

High-precision measurements of the co-polar correlation coefficient: non-Gaussian errors and retrieval of the dispersion parameter μ in rainfall

Article

Accepted Version

Keat, W. J., Westbrook, C. D. ORCID: <https://orcid.org/0000-0002-2889-8815> and Illingworth, A. J. ORCID: <https://orcid.org/0000-0002-5774-8410> (2016) High-precision measurements of the co-polar correlation coefficient: non-Gaussian errors and retrieval of the dispersion parameter μ in rainfall. *Journal of Applied Meteorology and Climatology*, 55 (7). pp. 1615-1632. ISSN 1558-8432 doi: <https://doi.org/10.1175/JAMC-D-15-0272.1> Available at <https://centaur.reading.ac.uk/62399/>

It is advisable to refer to the publisher's version if you intend to cite from the work. See [Guidance on citing](#).

To link to this article DOI: <http://dx.doi.org/10.1175/JAMC-D-15-0272.1>

Publisher: American Meteorological Society

All outputs in CentAUR are protected by Intellectual Property Rights law, including copyright law. Copyright and IPR is retained by the creators or other copyright holders. Terms and conditions for use of this material are defined in

the [End User Agreement](#).

www.reading.ac.uk/centaur

CentAUR

Central Archive at the University of Reading

Reading's research outputs online

1 **High-Precision Measurements of the Co-Polar Correlation**

2 **Coefficient: Non-Gaussian Errors and Retrieval of the Dispersion**

3 **Parameter μ in Rainfall**

4 **W. J. KEAT, * C. D. WESTBROOK, AND A. J. ILLINGWORTH**

Department of Meteorology, University of Reading, UK

* *Corresponding author address:* W. J. Keat, Department of Meteorology, University of Reading, Earley Gate, PO Box 243, Reading, RG6 6BB.

E-mail: w.j.keat@pgr.reading.ac.uk

5 ABSTRACT

6 The co-polar correlation coefficient (ρ_{hv}) has many applications, including hydrometeor clas-
7 sification, ground clutter and melting layer identification, interpretation of ice microphysics
8 and the retrieval of rain drop size distributions (DSDs). However, we currently lack the
9 quantitative error estimates that are necessary if these applications are to be fully exploited.
10 Previous error estimates of ρ_{hv} rely on knowledge of the unknown ‘true’ ρ_{hv} and implicitly
11 assume a Gaussian probability distribution function of ρ_{hv} samples. We show that fre-
12 quency distributions of ρ_{hv} estimates are in fact highly negatively skewed. A new variable:
13 $L = -\log_{10}(1 - \rho_{hv})$ is defined, which does have Gaussian error statistics, and a standard
14 deviation depending only on the number of independent radar pulses. This is verified using
15 observations of spherical drizzle drops, allowing, for the first time, the construction of rigor-
16 ous confidence intervals in estimates of ρ_{hv} . In addition, we demonstrate how the imperfect
17 co-location of the horizontal and vertical polarisation sample volumes may be accounted for.

18 The possibility of using L to estimate the dispersion parameter (μ) in the gamma drop
19 size distribution is investigated. We find that including drop oscillations is essential for
20 this application, otherwise there could be biases in retrieved μ of up to ≈ 8 . Preliminary
21 results in rainfall are presented. In a convective rain case study, our estimates show μ to
22 be substantially larger than 0 (an exponential DSD). In this particular rain event, rain rate
23 would be overestimated by up to 50% if a simple exponential DSD is assumed.

24 1. Introduction

25 The co-polar correlation coefficient, ρ_{hv} , between horizontal (H) and vertical (V) polari-
26 sation radar signals is a measure of the variety of hydrometeor shapes in a pulse volume. It is
27 therefore useful for applications such as identifying the melting layer (Caylor and Illingworth
28 1989; Brandes and Ikeda 2004; Tabary et al. 2006; Giangrande et al. 2008), ground clutter
29 (e.g. Tang et al. 2014), rain-hail mixtures (Balakrishnan and Zrnic 1990) and interpreting
30 polarimetric signatures in ice (e.g. Andrić et al. 2013), and potentially the retrieval of
31 the drop size distribution (DSD). The standard deviations of differential reflectivity (Z_{DR})
32 and differential phase shift (ϕ_{dp}) are both functions of ρ_{hv} (Bringi and Chandrasekar 2001).
33 Therefore, ρ_{hv} dictates both the quality of dual polarisation measurements and their weight-
34 ing in hydrometeor classification schemes (Park et al. 2009). In rainfall, ρ_{hv} is typically
35 0.98—1. Giangrande et al. (2008) use data where $\rho_{hv} < 0.97$ to identify the melting layer.
36 For hail, ρ_{hv} can be much lower due to the effects of Mie scattering. At present, quantita-
37 tive use of ρ_{hv} is hampered by a lack of rigorous confidence intervals accompanying the ρ_{hv}
38 estimates. Error estimates are available adopting an empirical approach (Illingworth and
39 Caylor 1991) or a linear perturbation technique (Liu et al. 1994; Torlaschi and Gingras 2003),
40 both of which implicitly assume a Gaussian probability distribution for the ρ_{hv} samples. We
41 will show that the distribution of ρ_{hv} samples is in fact non-Gaussian and highly negatively
42 skewed.

