Transreal proof of the existence of universal possible worlds

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Transreal arithmetic is *total*, in the sense that the fundamental operations of addition, subtraction, multiplication and division can be applied to any transreal numbers with the result being a transreal number [1]. In particular division by zero is allowed. It is proved, in [3], that transreal arithmetic is consistent and contains real arithmetic. The entire set of transreal numbers is a *total semantics* that models all of the semantic values, that is truth values, commonly used in logics, such as the classical, dialetheaic, fuzzy and gap values [2]. By virtue of the totality of transreal arithmetic, these logics can be implemented using total, arithmetical functions, specifically operators, whose domain and counterdomain is the entire set of transreal numbers.

Taking Wittgenstein's comments on logical space as a starting point, we develop a mathematically well defined notion of logical space. We begin by defining a Cartesian co-ordinate frame with a countable infinitude of transreal axes. We notionally tie each axis to a distinct, atomic proposition. With this arrangement every point is a distinct possible world whose co-ordinates are the semantic values of its propositions. Furthermore the points composing the whole of this space bijectively map the set of all possible worlds. In other words, each one of all possible worlds is a unique point in this *world space*. This allows us to rigorously apply topology to problems involving all possible worlds, including all logics because these appear in some possible worlds. Thus we provide both a universal metalogic and a foundation for particular universal logics.

We then introduce a more abstract space by taking each point in world space, that is we take each possible world, and use it as an axis in a very high dimensional space of functions. We call a particular subset of this space *proposition space*. In proposition space a given point, that is a given proposition, has, as co-ordinates, its semantic value in each possible world. When we apply mathematical or logical operations in this proposition space we are operating on all possible worlds at the same time.

We use linear transformations to define accessibility relations in world space and to define logical transformations in proposition space. In proposition space we define necessity and possibility as appear in modal logics and we establish a criterion to distinguish whether a proposition is or is not classical. In world space we establish our main result.

We extend standard results of topology to transreal spaces and prove that, in world space, there is a dense set of, at least countably many, hypercyclic, possible worlds that approximate every possible world, arbitrarily closely, by repeated application of a single, linear operator - the *backward shift.* In other words we prove the existence of a countable infinitude of worlds which approximate every possible world by repeated application of a single operator. That is we prove the existence of universal, possible worlds.

Proving existence is useful but it leaves many questions open. We mention just two. Are there any classical, hypercyclic worlds? Is there a construction for any hypercyclic world?

References

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