Scoring rules and competitive behavior in best value construction auctions

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SCORING RULES AND COMPETITIVE BEHAVIOR IN BEST-VALUE CONSTRUCTION AUCTIONS

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Abstract

This paper examines the extent to which engineers can influence the competitive behavior of bidders in Best Value or multi-attribute construction auctions, where both the (dollar) bid and technical non-price criteria are scored according to a scoring rule. From a sample of Spanish construction auctions with a variety of bid scoring rules, it is found that bidders are influenced by the auction rules in significant and predictable ways. The bid score weighting, bid scoring formula and abnormally low bid criterion are variables likely to influence the competitiveness of bidders in terms of both their aggressive/conservative bidding and concentration/dispersion of bids. Revealing the influence of the bid scoring rules and their magnitude on bidders' competitive behavior opens the door for the engineer to condition bidder competitive behavior in such a way as to provide the balance needed to achieve the owner's desired strategic outcomes.

ASCE subject headings: Bids; Construction management; Competition; Contractors

Keywords: Construction auctions; Scoring rule; Capped auctions; Economic bid weighting; Abnormally low bids criterion; Bid scoring formula; Competitive bidding.

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Introduction

Competitive bidding is the regular procurement method for many goods and services. Moreover, the requirement to ensure transparency, publicity and equality of opportunity in public procurement, means that clear procedures have to be followed by bidders (de Boer et al. 2001; Falagario et al. 2012; Panayiotou et al. 2004) to minimize the risk of unfair bias or corruption (Auriol 2006; Celentani and Ganuza 2002; Csáki and Gelléri 2005).

The traditional means of doing this is by the lowest bid auction, which assumes that the lowest (most competitive) bid is the best for the owner and therefore wins the auction (Ioannou and Leu 1993; Waara and Bröchner 2006; Wang et al. 2006). The lowest bid auction method provides the best incentive for cost reduction (Bajari and Tadelis 2001) and dominates both the public and private sectors in the United States (e.g. Art Chaovalitwongse et al. 2012; Shrestha and Pradhananga 2010), European Union (e.g. Bergman and Lundberg 2013; Rocha de Gouveia 2002) and many countries worldwide.

However, despite of its common use, the lowest bid auction method is considered by many to be a recipe for trouble (e.g. Holt et al. 1994a; Latham 1994; Williams 2003), especially when there is little work around and bidders are shaving their bids (Hatush and Skitmore 1998; Ioannou and Leu 1993; Oviedo-Haito et al. 2014). In fact, many previous studies point to the lowest bid often not being best bid in terms of final cost (Dawood 1994; Hatush and Skitmore 1998; Wong et al. 2001), time (Lambropoulos 2007; Shen et al. 2004; Shr and Chen 2003), quality (Asker and Cantillon 2008; Choi and Hartley 1996; Molenaar and Johnson 2003), or risk (Finch 2007).

In the construction sector, selection of the best *price-quality* bid in the form of Best Value auctions, also known as multi-attribute, multi-dimensional or two-envelope auctions (David et al.

2006; Karakaya and Köksalan 2011), has been promoted for a long time (Erickson 1968; Simmonds 1968). In Best Value auctions, bidders' proposals comprise two parts or envelopes: the economic (dollar) bid and the technical proposal, which contains purely non-price features. This way an optimum outcome (Choi and Hartley 1996; Wang et al. 2013) or the best value for money (Holt et al. 1995) is obtained for the owner, as the engineer seeks to maximize benefits for a certain dollar budget.

Traditionally in many countries, the engineer is both the auctioner (the agent who designs the auction rules and decides how the contract is to be awarded) and the auctioneer (the agent that implements the auction rules and awarding process) (Chen 2013). Therefore, the engineer is usually in charge of designing the scoring rules, which enable both the bids and technical proposals to be rated and ranked in order to select the best bidder (Ballesteros-Pérez et al. 2012a, 2012b). The term ‘Bid Scoring Formula (BSF)’ (also named Economic Scoring Formula) is used here to refer to the set of scoring rules that transform a bid into a bid score (Ballesteros-Pérez et al. 2012b; 2015b; 2015c), while ‘Technical Scoring Formula (TSF)’ denotes the set of scoring rules that transform a bidder’s technical proposal into a technical score. Each are then weighted by a respective weighting factor and the sum of the weighted bid score and weighted technical score provides the final overall score that determines the best bidder.

Having clarified this, the aim of this paper is to analyze the relationship between the BSF and competitive bidding behavior by means of a BSF dataset gathered in the Spanish construction industry. This is done by monitoring variations of the BSF subcomponents, called Scoring Parameters, in multiple auctions with similar characteristics.

The paper is divided into six remaining sections. The next section presents a literature review. This is followed by a section detailing the methodological elements needed to analyze the changes in bidding behavior associated with different BSF configurations. The fourth, fifth and sixth sections provide the calculations, results and validation tests. The last section, entitled “Discussion and Conclusions”, closes the paper in providing further insights into the problem analyzed.

Literature Review

The Bid Scoring Formula (BSF) is a mathematical expression that translates bids for an auction into scores. The BSF can also encompass another mathematical expression that determines which bids are abnormal or risky (Abnormally Low Bids Criterion, ALBC) when the engineer wants to set an approximate threshold beyond which bids will be disqualified (Ballesteros-Pérez et al. 2012a, 2012b).

However, despite extensive research on competitive bidding over the years (see Holt (2010) for a recent review), BSF selection remains a relatively poorly researched area. With very few exceptions, such as Dini et al. (2006) and Asker and Cantillon (2008, 2010), little has been done to bridge the gap between the theoretical analysis of scoring rules and their practical application in procurement practice (Bergman and Lundberg 2013). Likewise, abnormal (or unrealistically aggressive bidding) has also received very little attention in the literature to date (Ballesteros-Pérez et al. 2013b, 2015b; Chao and Liou 2007; Hidvégi et al. 2007; Skitmore 2002).

Therefore, very little is known of the relationship between BSFs and bidder behavior. As a result, BSF selection by auctioneers in practice is invariably a highly intuitive and subjective process (Holt et al. 1994a, 1994b) involving few theoretical or empirical considerations. This produces scoring rules that are often poorly designed (Bergman and Lundberg 2013) and affected by internal consistency and validity problems (Borcherding et al. 1991). Likewise, the allocation of weights to the bid and technical components of a proposal (which must be disclosed in the Request For Proposals) are generally based on subjective judgments (Lorentziadis 2010). Fixed criterion weights are often used, therefore, to ensure objectivity and reduce the risk of unfairness and corruption in the evaluation of proposals, providing they accurately reflect the relative importance of the evaluation factors of the engineer (Falagario et al. 2012). However, it is still possible to create an unfair evaluation system in which too much emphasis is placed on particular evaluation factors (Rapcsák et al. 2000) thus favoring, intentionally or otherwise, those bidders that score highly in these corresponding factors (Vickrey 1961).

Hence, at present, there is increasing attention paid to the criteria and weightings used to assess the dollar bids and associated technical proposals (Jennings and Holt 1998; Palaneeswaran and Kumaraswamy 2000). Nevertheless, there is as yet no regular prevailing method for assessing dollar bids or technical proposals for Best Value. Engineers frequently use the same BSF for all projects, but different engineers generally favor different BSFs (Ioannou and Leu 1993; Rocha de Gouveia 2002).

The European Union has addressed this issue (Bergman and Lundberg 2013; Rocha de Gouveia 2002), and the dubious actions taken by overly aggressive bidders to recover their subsequent losses – a recurring theme in the theoretical literature from as long ago as 1971 (Capen et al. 1971). In 1993, the European Union stated that quality was as important as price (European Union 2002), incorporating this into Directive 93/97/EEC which, for the first time, allowed an auction to be awarded to the Best Value bidder (Rocha de Gouveia 2002). Nevertheless, only since 1999 have clear recommendations been made for a more methodical, consistent and auditable appraisal of auctions to meet the Best Value criterion (Carter and Stevens 2007; Rocha de Gouveia 2002). These aim to remedy the shortcomings of the traditional lowest bid criterion by discouraging the undesirable effects of unrealistic or abnormally aggressive bids on the industry (Conti and Naldi 2008; Crowley and Hancher 1995).

However, the difficulty for researchers is that longitudinal data concerning bids and profit from individual bidders are limited due to confidentiality and competitive issues. Therefore, empirical analysis has been severely restricted to a small number of cases (Vanpoucke et al. 2014), the main conclusion to date being that the decision to bid aggressively or conservatively is very "complex" (Carter and Stevens 2007).

Hence, despite the current number of theoretical models from the economic theory of auctions, there is still a lack of fieldwork concerning the extent to which engineers are able to influence bidder competitiveness. The difficulties in obtaining appropriate data generally prevent any convincing conclusions to be reached. However, the use of Best Value auctions calls for the implementation of scoring rules in which both bid and technical criteria are involved. This situation provides an opportunity to examine how the responses of bidders change under a variety of scoring auction rule configurations. This is the point of departure of this research, which aimed to shed more light on this complex issue by examining evidence of the effect of different BSFs on bidder competitiveness.

Materials and Methods

Methodology Outline

Before studying how economic auction rules affect bidding competitiveness, it is necessary to state the problem in a way that will allow an effective analysis. First, an auction X is taken to exhibit a higher level of bidding aggressiveness compared to an auction Y when these two conditions occur simultaneously:

1. The *average bid* for auction X is proportionally lower than its *estimated cost* than for auction Y.

2. The *lowest bid* for auction X is proportionally lower than its *average bid* than for auction Y.

This means that, when comparing the results of two auctions X and Y of different economic sizes (e.g., different average bid values), the only way to be certain that X is more competitive than Y (i.e., X evidences more aggressive bidding) is by knowing that the ratio of their respective bid average and estimated cost is lower for auction X *and* the ratio between the lowest bid and the average bid is also lower for X. Fulfilling only one of the conditions – such as one auction having a proportionally lower average bid with the other having a proportionally lower lowest bid - makes it uncertain which is more competitive.