43 Natural rain drop size distributions can be described by a gamma distribution (Ulbrich
44 1983):

$$N(D) = N_0 D^\mu \exp \left[-\frac{(3.67 + \mu)}{D_0} D \right] \quad (1)$$

45 where D is the equivalent spherical drop diameter, N_0 is the intercept parameter, D_0 is the
46 median volume drop diameter and μ is the dispersion parameter (a measure of the drop size
47 spectrum shape). If $\mu = 0$, by exploiting the relationship between drop diameter and drop

48 axis ratio, D_0 can be estimated using Z_{DR} (Seliga and Bringi 1976). Higher μ correspond
 49 to more monodisperse drop size distributions. Since ρ_{hv} is sensitive to variations in drop
 50 shape, it can in principle be used to estimate μ (Jameson 1987), knowledge of which could
 51 improve dual polarisation and dual frequency (e.g. the Global Precipitation Measurement
 52 satellite) rain rate estimates. Figure 1 shows rain rate (R) per unit radar reflectivity (Z) as
 53 a function of Z_{DR} for simulated Gamma distributions with $\mu = -1, 0, 2, 4, 8, 12$ and 16 . The
 54 rain rate is sensitive to variability in the shape of the drop size spectrum; uncertainty in μ
 55 alone could introduce an error in the retrieved rain rate of up to 2.5 dB (almost a factor of
 56 2) for a given pair of Z and Z_{DR} observations.

57 It is difficult to obtain reliable estimates of μ from observations. Disdrometers suffer from
 58 undersampling of large drops, which cause μ values that are derived from the 3rd, 4th and 6th
 59 moments of the drop size distribution to be biased high (Johnson et al. 2014). Furthermore,
 60 disdrometers also undercount the number of drops < 0.5 mm (Tokay et al. 2001), which can
 61 also introduce a bias in estimates of μ . Estimating DSD parameters using radar is therefore
 62 preferable, due to the very large number of drops being sampled. Wilson et al. (1997)
 63 made radar observations dwelling in rain at elevation angles above 20° and report that the
 64 difference in the mean Doppler velocity at H and V polarisations provides an estimate of μ ,
 65 which were in the range of 1 to 11, and, once Z_{DR} exceeded 0.5 dB, all the values were above
 66 4. Doppler spectra of rain at vertical incidence with multiple wavelength radars, including
 67 wind profiler frequencies that respond to the clear air motion have been utilised to estimate
 68 μ (Williams 2002; Schafer et al. 2002). These experiments find μ ranges between 0 and 18,
 69 but is typically 0—6. Unal (2015) fits observed Doppler spectra to theoretical drop spectra
 70 at S-band, and retrieves μ in the range of -1—5. The disadvantage of these techniques is that
 71 they use high elevation angles; for operational monitoring of surface rainfall, measurements
 72 at low elevation angles are preferable. This motivates the use of ρ_{hv} to derive μ in rainfall.
 73 Illingworth and Caylor (1991) and Thurai et al. (2008) inferred μ from the decrease in
 74 ρ_{hv} as Z_{DR} increases. The difficulty here is that any mis-matches in the H and V beams

75 will introduce an uncorrelated noise component, so that even for perfectly spherical drizzle
76 droplets, where the “true” ρ_{hv} is unity, the radar will always detect a value less than one (we
77 will call this maximum obtainable level of ρ_{hv} “ f_{hv}^{max} ”, see Section 5). From measurements in
78 rain at short range, Illingworth and Caylor (1991) inferred μ values, which if corrected with
79 an estimate of f_{hv}^{max} were in the range 0–2, but even for long dwells the estimated errors
80 in μ were quite large. Thurai et al. (2008) analysed ρ_{hv} measurements from an operational
81 radar and obtained estimates of μ in the range of 1–3, however their approach relies on
82 empirically derived relationships between ρ_{hv} and DSD widths from 2 dimensional video
83 disdrometer (2DVD) measurements. Furthermore, the technique is only valid for intense
84 rain ($Z_{DR} \geq 2$ dB and $\rho_{hv} < 0.98$).

