maastricht, The Netherlands. ISBN 9-0621-6576-1.

Wu, R. and Beal, D.F. (2001). Fast, Memory-Efficient Retrograde Algorithms. *ICGA Journal*, Vol. 24, No. 3, pp. 147-159.