On the other hand, an auction X is defined as having a higher level of bid dispersion compared to auction Y if the following three conditions occur simultaneously:

1. the lowest bid is proportionally lower in auction X than in auction Y,
2. the highest bid is proportionally higher in auction X than in auction Y, and
3. the bid standard deviation is proportionally higher in auction X than in auction Y.

This case is easier to understand, since an auction X will inevitably have a higher bid dispersion – equivalent to a lower bid concentration – compared to an auction Y, which might also have a different economic size, when the relative proportional distances between the highest bid/average bid, the average bid/lowest bid and the bid standard deviation/average bid are simultaneously higher in auction X.

Therefore, the variations of the relative values of estimated cost, bid average, lowest bid, highest bid and bid standard deviation are the key variables to be monitored. These are named here Scoring Parameters, since they coincide with the variables usually found in BSFs. For instance, examples of BSFs commonly found in practice are:

$$S_i = \frac{b_{\max} - b_i}{b_{\max} - b_{\min}} \qquad S_i = \frac{b_{\min}}{b_i} \qquad S_i = \frac{b_m + 3s - b_i}{6s}$$

Where S_i is the bid score (expressed on a scale of 0 to 1) produced by bidder i's bid (b_i) in an auction, where $b_{\min}$, b_m, $b_{\max}$ and s are the minimum bid (lowest bid), the average (mean) bid, the maximum (highest) bid and the bid standard deviation respectively of an auction (see "Notation List").

Scoring Rules Dataset

The dataset analyzed comprises 124 auction specification documents with 47 different groups of BSFs and ALBC for different Spanish owners, and enough auction data to enable a first quantitative analysis to be made. This is displayed in Table 1 and the terminology used will be explained later. The data are quite representative of the Spanish bidding system, as they comprise auctions from public authorities (city councils, local councils, semi-public entities, universities, ministries, etc.) and private companies.

The dataset spans 5 years. Ideally, a good dataset should comprise as many auctions as possible within the shortest time. However, in order to be representative of the wide variety of scoring rules applied by many organizations, many of which are national bodies and do not regularly conduct construction auctions, it has been necessary to extend this time to 5 years (2003-2008). The period chosen seems to be in line with other similar auction datasets; for example, a very recent study making use of twelve international auction datasets for modeling the number of bidders in construction auctions (Ballesteros-Pérez et al. 2015a) spanning from 2 to 10 years, making our 5-year scoring rule dataset length quite reasonable. Spain enjoyed a period of economic prosperity from approximately 1997 to 2008 and hence the dataset is not expected to be influenced by a volatile market. As is seen later in the "Test of the Model" section, as soon as market conditions change, the bidders' behavior also gradually changes too. Seven more Spanish auctions from 2009 and 2010 – a period in which the European Union and

Spanish economic recession began – are compared to the model developed for the first 124 auctions, showing that bidders in an economic downturn tend to be more aggressive in situation of work scarcity.

The 124-auction dataset comprises a wide range of civil works (irrigation systems, desalination and waste water treatment plants, drinking water treatment stations and water supply systems, sewage lines and pumping stations, libraries, landfill sites, and small road networks) together with operation and maintenance services (dams, airports, touristic beaches, waste management, cinema studios, hospitals, seaports, amusement parks, university technological equipment) all involving construction or reconstruction activities to some extent. The more recent seven-auction dataset comprises buildings and hydraulic civil work auctions.

Terminology

For the sake of clarity, several terms used later are defined first. Each group of *n* auctions under the 47 different combinations of BSFs and ALBC in the 124 dataset is classified as what are called ‘capped tenders’ (in British English) or ‘capped auctions’ (in American English). In this form of auction, the engineer sets an upper bid limit (*A*) (sometimes also called ceiling price), which is stated in the auction specifications and against which bidders must underbid. That is, in capped auctions, bidders offer a ‘drop’ (d_i) from the bid limit (*A*). The relationship between the monetary bids (b_i) and drops (d_i) in these auctions is straightforward as

$$d_i = 1 - \frac{b_i}{A} \tag{1}$$

Therefore, in capped auctions, bids can be equally analyzed as monetary bids (b_i ranging from 0 to *A*) or as drops (d_i ranging from 0 to 1 or, equally, from 0% to 100%). In uncapped auctions – auctions in which the engineer does not set a maximum or a minimum price and in which bidders can freely submit the bids they want – the bids can only be expressed as monetary bids (b_i), since there is no set limit from which calculate the drop.

It is quite usual that some countries use the capped bidding approach while others resort to the uncapped approach. However there is a large number of countries that adopt both approaches depending on their respective traditions, preferences or specific needs (Ballesteros-Pérez et al. 2010). In this case, capped bidding is used more frequently whenever there is a previous and well-developed project that clearly defines the scope of the works to be carried out. On the other hand, when the request for proposals invites the bidders to submit a bid for the design, build and sometimes the operation of the works auctioned, it is often more convenient to resort to uncapped bidding since the scope of work is less defined.

Here, for the comparison of bids in different auctions with different initial upper limits (A), it is preferable to use drops rather than monetary-based bids, although the results are not expected to be different for uncapped auctions. Using drops always also has the advantage of involving the same 0 to 1 scale for analyzing the scoring parameter variations and therefore also range from 0 to 1 when expressed in drops, since the bidders' drops (d_i) themselves also range within that interval of variation (Ballesteros-Pérez et al. 2014). Therefore, the Scoring Parameters of mean bid, maximum bid, minimum bid and bid standard deviation can be expressed either in monetary-based values (b_m, b_{max}, b_{min} and s, ranging from 0 to A) or in their respective drop-based version in capped auctions (d_m, d_{min}, d_{max} and σ, ranging from 0 to 1 and obtained replacing the b_m, b_{min}, b_{max} and s values respectively in Equation 1 when the auction maximum price limit A has been set).

Furthermore, there are four aspects of scoring methods that can be analyzed (Ballesteros-Pérez et al. 2015c): (a) the way the bid score is calculated (BSF); (b) the way the technical score is calculated (TSF); (c) the way the weights the bid and the technical scores are set; and (d) how the ALBC is defined. Since this paper only focuses the on the bid score, (b) is ruled out, and the three main variables become the BSF, bid score weighting and ALBC. Table 1 shows these three variables for the dataset under study. From right to left these are the Bid Scoring Formulas (BSF), ALBC width (t_k), and bid weighting (w_k). The latter represents the weight of the bid score (with $0 \leq w_k \leq 1$) versus the technical score (which generally equals $1-w_k$) in a multi-attribute or Best Value auction. The former is related to the unique generic mathematical expression of ALBC found in the dataset, which is $b_{abn} = (1-t_k)b_m$ (in monetary bids) or, alternatively, $d_{abn} = 1-(1-t_k)(1-d_m)$ (when expressed in drops by means of replacing in the former variables b_m and b_{abn} by $(1-d_m)A$ and $(1-d_{abn})A$ respectively according to Equation 1). This is the most common mathematical expression in use in European Union countries for setting a cut-off limit beyond which all bids are ineligible. The variable b_{abn} (d_{abn}) denotes the abnormal bid (drop) threshold value below (above) which every bid b_i (d_i) is disqualified; whereas variable t_k (ALBC width) is a parameter set by the engineer for a BSF in many ways –Belgium, France, Italy and Spain, for example, use ranges mostly varying between t_k=0.10 and 0.15) (European Union 1999). As will be seen later, both w_k and t_k variables are important parameters for promoting bidding competitiveness.

Scoring Parameter Relationships

The bid scoring rules comprise, in addition to the weighting factor, two mathematical expressions: (1) the Bid Scoring Formula (BSF), which are expressions similar to the ones shown in Table 1 formulated as a function of bidder *i*'s bid b_i (or d_i when expressed in drops) and generally with at least one or more Scoring Parameters (b_m, b_{max}, b_{min} and *s*, in monetary bids, or, analogously, in drops, d_m, d_{min}, d_{max} and σ, respectively); and (2) the Abnormally Low Bids Criteria (ALBC) which are the mathematical expression of a cut-off limit beyond which, any bid b_i, or its equivalent drop d_i, are no longer eligible. The first converts the bids b_i (or d_i) into scores, whereas the ALBC determines which bids are ex-ante ineligible as being too cheap or too expensive.

Now, the mathematical expressions of almost all BSFs and ALBC are defined by a combination of one or more Scoring Parameters (SP): b_m, b_{max}, b_{min} and s, or d_m, d_{min}, d_{max} and σ (Ballesteros-Pérez et al. 2015c), which are variables that are only known after the auction has taken place and the price bids are known. Hence, these SP constitute, at the same time, a descriptive set of auction bid statistics (average, minimum, maximum and standard deviation) to calculate the bidders' scores.

Therefore, if the variations of these individual SP can be traced with respect to the BSF and ALBC settings, it is possible to identify when an auction is more aggressive/conservative and more concentrated/dispersed. For example, translating what was said in the "Methodology Outline", an auction X is more aggressive than another auction Y when the ratios b_o/b_m (equivalent to d_m/d_o) and b_{min}/b_m (equivalent to d_{max}/d_m) are lower for auction X, where b_o and d_o are the estimated cost of the auction expressed in money or drops, respectively. Analogously, an auction X evidences a higher level of bid dispersion when these three ratios: b_{min}/b_m, b_{max}/b_m and s/b_m (or equivalently in drops d_{max}/d_m, d_{min}/d_m and σ/d_m) are larger in auction X compared to auction Y.