85 The aim of this paper is to define a new variable, $L = -\log_{10}(1 - \rho_{hv})$, that has Gaussian
86 error statistics with a width predictable from the number of independent radar pulses. This
87 can be readily estimated by using the observed Doppler spectral width (σ_v). We will then
88 present measurements of L in rainfall as a function of Z_{DR} , and retrieve estimates of μ by
89 comparing these with predicted L and Z_{DR} for various three-parameter gamma distributions.
90 The possibility of using this technique to retrieve μ using operational radars is then discussed.

91 2. The Co-Polar Correlation Coefficient (ρ_{hv})

92 ρ_{hv} is defined as (Doviak and Zrnic 2006):

$$\rho_{hv} = \frac{\langle S_{VV} S_{HH}^* \rangle}{\sqrt{\langle |S_{HH}|^2 \rangle \langle |S_{VV}|^2 \rangle}} \quad (2)$$

93 where $\langle S_{HH} \rangle$ and $\langle S_{VV} \rangle$ are the co-polar elements of the backscattering matrix averaged
94 over an ensemble of scatterers for the H and V polarisations respectively, and * indicates the
95 complex conjugate. It can be estimated by correlating successive power or complex (I and
96 Q) measurements. Examples of power time-series in (a) drizzle and (b) heavier rainfall from

97 the 3 GHz Chilbolton Advanced Meteorological Radar (CAMRa) are shown in figure 2. The
 98 radar is a coherent-on-receive magnetron system, transmitting and receiving alternate H and
 99 V polarised pulses with a pulse repetition frequency (PRF) of 610 Hz. A cubic polynomial
 100 interpolation is used to estimate the H power at the V pulse timing and the V power at the
 101 H pulse timing. Its narrow one-way half power beamwidth (0.28°) makes it capable of very
 102 high resolution measurements. The full capabilities of this radar are discussed in Goddard
 103 et al. (1994). The observed fluctuating signals in Figure 2 are caused by the superposition
 104 of the backscattered waves from each drop in the sample volume; the rate of fluctuation
 105 is determined by the Doppler spectral width. For drizzle, since the drops are spherical,
 106 $Z_{DR} = 0$ dB, and the H and V signals are almost perfectly correlated: $\rho_{hv} = 0.995$. For
 107 heavier rainfall, a systematically lower V power is received ($Z_{DR} = 1.1$ dB), and the signals
 108 are visibly less correlated ($\rho_{hv} = 0.987$), due to the broader axis ratio distributions in the
 109 sample volume.

110 These estimates of ρ_{hv} are derived from a finite number of reshufflings, and therefore
 111 there is some uncertainty in them. In what follows, we quantify this uncertainty.

112 **3. Theoretical Measurement Error in Estimated Corre-** 113 **lation of Time-Series**

114 Figure 3a shows the distribution of estimates of the correlation coefficient, $\hat{\rho}_{hv}$ (calculated
 115 from a finite length time-series), as distinct from the “true” co-polar correlation coefficient,
 116 $\overline{\rho_{hv}}$ (that would be measured for a time-series of infinite length). The data was collected
 117 during a 1.5° elevation dwell in drizzle ($Z_{DR} < 0.1$ dB), with very high SNR (> 40 dB) on
 118 6 February 2014. Each $\hat{\rho}_{hv}$ is calculated from 64 H and V pulse pairs (0.21s dwell) from a
 119 single 75 m range gate with $\sigma_v = 1.1 \pm 0.1$ ms⁻¹. The distribution of $\hat{\rho}_{hv}$ has a peak that is
 120 close to $\overline{\rho_{hv}}$ (which is < 1 , see Section 5d), but exhibits a very long tail at lower $\hat{\rho}_{hv}$, while
 121 there are no data with $\hat{\rho}_{hv} > 1$. Clearly, this distribution is not Gaussian and the negative

122 skewness will negatively bias the mean of many ρ_{hv} samples compared to the true value of
 123 ρ_{hv} .

124 Fisher (1915) states that sample correlation coefficients ($\hat{\rho}$) of a “true” correlation coef-
 125 ficient ($\bar{\rho}$) calculated from a finite number of Gaussian random variables are skewed for $\bar{\rho} \neq$
 126 0. However, the variable:

$$\hat{F} = \frac{1}{2} \ln \left(\frac{1 + \hat{\rho}}{1 - \hat{\rho}} \right) \quad (3)$$

127 is Gaussian, with a mean of:

$$\bar{F} = \frac{1}{2} \ln \left(\frac{1 + \bar{\rho}}{1 - \bar{\rho}} \right) \quad (4)$$

128 and standard error of:

$$\sigma_F = \frac{1}{\sqrt{N - 3}} \quad (5)$$

129 where N is the number of independent samples used to calculate $\hat{\rho}$.