The problem is that these SP ratios do not follow a linear relationship, because the SP variation itself is not generally linear either; thus, its relative variations must be carefully measured and compared. This is the aim of the present section, describing the major features of the SP and how they are interconnected with each other, so their relative variations can be properly registered and used later for linking them to more aggressive/conservative bidding behavior and to a higher concentration/dispersion of bids.

Therefore, as noted above, in both uncapped and capped auctions, the Scoring Parameters have particular mathematical relationships with each other; however, from now on, only SP relationships expressed in drops will be considered. These relationships are described and justified in Ballesteros-Pérez et al. (2012a, 2013a, 2015c) and, when they are expressed as a function of the scoring parameter mean drop (d_m), they are as described in the first column of Figure 1. As can be seen, each of these expressions is known when the respective 'regression coefficients' (λ, α, β and γ, respectively by rows) is determined.

Specifically, these four regression coefficients have the following meanings:

- λ relates the estimated cost (d_o) to the mean bid (d_m) when expressed in drops. The larger this coefficient is, the larger the mean drop will be compared to the estimated cost (aggressive bidding); whereas the smaller is λ, the mean drop will also be smaller (more conservative bidding).
- α relates the mean bid (d_m) to the maximum drop (d_{max}). The larger this coefficient is in a particular auction, the closer is d_{max} to d_m , meaning more conservative bidding. We therefore use '$-\alpha$' instead of '$+\alpha$', because '$-\alpha$' will be read the same way as λ is read (the larger $-\alpha$ denoting more aggressive bidding). This coefficient also indirectly means the concentration/dispersion of bids, since the distance between the lowest and the average value of bids indicates how dispersed the bids are.
- β is a very similar coefficient to '$-\alpha$', sharing the same mathematical expression, but relating the highest bid (lowest or minimum drop d_{min}) to d_m. The larger β is, the further d_{min} will be

located from d_m and *vice versa*. Thus, this coefficient allows analysis of the concentration (with small β values) or dispersion (with large β values) of a bids in the same way as coefficient α.

- γ connects the bids standard deviation (σ) with the mean bid (d_m), but is expressed in drops. Again, the bigger is γ, the greater is the dispersion of bids.

The expressions for calculating the 'regression coefficient averages' ($\bar{\lambda}$, $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$) are shown in the second column of Figure 1; further details and justification of the regression coefficient mathematical expressions can be found in Ballesteros-Pérez et al (2015c). These expressions are formulated as a function of the scoring parameter values obtained for the number of *n* auctions in Table 1 (the complete auction data having not been displayed for the sake of brevity), which share the same BSF description (coded as ID in Table 1). The 'regression coefficient averages', however, are presented in the last four columns of Table 3, while a numerical example is also given in Table 2.

The third and last column in Figure 1 displays how each regression coefficient average potential value is associated with different levels of bidding aggressiveness and/or dispersion. In particular, each graph represents how different intervals of the regression coefficient values produce different curves. These indicate how the relative distances or ratios between d_o, d_{max}, d_{min} or σ, respectively, to d_m, evolve. Table 2 shows a numerical example detailing how the four average regression coefficients are calculated according to the second column of Figure 1 for a particular BSF (BSF *ID=1* from Table 1) with two auctions (*n*=2).

All the variables used in Table 2 have been introduced above with, as noted earlier, d_o corresponding to the estimated cost for each auction expressed in drops. This value was given by the same bidder for each of the 124 auctions, i.e., unlike d_{max}, d_m, d_{min} and σ, it cannot be derived from the list of bids submitted by the bidders in each auction.

In short, these ‘regression coefficient averages’ are important as they are the variables whose variations allow the comparisons between pairs of scoring parameters, which allows us to compare more aggressive with more conservative bidding (and more dispersed bids with more concentrated bids), for different auctions with different BSFs as stated in the “Methodology Outline” sub-section.

Hypotheses

The strategy is to study how different BSF features affect the ‘regression coefficient average’ values of $\bar{\lambda}$, $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$. In doing this, coefficient $\bar{\alpha}$ will be replaced by $-\bar{\alpha}$, since this better aligns its direction of variation with the rest of scoring parameters.

The central block in Table 3 (second to fourth columns) presents the three variables most influential on the regression coefficient averages: the bid weighting (w_k), ALBC width (t_k) and the BSF (simplified by its gradient g_k) (Ballesteros-Pérez et al. 2015c). As explained earlier, the value of w_k indicates the importance of the bid (S_i) relative to the technical proposal (T_i). It ranges from 0 (when the engineer is only interested in the technical proposal) to 1 (when the engineer is only interested in the bid value: an auction where the only selection mechanism is the highest drop or lowest bid). When $0 < w_k < 1$, the proposals are evaluated according to a mixture of economic (bid) and technical criteria.

The ALBC width is a measurement of how narrow the cut-off for unrealistic ineligible bids is in terms of relative distance, t_k, from the mean drop d_m. Usual values found for this variable in European Union countries range from 0.04 to 0.25 whenever an ALBC is implemented. Otherwise, when there is no ALBC ($\nexists\, t_k$), t_k is considered as 1 (cut-off always at zero).

Finally, the BSF gradient is concerned with the bidders' perception of how quickly they score reduces as a function of how far apart they are from the best-scored bid (theoretically from the first ranked bidder, see last column of Figure 2). This is easily visualized by plotting the S_i curve for an auction. However, the interest is really in the shape of the curve: (1) a concave curve indicating the bid score-reduction is larger near the best bid; (2) a convex curve indicating the bid score reduction is smaller near the best bid; and (3) a linear curve indicating the bid score reduction is constant no matter what the distance to the best bid.

The expectation now is that, with a higher bid score weighting (w_k), bidders will bid lower (with bigger drops) in order to win the auction as they have less possibility of gaining any advantage through having a superior technical proposal. Similarly, when the ALBC width is wide (larger values of t_k) and excludes very few bidders, bidder behavior is expected to be more aggressive since there is less chance of being disqualified for bidding too low. Analogously, concerning the BSF gradient, bidders whose d_i values are close to the maximum drop d_{max}, are more likely to compete strongly whenever they feel that their score will be reduced even though their bids are quite similar; this only happens with concave BSF gradients. This increased bidding aggressiveness for auctions with a specific combination of w_k , t_k and g_k values will therefore be demonstrated for a set of auctions if the $\bar{\lambda}$ and $-\bar{\alpha}$ values are larger than for auctions with different w_k , t_k and g_k values.

Calculations

In order to validate and measure the extent to which conservative-aggressive bidding is actually influenced by the three independent variables of bid score weighting w_k (now X_1), ALBC width t_k (now X_2), and BSF gradient (now X_3), that is, to what extent different values of X_1, X_2 and X_3

can alter the values of $\bar{\lambda}$, $-\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$, four multiple linear regression analyses are carried out (one for each 'regression coefficient average': $\bar{\lambda}$, $-\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$, as a function of the three independent variables X_1, X_2 and X_3 identified above). The aim of this approach is to determine if the regression coefficient averages ($\bar{\lambda}$, $-\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$, now dependent variables Y_1, Y_2, Y_3 and Y_4, respectively) are actually conditioned by the three variables X_1, X_2 and X_3, whose test results of their interdependence will be presented later in Figure 3 as Covariation and Correlation matrices.

To do this, we use a simple trichotomic scoring (-1, 0, +1) as in Figure 2, according to particular pre-set levels by rows of the three independent variables involved. In particular, possible values of independent variable X_1 (w_k) are divided into three equally wide intervals each of which depicts the situation of a bid up to 33.3%, 66.7% and 100.0% respectively of the overall score (technical + bid) since this variable can range from 0 to 100%. Independent variable X_2 (t_k) variation is divided again into three intervals. In this case however, despite t_k also theoretically ranging from 0 to 1, the usual values implemented in European Union countries range from 0.00 to 0.25 as noted above, so it was found preferable to adapt the three intervals to the most common range of actual t_k values found in practice (t_k up to 0.05, 0.15 and 1.00). Finally, independent variable X_3 (g_k) was directly classified according to the three only possible shapes the BSF curve can have: concave, convex or constant (linear).

This way, according to the three main column values shown in the second and central block of Table 3, the trichotomic scoring for variables X_1, X_2 and X_3 can be assigned according to the three levels from Figure 2, whereas the results of this assignment to the three independent variables w_k, t_k and g_k is shown on the right block of Table 3 in columns '*X₁*', '*X₂*' and '*X₃*', respectively.

Analogously, the regression coefficient average values for $\bar{\lambda}$, $-\overline{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$, are shown on the right block of Table 3 in columns Y_1, Y_2, Y_3 and Y_4, respectively. These are calculated according to the expressions shown in Figure 1 (column 'Calculation') and as exemplified in Table 2 for each different set of *n* auctions with the same ID from Table 1.

Independent variables w_k and t_k are ratios from 0 to 1 and, therefore, they could be used as continuous variables. However, the four multivariate analyses performed here opted instead for three-level categorical variables. The reason is that preliminary analyses (not included here due to lack of space) indicated *non-linear* contributions of w_k and t_k. Unfortunately, non-linear analyses usually require far more data when the contribution of each independent variable is still to be researched, and the present dataset is not extensive enough to allow such an extensive analysis. However, the adopted three-level system equally allows two important aspects to be analyzed: the degree of contribution of each independent variable (w_k, t_k and g_k) as well as the direction in which each variable influences bidding behavior. Both facets are of primary importance in providing the first set of results and concluding where future research is still required.

Results

The results of the four regression analyses performed – one for each dependent variable, that is, Y_1 (coefficient $\bar{\lambda}$), Y_2 (coefficient $-\overline{\alpha}$), Y_3 (coefficient $\bar{\beta}$) and Y_4 (coefficient $\bar{\beta}$) – are shown in Figure 3 arrayed horizontally, along with other intermediate calculations. However, the most representative results are the coefficients of determination (R^2) and significance tests for each Y_i's multiple linear regression coefficient (M_i), both checked as a group (M_0 to M_3 together passing the F-Fisher test) and individually (each M_i passing the Student t-test). The covariance and correlation matrices are also provided at the bottom of Figure 3.

Summarizing the results of Figure 3, four major conclusions can be stated. First, all the coefficients of determination (R^2) in Figure 3 are large enough to indicate that there is a moderate or high degree of correlation between the independent variables selected ($X_1 = w_k$, $X_2 = t_k$ and $X_3 = g_k$) and each of the dependent variables ($Y_1 = \overline{\lambda}$, $Y_2 = -\overline{\alpha}$, $Y_3 = \overline{\beta}$ and $Y_4 = \overline{\gamma}$). This means that the bid score weighting (w_k), ALBC width (t_k) and BSF gradient (g_k) are correctly identified as significant and influential variables.

Second, the multiple linear regression coefficient values M_1, M_2 and M_3 (but for the coefficient M_3 when relating '$Y_3 = \overline{\gamma}$') are positive, meaning that Figure 2 is therefore correctly ordered, i.e., from the scenario where bidders' bid more aggressively and more dispersed in the top row (row with scoring +1), to more conservative bidding with more concentration in the bid values in the bottom row (row with scoring -1).

Third, the covariance and correlation factors found in the covariance and correlation matrices outside the diagonal between the independent variables ($X_1 = w_k$, $X_2 = t_k$ and $X_3 = g_k$) themselves are generally small. The only exception is the comparatively larger 0.271 correlation between independent variables X_1 and X_3. This significant, but still moderately weak, correlation originates when auctioners implement BSF for a Best Value or multi-attribute auction and they have the common habit of using high bid score weightings (X_1=+1) along with concave BSF gradients (X_3=+1), as well as low bid score weightings (X_1=-1) with convex BSF gradients (X_3=+1); the first combination promotes bidding aggressiveness, whereas the second promotes bidding conservativeness. Nevertheless, the relatively small correlation factors suggests that, even though there is some combined effect of the three independent variables, they are expected to be minor, i.e., every variable depicts a relatively independent single component that affects bidding behavior.

Conversely, it is worth highlighting that the regression analysis found the linearity assumption to be reasonably satisfied. However, as noted above, this was not necessarily because the correlations among variables analyzed behave linearly. The data has been organized into a three-level ordinal scale that does not provide any information for the possible development of underlying mathematical functions that might have been identified by working with continuous variables in a larger BSF database. This issue remains in need of further research.

Fourth, Figure 4 shows the Q-Q plots of the standardized residuals for the four multiple linear regression analyses. As can be easily seen, most data fit a straight line, indicating that the residuals follow approximately a Normal distribution.

Finally, the last step was to carry out an Analysis of Variance (ANOVA) – summarized in Figure 5 – to test if the multiple regression linear coefficients 'M_i' values were significantly different from each other in order to rank the three independent variables ($X_1 = w_k$, $X_2 = t_k$ and $X_3 = g_k$) by decreasing the order of importance. Initially, inspection of the coefficients M_1's, M_2's and M_3's values in Figure 3 revealed that $M_1 > M_2 > M_3$ for Y_1 and Y_2, and that $M_2 > M_1 > M_3$ for Y_3 and Y_4, so the bid score weighting and ALBC width may be equally important, but both having more influence when compared to the BSF gradient.

In particular, an ANOVA was carried out by studying the Fisher's Least Significant Difference (LSD) intervals, which is a statistical method for comparing the means of several variables and does not require correction for multiple comparisons. The main results of this analysis are shown in Figure 5.

The major results from the ANOVA also indicated that both the bid score weighting and ALBC width are almost always more important than the BSF gradient (their Fisher LSD intervals rarely intersect), whereas the bid score weighting was not always more influential than the ALBC width (since their Fisher's LSD intervals are partially overlapped for most *Y* variables). Therefore, the results of this latter analysis confirm that the variables bid score weighting, ALBC width and BSF gradient are already ranked in decreasing order of importance, but the first two almost always have a quite similar influence on bidder behavior.

Summarizing, as said in the "Hypotheses", the expectation was that the higher the bid scoring weighting ($X_1 = w_k$), the lower the bidders would bid, as they would have had less possibility of gaining any advantage through having a superior technical proposal. Similarly, when the ALBC is lenient (because it excludes very few bidders by a very large or even non-existent $X_2 = t_k$ value), bidder behavior was expected to be more aggressive since there is less chance of being disqualified for bidding too low. Analogously, it was claimed that bidders who are close to the lowest (maximum drop) would be more likely to compete strongly with concave BSF curves as they would feel that their score might be reduced even though their bids are quite similar.

Hence, for example, it can be seen that BSF ID=6 from Table 1, with all the trichotomic variables set at -1 (low w_k, narrow t_k and convex g_k), causes a higher level of bidding conservativeness and bid concentration as demonstrated by its small Y values from Table 3. Conversely, the traditional lowest-wins auction with no ALBC ($\nexists t_k$), which is perfectly concave and is actually represented by BSF ID=36 in Table 1, produces on average the largest $\bar{\lambda}$, $-\overline{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ values in Table 3. That is, it generates the highest bidding aggressiveness and bid dispersion. This accords well with the literature concerning traditional bidding and the very *raison d'être* for the introduction of BSF and non-price features in general.

Test of the Model

For an additional check, several more recent auctions were gathered from the same country (Spain) where the original auctions for developing the Multiple Linear Regression Analysis were collected. This new sub-dataset comprises a total of seven buildings and hydraulic civil work auctions from years 2009 and 2010 grouped under three sets of auctions with common BSF features in each of the three groups. Results of actual versus estimated $\overline{\lambda}$, $-\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$ values by using M_0, M_1, M_2 and M_3 values according to Figure 3 (left column) are presented in Table 4.

As can be seen, per-unit deviations between actual and estimated values generally remain below 10%. However, there are two exceptions for $\overline{\lambda}$ (the regression parameter that specifies the linear relationship between d_o and d_m) with deviations up to 20%. It must be noted however, that years 2009 and 2010 were the first officially considered in the economic recession in Spain; hence, it is expected that with equivalent cost estimates (d_o) the bidders bid more aggressively (lower mean bids, d_m) compared to the previous period of 2003-2008. However, these deviations were found only for the dependent variable Y_1 ($\overline{\lambda}$), not for the other three ($-\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$). Therefore, overall, it can be considered as a highly satisfactory result.

Discussion and Conclusions

There are many scoring formulas currently in use for evaluating bid proposals in Best Value auctions. These affect bidder conservativeness-aggressiveness in profound ways but their design in practice is invariably a highly intuitive process, involving few theoretical or empirical considerations. To date, the vast literature of theoretical competitive models has relied almost exclusively on a combination of the foundational axioms of economics and intuition together with scarce experimental results that many perceive as being of uncertain veracity. The contribution here adds to the relatively tiny amount of complementary field studies in this area, providing some confidence in the theoretical developments so far.

In this paper, an analysis aimed at bridging this gap through the empirical study of a sample of 131 Spanish procurement auctions is provided in order to establish the changes in bidding competitiveness that occur, at least partially, in response to the mathematical scoring rule chosen by the engineer in the auction specifications. In doing this, three major variables are hypothesized as being likely to influence the competitiveness of bidders in terms of both their aggressive/conservative bidding and concentration/dispersion of their bids. These variables are the bid score weighting (how relatively important is the bid in contrast with the technical proposal), the ALBC measured by its width (how narrow is the cut-off that sets a threshold beyond which a bid is disqualified), and the BSF measured by its gradient (the concavity, linearity or convexity of the scoring curve that makes bidders realize how quickly their score decreases the more they exceed the lowest bid). For example, aggressive bidding is expected to occur with a high bid score weighting (hardly any non-price features allowed), no abnormal bid detection and a concave scoring curve. From this, it is easy to show that the traditional lowest-wins auction prompts the most aggressive behavior from bidders and, hence, all the negative outcomes associated with aggressive bidding.

In terms of industry practice, the findings concern both the bidders and the entities that design and/or eventually award the auctions. On one hand, bidders can benefit from understanding how different BSF and ALBC mathematical configurations force them to submit more competitive price bids, that is, to renounce to higher profits for the sake of obtaining higher scores. Indeed, bidders who understand these effects even before their first bidding experience might gain a clear competitive edge over their rivals.

On the other hand, the findings of the research indicate the potential for individual engineers or owners to control the aggressiveness of bidders' bids to a level that strikes a desired balance between the monetary costs of under-competitiveness and the increased risk of problems associated with over-competitiveness. Previous research into optimal auction design is far from incorporating such practical issues as non-price features, unrealistic bid detection and actual individual auctioneer risk preferences. The conceptual framework developed in this paper, therefore, offers a potential means of doing this through the design of enhanced scoring formulas for individual engineers. In its present form, however, the analysis is restricted to providing a general qualitative configuration. The next logical step is the development of a quantitative means of determining how small variations in the BSF mathematical expressions might affect the level of bidder aggressiveness and bid dispersion for a future Best Value auction. This could be done, for example, by unbalancing the importance of the bid versus the technical proposal, adjusting the ALBC width or just by implementing BSF curves with different levels of concavity/convexity. All this is with the intention of promoting an equilibrium between competitiveness and risk among bidders' bids, since in public construction contract auctions, for instance, both practitioners and researchers are aware that overly conservative bidding tends to waste public funds (i.e., a situation in which bidders make unreasonably high profits when winning the auction), whereas overly aggressive bidding causes problems such as poor quality, prolonged construction duration and 'false economy', that are said to ruin the health of the entire industry in the long run (Drew and Skitmore 1997; Flanagan et al. 2007).

For future empirical research, the analysis needs to be repeated in other contexts in order to study whether the importance, and the order of importance, of the three variables identified influence bidder behavior to the same extent, regardless of other uncontrolled variables. Also needed is an examination of the indirect effects of scoring technical proposals. For instance, recent empirical studies have found that, whenever the score for technical proposals is increased, bidders are encouraged to be more innovative and hence more focused on cost savings (Pellicer et al. 2014), an issue that may also eventually be reflected in the monetary component of the auction. In addition, analysis of a much larger dataset would help measure quantitatively, and with higher accuracy, how the particular configuration of scoring rules influences bidder behavior in other industries.

Notation List

The following variables are used in this paper.

A	Maximum price possible to be submitted in a capped tender/auction
b_{abn}	Abnormal bid threshold (expressed in money)
b_i	Bidder *i*'s bid (expressed in money)
b_m	Mean (average) bid (expressed in money)
b_{max}	Maximum (highest) bid (expressed in money)
b_{min}	Minimum (lowest) bid (expressed in money)
b_o	Estimated cost, expressed in bid (in money)
d_{abn}	Abnormal drop threshold (expressed in /1)
d_i	Bidder *i*'s drop (expressed in /1)
d_m	Mean drop (average bid) (expressed in /1)
d_{max}	Maximum drop (lowest bid) (expressed in /1)
d_{min}	Minimum drop (highest bid) (expressed in /1)
d_o	Estimated cost, expressed in drop (in /1)

g_k Bid Scoring Formula curve gradient in auctions with the same BSF *ID* and converted into a X_3 later (in trichotomic score)

$M_0 \ldots M_3$ Multiple linear regression coefficients relating X_1, X_2 and X_3 with each of the four Y_1, Y_2, Y_3 and Y_4 independent variables.

n Number of auctions with the same combination with the same BSF and ALBC and engineer

s Bid standard deviation (expressed in money)

S_i Score awarded to bidder i as a function of b_i or d_i (expressed in /1)

T_i Score awarded to bidder i as a function of its Technical proposal (in /1)

t_k Abnormally low bids criterion (ALBC) width in auctions with the same BSF *ID* (expressed in /1) and converted into a X_2 later (in trichotomic score)

w_k Bid score weighting in auctions with the same BSF *ID* (expressed in /1) and converted into a X_1 later (in trichotomic score)

α Regression parameter that specifies the parabolic relationship between d_{max} and d_m in drops (or b_{min} and b_m in bids)

$\bar{\alpha}$ Average of the n values of α with the same ID (k value), renamed later as $-Y_2$

β Regression parameter that specifies the parabolic relationship between d_{min} and d_m in drops (or b_{max} and b_m in bids)

$\bar{\beta}$ Average of the n values of β with the same *ID* (k value), renamed later as Y_3

γ Regression parameter that specifies the mathematical relationship between σ and d_m in drops (or s and b_m in bids)

$\bar{\gamma}$ Average of the n values of γ with the same *ID* (k value), renamed later as Y_4

λ Regression parameter that specifies the linear relationship between d_o and d_m in drops (or b_o and b_m in bids)

$\overline{\lambda}$ Average of the *n* values of λ with the same *ID* (*k* value), renamed later as Y_1

σ Drop standard deviation (expressed in /1)

Standard statistical variables, such the ones used in Figures 3 and 5 (e.g. R^2, *SE*, *F*, *t*, *df*), are not displayed.

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ID	n	w_k	t_k	BSF description
1	2	0.50	$\nexists t_k$	$S_i = 1 \; if \; d_i > 0.9d_m + 0.1$ $S_i = 0.9 \; if \; 0.9d_m + 0.1 \geq d_i > d_m$ $S_i = 0.8 \; if \; d_m \geq d_i > 1.1d_m - 0.1$ $S_i = 0 \; if \; 1.1d_m - 0.1 \geq d_i > 0$
2	2	0.40	0.10	$S_i = 0.99 + 2(d_i - d_m) -$ $-1.8\lvert d_i - d_m \rvert$ $-0.2\lvert 0.05 + d_m - d_i \rvert$
3	45	0.45	0.05	$S_i = 1 - \frac{d_{max} - d_i}{1 - d_{max}}$
4	1	0.50	0.05	$S_i = 1 \; if \; d_i > d_m$ $S_i = 1 - 0.02(d_i - d_m)$ $if \; d_m \geq d_i > d_m - 0.05$ $S_i = 0.8(d_m - d_{min} - 0.05)/(d_i - d_{min})$ $if \; d_m - 0.05 \geq d_i > d_{min}$
5	2	0.30	0.06	$S_i = 1 - \frac{0.6}{1 - d_m}\left(\frac{1 - 10d_i + 9d_m}{1 - 9d_m}\right)^2$
6	2	0.30	0.04	$S_i = 1 - \frac{0.6}{1 - d_m}\left(\frac{1 - 10d_i + 9d_m}{1 - 9d_m}\right)^2$
7	1	0.28	0.04	$S_i = 0.30 + 0.70\frac{d_i}{d_{max}}$
8	1	0.55	0.10	$S_i = d_i / d_{max}$
9	1	0.40	0.10	$S_i = 1 \; if \; d_i \geq 0.08$ $S_i = {d_i}/{0.08} \; if \; 0 \leq d_i < 0.08$
10	1	0.40	0.10	$S_i = 1 - \frac{d_{max} - d_i}{1 - d_{max}}$
11	1	0.40	0.10	$S_i = 1 - \frac{d_{max} - d_i}{1 - d_{max}}$
12	2	0.50	$\nexists t_k$	$S_i = 1 \; if \; d_i > 0.9d_m + 0.1$ $S_i = 0.9 \; if \; 0.9d_m + 0.1 \geq d_i > d_m$ $S_i = 0.8 \; if \; d_m \geq d_i > 1.1d_m - 0.1$ $S_i = 0 \; if \; 1.1d_m - 0.1 \geq d_i > 0$
13	3	0.30	$\nexists t_k$	$S_i = d_i / d_{max}$
14	1	0.40	$\nexists t_k$	$S_i = 1 - 1.5\frac{d_{max} - d_i}{1 - d_{max}}$
15	1	0.50	$\nexists t_k$	$S_i = 1 \quad if$ $d_i > 0.90d_m + 0.10$ $S_i = d_i / 0.1 + 0.9d_m \quad if$ $d_i < 0.90d_m + 0.10$
16	1	1.00	0.10	$S_i = 1 \; if \; d_i = d_{max}$ $S_i = 0 \; if \; d_i \neq d_{max}$
17	3	0.20	$\nexists t_k$	$S_i = 1 - 0.2\frac{d_{max} - d_i}{d_{max} - d_m} \; if \; d_m \leq d_i \leq d_{max}$ $S_i = 0.8\left(1 - \frac{d_m - d_i}{d_m - d_{min}}\right) if \; d_{min} \leq d_i \leq d_m$
18	3	0.50	$\nexists t_k$	$S_i = 1 \quad if \; d_i > 0.90d_m + 0.10$ $S_i = \frac{d_i}{0.1 + 0.9d_m} \quad if \; d_i < 0.90d_m + 0.10$
19	1	0.13	$\nexists t_k$	$S_i = 1 - 0.5\frac{d_{max} - d_i}{1 - d_{max}}$
20	1	0.40	$\nexists t_k$	$S_i = 1 - \frac{d_{max} - d_i}{1 - d_{max}}$
21	1	0.40	0.20	$S_i = 0.5 + 0.5\frac{d_i - d_{min}}{d_{max} - d_{min}}$
22	2	0.45	$\nexists t_k$	$S_i = 1 + \frac{d_i - d_{max}}{d_{max} + 0.5 - 1.5d_m}$
23	4	0.45	0.10	$S_i = 1 - \frac{0.6}{1 - d_m}\left(\frac{1 - 10d_i + 9d_m}{1 - 9d_m}\right)^2$
24	1	0.50	0.10	$S_i = 1 \; if \; d_i \geq 0.20$ $S_i = d_i / 0.20 \; if \; 0 \leq d_i < 0.20$
25	4	0.35	0.10	$S_i = 1 - \frac{0.6}{1 - d_m}\left(\frac{1 - 10d_i + 9d_m}{1 - 9d_m}\right)^2$
26	2	0.50	$\nexists t_k$	$S_i = 1 - 0.2\frac{d_{max} - d_i}{d_{max} - d_m}$
27	1	0.40	0.20	$S_i = d_i / 0.20$
28	1	0.30	$\nexists t_k$	$S_i = 1 - 5\frac{d_{max} - d_i}{1 - d_{max}}$
29	1	0.40	0.15	$S_i = 1 \; if \; d_i \geq 0.10$ $S_i = d_i / 0.10 \; if \; 0 \leq d_i < 0.10$
30	2	0.30	0.10	$S_i = \frac{1 - d_{max}}{1 - d_i}$
31	2	0.35	0.10	$S_i = 1 - \frac{0.6}{1 - d_m}\left(\frac{1 - 10d_i + 9d_m}{1 - 9d_m}\right)^2$
32	1	0.40	0.10	$S_i = d_i / d_{max}$
33	1	0.35	$\nexists t_k$	$S_i = 1 \; if \; d_m \leq d_i \leq d_{max}$ $S_i = d_i / d_m \; if \; 0 \leq d_i < d_m$
34	3	0.30	0.18	$S_i = 1 - \frac{d_{max} - d_i}{1 - d_{max}}$
35	1	0.40	$\nexists t_k$	$S_i = 1 - 0.25\frac{d_{max} - d_i}{d_{max} - d_m}$
36	1	1.00	$\nexists t_k$	$S_i = 1 \; if \; d_i = d_{max}$ $S_i = 0 \; if \; d_i \neq d_{max}$
37	3	0.51	0.10	$S_i = 0.745 - 0.255\frac{d_m - d_i}{d_{max} - d_m}$ $if \; d_m \leq d_i \leq d_{max}$ $S_i = 0.745\frac{d_m - d_i}{1 - d_m} \; if \; d_{min} \leq d_i \leq d_m$
38	5	0.35	0.10	$S_i = d_i / d_{max}$
39	3	0.20	$\nexists t_k$	$S_i = 1 - 0.2\frac{d_{max} - d_i}{d_{max} - d_m} \; if \; d_m \leq d_i \leq d_{max}$ $S_i = 0.8\left(1 - \frac{d_m - d_i}{d_m - d_{min}}\right) if \; d_{min} \leq d_i \leq d_m$
40	1	0.50	0.10	$S_i = d_i / d_{max}$
41	2	0.40	$\nexists t_k$	$S_i = 1 - \frac{d_{max} - d_i}{d_{max}}$
42	1	0.70	0.04	$S_i = 0.30 + 0.70\frac{d_i}{d_{max}}$
43	1	0.55	0.10	$S_i = d_i / d_{max}$
44	1	0.70	0.10	$S_i = d_i / d_{max}$
45	1	0.33	0.20	$S_i = 1 - 0.75\frac{d_{max} - d_i}{d_{max}}$
46	1	0.30	$\nexists t_k$	$S_i = d_i / d_{max}$
47	2	0.60	0.25	$S_i = 1 - \frac{d_{max} - d_i}{1 - d_{max}}$

Table 1: BSFs and ALBCs dataset

BSF ID (k)	***Auction ID***	***Upper Price limit (A)***	***Auction ID***	***Upper Price limit (A)***	***n (∑Auction IDs)***
1	1	320,032.00 €	2	1,585,015.00 €	2
Bidder (i)	***Bid (monetary value) (b_i)***	***Drop (/1 value) (d_i)***	***Bid (monetary value) (b_i)***	***Drop (/1 value) (d_i)***	
Lowest = 1	173,361.33 €	0.458	683,152.58 €	0.569	
2	198,419.84 €	0.380	767,798.23 €	0.516	
3	201,620.16 €	0.370	810,121.06 €	0.489	
4	204,820.48 €	0.360	852,443.89 €	0.462	
5	208,020.80 €	0.350	871,758.25 €	0.450	
6	211,221.12 €	0.340	871,758.25 €	0.450	
7	216,021.60 €	0.325	894,766.72 €	0.435	
8	217,621.76 €	0.320	935,158.85 €	0.410	
9	221,587.19 €	0.308	937,089.54 €	0.409	
10	224,022.40 €	0.300	951,009.00 €	0.400	
11	230,423.04 €	0.280	979,412.37 €	0.382	
12	279,227.92 €	0.128	1,014,409.60 €	0.360	
13			1,021,735.20 €	0.355	
Highest =14			1,233,349.34 €	0.222	
Scoring Parameters (SP)					
d_o		0.235		0.358	
d_{max}		0.458		0.569	
d_m		0.327		0.422	
d_{min}		0.128		0.222	
σ		0.066		0.072	
Regression coefficients (calculated according to Figure 1, 2nd column)					***Averages***
λ		1.136		1.111	$\bar{\lambda}$ = **1.123**
α		0.599		0.602	$\bar{\alpha}$ = **0.601**
β		0.905		0.821	$\bar{\beta}$ = **0.863**
γ		0.182		0.221	$\bar{\gamma}$ = **0.202**

Table 2: Example of BSF ID=1's Regression Coefficient (λ, $-\alpha$, β and γ) calculations

ID (k)	BS Weigh. (w_k)	ALBC width (t_k)	BSF Gradient (g_k)	X_1 $f(w_k)$	X_2 $f(t_k)$	X_3 $f(g_k)$	Y_1 ($\bar{\lambda}$)	Y_2 ($-\bar{\alpha}$)	Y_3 ($\bar{\beta}$)	Y_4 ($\bar{\gamma}$)
1	0.50	$\nexists t_k$	Convex	0	1	-1	1.123	0.601	0.863	0.202
2	0.40	0.10	Convex	0	0	-1	1.070	0.589	0.561	0.140
3	0.45	0.05	Constant	0	0	0	1.070	0.590	0.630	0.130
4	0.50	0.05	Convex	0	0	-1	0.990	0.551	0.693	0.159
5	0.30	0.06	Convex	-1	0	-1	0.835	0.327	0.422	0.134
6	0.30	0.04	Convex	-1	-1	-1	0.641	0.227	0.291	0.104
7	0.28	0.04	Constant	-1	-1	0	0.703	0.278	0.329	0.076
8	0.55	0.10	Constant	0	0	0	1.078	0.564	0.693	0.149
9	0.40	0.10	Convex	0	0	-1	1.060	0.524	0.706	0.165
10	0.40	0.10	Constant	0	0	0	1.100	0.651	0.634	0.140
11	0.40	0.10	Constant	0	0	0	1.045	0.620	0.660	0.177
12	0.30	0.10	Convex	-1	0	-1	0.764	0.323	0.432	0.134
13	0.30	$\nexists t_k$	Constant	-1	1	0	1.082	0.541	0.728	0.131
14	0.40	$\nexists t_k$	Constant	0	1	0	1.283	0.777	0.789	0.187
15	0.50	$\nexists t_k$	Convex	0	1	-1	1.088	0.653	0.780	0.169
16	1.00	0.10	Concave	1	0	1	1.448	0.892	0.865	0.191
17	0.20	$\nexists t_k$	Convex	-1	1	-1	0.884	0.459	0.644	0.165
18	0.50	$\nexists t_k$	Convex	0	1	-1	1.088	0.614	0.764	0.173
19	0.13	$\nexists t_k$	Constant	-1	1	0	1.113	0.551	0.553	0.158
20	0.40	$\nexists t_k$	Constant	0	1	0	1.170	0.706	0.780	0.205
21	0.40	0.20	Constant	0	1	0	1.321	0.733	0.913	0.144
22	0.45	$\nexists t_k$	Constant	0	1	0	1.346	0.696	0.913	0.153
23	0.45	0.10	Convex	0	0	-1	0.940	0.551	0.620	0.150
24	0.50	0.10	Constant	0	0	0	1.100	0.577	0.574	0.143
25	0.35	0.10	Convex	0	0	-1	1.010	0.535	0.640	0.134
26	0.50	$\nexists t_k$	Constant	0	1	0	1.346	0.696	0.888	0.189
27	0.40	0.20	Constant	0	1	0	1.207	0.777	0.747	0.191
28	0.30	$\nexists t_k$	Constant	-1	1	0	1.050	0.530	0.585	0.129
29	0.40	0.15	Convex	0	1	-1	1.100	0.700	0.730	0.180
30	0.30	0.10	Constant	-1	0	0	0.924	0.398	0.494	0.136
31	0.35	0.10	Convex	0	0	-1	0.980	0.578	0.713	0.131
32	0.40	0.10	Constant	0	0	0	1.034	0.632	0.667	0.167
33	0.35	$\nexists t_k$	Convex	0	1	-1	1.229	0.719	0.706	0.178
34	0.30	0.18	Constant	-1	1	0	1.124	0.562	0.741	0.123
35	0.40	$\nexists t_k$	Constant	0	1	0	1.283	0.681	0.822	0.158
36	1.00	$\nexists t_k$	Concave	1	1	1	1.701	1.102	1.091	0.204
37	0.51	0.10	Convex	0	0	-1	1.050	0.556	0.581	0.126
38	0.35	0.10	Constant	0	0	0	1.034	0.651	0.693	0.155
39	0.20	$\nexists t_k$	Convex	-1	1	-1	0.941	0.464	0.592	0.173
40	0.50	0.10	Constant	0	0	0	1.177	0.670	0.581	0.179
41	0.40	$\nexists t_k$	Constant	0	1	0	1.245	0.733	0.813	0.178
42	0.70	0.04	Constant	0	-1	0	0.930	0.535	0.500	0.114
43	0.55	0.10	Constant	0	0	0	1.144	0.583	0.739	0.135
44	0.70	0.10	Constant	1	0	0	1.081	0.667	0.623	0.128
45	0.33	0.20	Constant	-1	1	0	1.008	0.498	0.605	0.152
46	0.30	$\nexists t_k$	Constant	-1	1	0	1.040	0.498	0.676	0.128
47	0.60	0.25	Constant	0	1	0	1.219	0.681	0.772	0.148

Table 3: Analysis of BSFs

ID (k)	Nº auctions (n)	BSF description	Bid Score Weighting (w_k)	ALBC width (t_k)	BSF Gradient (g_k)
1	3	$S_i = 1 - \frac{0.6}{1-d_m}\left(\frac{1-10d_i+9d_m}{1-9d_m}\right)^2$	0.50	0.10	Convex
2	2	$S_i = 1 - 1.5\frac{d_{max}-d_i}{1-d_{max}}$	0.30	0.04	Constant
3	2	$S_i = 1\ if\ d_i = d_{max}$ $S_i = 0\ if\ d_i \neq d_{max}$	1.00	0.10	Concave

ID (k)	X_1 $f(w_k)$	X_2 $f(t_k)$	X_3 $f(g_k)$	Estimated Y_1 ($\bar{\lambda}$)	Estimated Y_2 (-$\bar{\alpha}$)	Estimated Y_3 ($\bar{\beta}$)	Estimated Y_4 ($\bar{\gamma}$)	Actual Y_1 ($\bar{\lambda}$)	Actual Y_2 (-$\bar{\alpha}$)	Actual Y_3 ($\bar{\beta}$)	Actual Y_4 ($\bar{\gamma}$)	Deviations (/1) Y_1 ($\bar{\lambda}$)	Deviations (/1) Y_2 (-$\bar{\alpha}$)	Deviations (/1) Y_3 ($\bar{\beta}$)	Deviations (/1) Y_4 ($\bar{\gamma}$)
1	0	0	-1	0.977	0.539	0.616	0.150	1.092	0.549	0.677	0.154	0.12	0.02	0.10	0.03
2	-1	-1	0	0.740	0.296	0.326	0.093	0.888	0.287	0.334	0.087	0.20	0.03	0.02	0.07
3	1	0	1	1.413	0.887	0.852	0.176	1.425	0.965	0.902	0.186	0.01	0.09	0.06	0.06

Table 4: Validation of the Multiple Linear Regression expressions with a recent sub-set of auctions

Figure 1

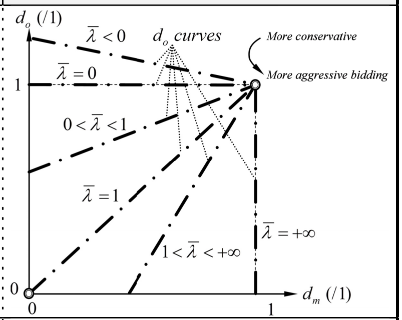
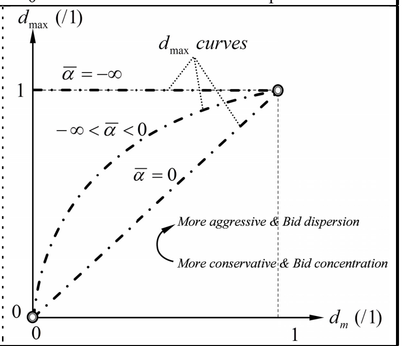
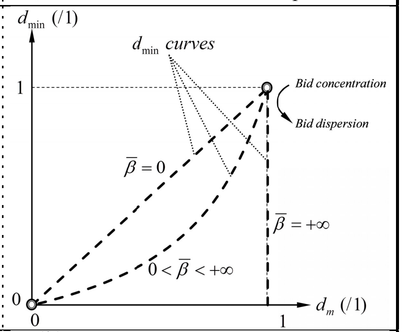
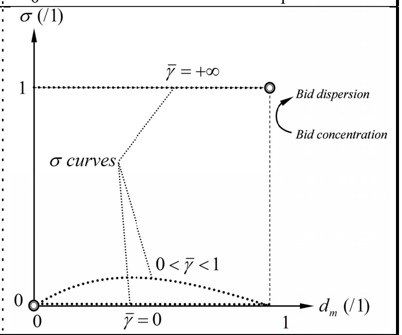
<table>
<tr><th>SP relationships (Capped auctions)</th><th colspan="2">Regression coefficient averages</th></tr>
<tr><td>Central parameter for comparisons with the rest of SP:
Mean (average) drop
d_m (with $0 \le d_m \le 1$)</td><td>Calculation
(for the n auctions with the same BSF ID)</td><td>Interpretation
(aggressive vs conservative bidding)
(bid dispersion vs bid concentration)</td></tr>
<tr><td>Estimated cost drop
d_o
$d_o = f(d_m) = 1 + (d_m - 1)\cdot\lambda$
(The relationship between d_o and d_m is usually presented the other way around, that is, as $d_m = f(d_o)$, but here it is presented as above for the sake of simplicity)</td><td>$\bar{\lambda} = \frac{1}{n}\sum_{j=1}^{j=n}\lambda_j = \frac{1}{n}\sum_{j=1}^{j=n}\frac{d_{oj}-1}{d_{mj}-1}$
$-\infty \le \bar{\lambda} \le +\infty$ ▶
(bid aggressiveness bid conservativeness)</td><td>
</td></tr>
<tr><td>Maximum drop (lowest bid)
$d_{\max}$
$d_{\max} = f(d_m) = \alpha d_m^{\,2} + (1-\alpha) d_m$
(Potential relationship expressions are also found in Ballesteros-Pérez et al. 2012b)</td><td>$\bar{\alpha} = \frac{1}{n}\sum_{j=1}^{j=n}\alpha_j = \frac{1}{n}\sum_{j=1}^{j=n}\frac{d_{\max j}-d_{mj}}{d_{mj}^{\,2}-d_{mj}}$
$-\infty \le \bar{\alpha} \le 0$ ▶
(bid aggressiveness bid conservativeness)
(bid dispersion bid concentration)</td><td>
</td></tr>
<tr><td>Minimum drop (highest bid)
$d_{\min}$
$d_{\min} = f(d_m) = \beta d_m^{\,2} + (1-\beta) d_m$
(Potential relationship expressions are also found in Ballesteros-Pérez et al. 2012b)</td><td>$\bar{\beta} = \frac{1}{n}\sum_{j=1}^{j=n}\beta_j = \frac{1}{n}\sum_{j=1}^{j=n}\frac{d_{\min j}-d_{mj}}{d_{mj}^{\,2}-d_{mj}}$
$0 \le \bar{\beta} \le +\infty$ ▶
(bid concentration bid dispersion)</td><td>
</td></tr>
<tr><td>Drop standard deviation
σ
$\sigma = f(d_m) = \gamma(d_m^{\,1/3} - d_m)$</td><td>$\bar{\gamma} = \frac{1}{n}\sum_{j=1}^{j=n}\gamma_j = \frac{1}{n}\sum_{j=1}^{j=n}\frac{\sigma_j}{d_{mj}^{\,1/3}-d_{mj}}$
$0 \le \bar{\gamma} \le +\infty$ ▶
(bid concentration bid dispersion)</td><td>
</td></tr>
</table>

Figure 2

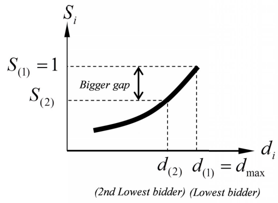
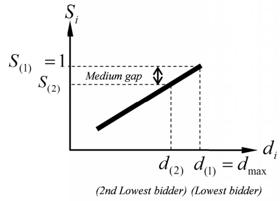
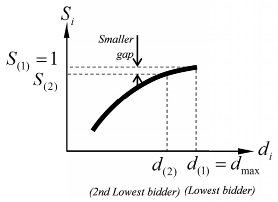
Score	X_1 ***Bid Score Weighting*** (w_k)	X_2 ***ALBC width*** (t_k)	X_3 ***BSF Gradient*** (g_k)
+1	$\frac{2}{3} < w_k \leq 1$ (The technical bid weighting is underrated)	$0.15 \leq t_k \leq 1$ (lenient abnormally low bids criterion; cases with $\nexists t_k$ included here)	*Concave*  (higher score loss near the best scored bidder)
0	$\frac{1}{3} < w_k \leq \frac{2}{3}$ (Balance between the bid and the technical proposal weigting)	$0.05 \leq t_k < 0.15$ (balanced abnormally low bids criterion)	*Constant*  (linearly proportional score loss from the best scored bidder)
-1	$0 \leq w_k \leq \frac{1}{3}$ (The bid weighting is underrated)	$0 \leq t_k < 0.05$ (extremely narrow abnormally low bids criterion)	*Convex*  (softer score loss near the best scored bidder)

Figure 3

<u>Coefficient λ's Multiple Linear regression</u> $Y_1 = \lambda = M_0 + M_1 * X_1 + M_2 * X_2 + M_3 * X_3$

M_0 = **1.099**	SE_{M_0} = 0.013	F_Y-value =	116.523	F_{fisher} (α=5%)	3.438	F_Y-value > $F_{fisher\ (\alpha=5\%)}$?	OK
M_1 = **0.193**	SE_{M_1} = 0.018	t_{M_1}-value =	10.910	(with df_1 and df_2)		t_{M_1}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_2 = **0.166**	SE_{M_2} = 0.015	t_{M_2}-value =	10.910	$t_{student}$ (α=5%)	2.017	t_{M_2}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_3 = **0.122**	SE_{M_3} = 0.018	t_{M_3}-value =	6.959	(with $df = df_2$)		t_{M_3}-value > $t_{student\ (\alpha=5\%)}$?	OK
R^2 = **0.890**	SE_Y = 0.063	n =	47	df_1 =	3	df_2 = 43	

<u>Coefficient -α's Multiple Linear regression</u> $Y_2 = -\alpha = M_0 + M_1 * X_1 + M_2 * X_2 + M_3 * X_3$

M_0 = **0.613**	SE_{M_0} = 0.009	F_Y-value =	182.709	F_{fisher} (α=5%)	3.438	F_Y-value > $F_{fisher\ (\alpha=5\%)}$?	OK
M_1 = **0.200**	SE_{M_1} = 0.012	t_{M_1}-value =	16.976	(with df_1 and df_2)		t_{M_1}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_2 = **0.117**	SE_{M_2} = 0.010	t_{M_2}-value =	11.526	$t_{student}$ (α=5%)	2.017	t_{M_2}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_3 = **0.074**	SE_{M_3} = 0.012	t_{M_3}-value =	6.317	(with $df = df_2$)		t_{M_3}-value > $t_{student\ (\alpha=5\%)}$?	OK
R^2 = **0.927**	SE_Y = 0.042	n =	47	df_1 =	3	df_2 = 43	

<u>Coefficient β's Multiple Linear regression</u> $Y_3 = \beta = M_0 + M_1 * X_1 + M_2 * X_2 + M_3 * X_3$

M_0 = **0.653**	SE_{M_0} = 0.013	F_Y-value =	72.100	F_{fisher} (α=5%)	3.438	F_Y-value > $F_{fisher\ (\alpha=5\%)}$?	OK
M_1 = **0.162**	SE_{M_1} = 0.018	t_{M_1}-value =	9.032	(with df_1 and df_2)		t_{M_1}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_2 = **0.166**	SE_{M_2} = 0.015	t_{M_2}-value =	10.750	$t_{student}$ (α=5%)	2.017	t_{M_2}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_3 = **0.038**	SE_{M_3} = 0.018	t_{M_3}-value =	2.117	(with $df = df_2$)		t_{M_3}-value > $t_{student\ (\alpha=5\%)}$?	OK
R^2 = **0.834**	SE_Y = 0.064	n =	47	df_1 =	3	df_2 = 43	

<u>Coefficient γ's Multiple Linear regression</u> $Y_4 = \gamma = M_0 + M_1 * X_1 + M_2 * X_2 + M_3 * X_3$ → $Y_4 = \gamma = M_0 + M_1 * X_1 + M_2 * X_2$

M_0 = **0.146**	SE_{M_0} = 0.004	F_Y-value =	19.202	F_{fisher} (α=5%)	3.438	F_Y-value > $F_{fisher\ (\alpha=5\%)}$?	OK
M_1 = **0.025**	SE_{M_1} = 0.005	t_{M_1}-value =	4.830	(with df_1 and df_2)		t_{M_1}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_2 = **0.027**	SE_{M_2} = 0.004	t_{M_2}-value =	6.091	$t_{student}$ (α=5%)	2.017	t_{M_2}-value > $t_{student\ (\alpha=5\%)}$?	OK
M_3 = **-0.004**	SE_{M_3} = 0.005	t_{M_3}-value =	-0.749	(with $df = df_2$)		t_{M_3}-value > $t_{student\ (\alpha=5\%)}$?	No
R^2 = **0.573**	SE_Y = 0.019	n =	47	df_1 =	3	df_2 = 43	

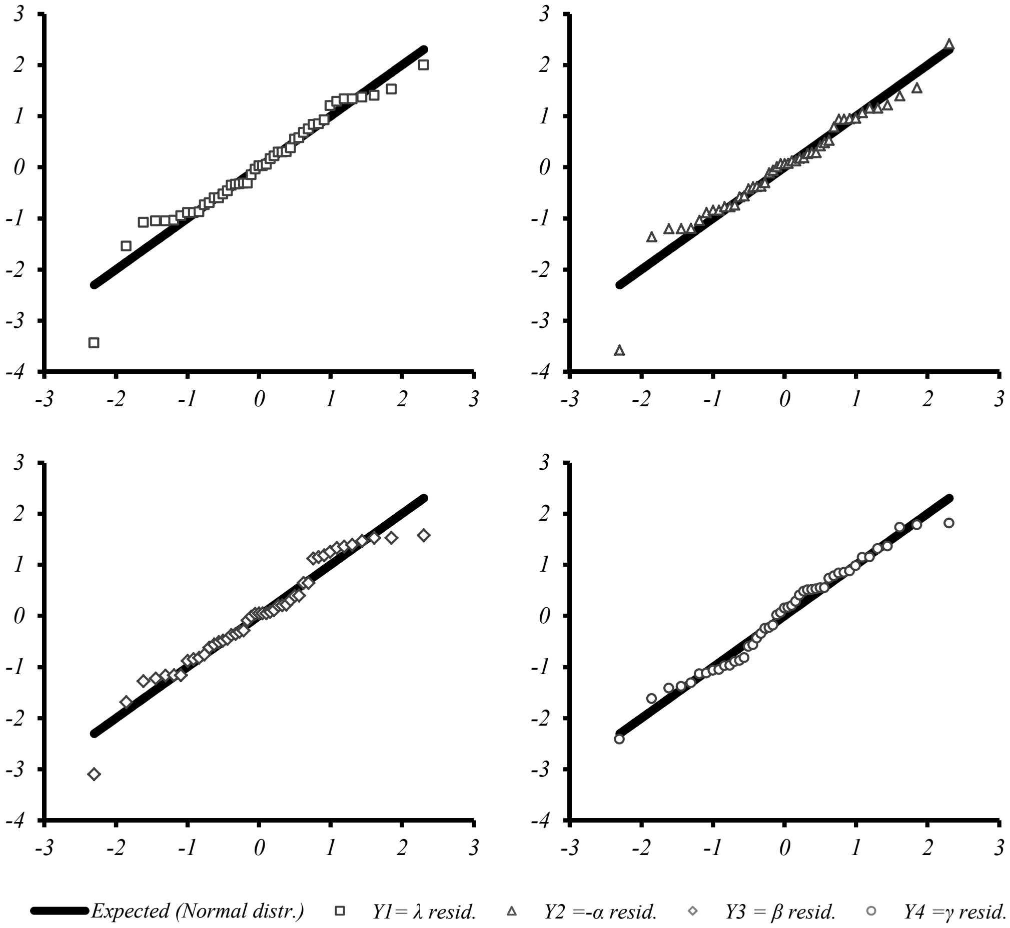
<u>Covariance Matrix (CvM)</u>

	X_1	X_2	X_3
X_1	0.302	-0.016	0.083
X_2	-0.016	0.380	0.030
X_3	0.083	0.030	0.309

<u>Correlation Matrix (CrM)</u>

	X_1	X_2	X_3
X_1	1.000	-0.048	0.271
X_2	-0.048	1.000	0.088
X_3	0.271	0.088	1.000

Figure 4



3
2
1
0
-1
-2
-3
-4
-3 -2 -1 0 1 2 3
Expected (Normal distr.)
Y1= λ resid.
Y2 =-α resid.
Y3 = β resid.
Y4 =γ resid.

Figure 5

Coefficient λ's M_1, M_2 and M_3's LSDs — $Y_1 = \lambda = M_0 + \mathbf{M_1} * X_1 + \mathbf{M_2} * X_2 + \mathbf{M_3} * X_3$

						LB LSD intervals	UP LSD intervals	Observations:
M_1 =	0.193	SE_{M_1} =	0.018	s_{M_1} =	0.019	0.166	0.220	M_1's and M_2's LSD intervals intersect, as
M_2 =	0.166	SE_{M_2} =	0.015	s_{M_2} =	0.018	0.141	0.191	M_2's with M_3's. Hence, X_1's M_1 value seems
M_3 =	0.122	SE_{M_3} =	0.018	s_{M_3} =	0.019	0.095	0.149	more important than X_3's M_3 value.
n =	47	N =	141	t_{N-1} (α=5%) =	1.977			

Coefficient -α's M_1, M_2 and M_3's LSDs — $Y_2 = -\alpha = M_0 + \mathbf{M_1} * X_1 + \mathbf{M_2} * X_2 + \mathbf{M_3} * X_3$

						LB LSD intervals	UP LSD intervals	Observations:
M_1 =	0.200	SE_{M_1} =	0.012	s_{M_1} =	0.016	0.178	0.222	No LSD intervals intersect,
M_2 =	0.117	SE_{M_2} =	0.010	s_{M_2} =	0.015	0.0964	0.138	then, X_1 is more important than X_2
M_3 =	0.074	SE_{M_3} =	0.012	s_{M_3} =	0.016	0.052	0.0958	and, X_2 is more important than X_3.
n =	47	N =	141	t_{N-1} (α=5%) =	1.977			

Coefficient β's M_1 and M_2's LSDs — $Y_3 = \beta = M_0 + \mathbf{M_1} * X_1 + \mathbf{M_2} * X_2 + \mathbf{M_3} * X_3$

						LB LSD intervals	UP LSD intervals	Observations:
M_1 =	0.162	SE_{M_1} =	0.018	s_{M_1} =	0.020	0.134	0.189	M_1's and M_2's LSD intervals intersect,
M_2 =	0.166	SE_{M_2} =	0.015	s_{M_2} =	0.018	0.140	0.191	then, X_1 and X_2 are equally important.
M_3 =	0.038	SE_{M_3} =	0.018	s_{M_3} =	0.019	0.010	0.065	Both are more important than X_3.
n =	47	N =	141	t_{N-1} (α=5%) =	1.977			

Coefficient γ's M_1 and M_2's LSDs — $Y_4 = \gamma = M_0 + M_1 * X_1 + M_2 * X_2 + M_3 * X_3 \rightarrow Y_4 = \gamma = M_0 + \mathbf{M_1} * X_1 + \mathbf{M_2} * X_2$

						LB LSD intervals	UP LSD intervals	Observations:
M_1 =	0.025	SE_{M_1} =	0.005	s_{M_1} =	0.011	0.010	0.040	M_1's and M_2's LSD intervals intersect,
M_2 =	0.027	SE_{M_2} =	0.004	s_{M_2} =	0.010	0.014	0.041	then, X_1 and X_2 are equally important.
M_3 =	-0.004	SE_{M_3} =	0.005	s_{M_3} =	0.010	-0.019	0.011	X_3 was deemed meaningless.
n =	47	N =	141	t_{N-1} (α=5%) =	1.977			

Cell Formulae

LB LSD intervals:	Lower Bound of Fisher's Least Significant Difference Intervals	$LB = M_i - 0.707 * t_{N-1} * s_{M_i}$
UB LSD intervals:	Upper Bound of Fisher's Least Significant Difference Intervals	$UB = M_i + 0.707 * t_{N-1} * s_{M_i}$

Figure Caption List

Figure 1: Scoring Parameter relationships in capped auctions

Figure 2: Trichotomic scoring of the three independent BSF variables w_k , t_k and g_k

Figure 3: Multiple linear regression analysis

Figure 4: Normality test of Residuals (Q-Q plots)

Figure 5: Least Significant Difference intervals analysis