130 This is directly applicable to estimates of the radar co-polar correlation coefficient, by
 131 realising that the I and Q samples that are used to estimate ρ_{hv} are Gaussian random
 132 variables (Doviak and Zrnic 2006). Noting that ρ_{hv} in meteorological targets is always close
 133 to unity so that fractional changes in $(1 - \rho_{hv})$ are always much greater than $(1 + \rho_{hv})$,
 134 Equation 3 can be written as:

$$\hat{F} \approx \frac{1}{2} \ln 2 - \frac{\ln 10}{2} \log_{10}(1 - \rho_{hv}) \quad (6)$$

135 Since \hat{F} is normally distributed, the quantity:

$$\hat{L} = -\log_{10}(1 - \rho_{hv}) \quad (7)$$

136 is also normally distributed, with a mean:

$$\bar{L} = -\log_{10}(1 - \overline{\rho_{hv}}) \quad (8)$$

137 and standard deviation of:

$$\sigma_L = \frac{2}{\ln 10} \times \frac{1}{\sqrt{N_{IQ} - 3}} \quad (9)$$

138 for $N_{IQ} \gg 3$, where N_{IQ} is the number of independent I and Q samples used to calculate
 139 $\hat{\rho}_{hv}$. Despite having similar characteristics, L is preferred over the use of F as it has the
 140 convenient property that $\rho_{hv} = 0.9, 0.99$ and 0.999 correspond to $L = 1, 2$ and 3 respectively
 141 and therefore is more intuitive. Illingworth and Caylor (1991) plotted their $\hat{\rho}_{hv}$ data as
 142 $\log_{10}(1 - \hat{\rho}_{hv})$ and their histograms also appear Gaussian in shape. Figure 3b illustrates the
 143 effect of the transform $\hat{L} = -\log_{10}(1 - \hat{\rho}_{hv})$ on the distribution in Figure 3a. The histogram
 144 is now symmetrical, and bell shaped. A Gaussian curve with an equal mean and standard
 145 deviation to the \hat{L} PDF is overplotted and is an excellent fit to the data, showing that the
 146 distributions are indeed Gaussian (and Quantile - Quantile plots, not shown here for brevity,
 147 confirm this).

148 To determine the number of independent I and Q samples, N_{IQ} , we consider the auto-
 149 correlation function for I and Q samples given by Doviak and Zrnic (2006):

$$R_{IQ}(nT_s) = \exp \left[-8 \left(\frac{\pi \sigma_v n T_s}{\lambda} \right)^2 \right] \quad (10)$$

150 where T_s is the time spacing between pulses of the same polarisation and nT_s is the total
 151 time lag. Following the definition of Papoulis (1965), the time to independence for I and Q
 152 samples for large N_{IQ} can be shown to be:

$$\tau_{IQ} = \frac{\lambda}{2\sqrt{2}\pi\sigma_v} \quad (11)$$

153 where λ is the radar wavelength and σ_v is the Doppler spectral width. This is a factor of $\sqrt{2}$
 154 smaller than the more often used time to independence for reflectivity samples. The number

155 of independent I and Q pulses per ρ_{hv} sample can therefore be estimated by:

$$N_{IQ} = \frac{T_{dwell}}{\tau_{IQ}} = \frac{2\sqrt{2\pi}\sigma_v T_{dwell}}{\lambda} \quad (12)$$

156 where T_{dwell} is the dwell time.

157 The result (Equation 9) is significant as it shows that a confidence interval for any
 158 measurement of ρ_{hv} can be calculated solely in terms of the number of independent I and
 159 Q samples used to estimate it, which in turn can be readily estimated using the observed
 160 Doppler spectral width and Equation 12. Furthermore, when multiple samples of \hat{L} are
 161 averaged, no bias is introduced to estimates of ρ_{hv} because of the non-linear transform. We
 162 expand this point in Section 4.

163 To estimate confidence intervals for measurements of ρ_{hv} , one must:

- 164 • Apply the transform $\hat{L} = -\log_{10}(1 - \rho_{hv})$
- 165 • Calculate the standard deviation of \hat{L} using Equation 9.
- 166 • Apply the inverse transform $1 - 10^{-(\hat{L} \pm \sigma_L)}$ to obtain upper and lower confidence intervals
 167 (where σ_L will contain the true value 68% of the time and $2\sigma_L$ 98%).

168 More conveniently, one can simply transform ρ_{hv} data to \hat{L} and use this for any subsequent
 169 analysis, with confidence intervals of $\hat{L} \pm \sigma_L$. This is the approach we follow in the rest
 170 of this paper. Although we are focusing on data with very high signal-to-noise (SNR) in
 171 this paper, the theory above should also be valid for weak SNR data, providing that noise
 172 introduced is also Gaussian in the I and Q samples.

173 This theoretical prediction was tested by comparing estimates of σ_L using data collected
 174 in homogeneous drizzle ($Z_{DR} < 0.1$ dB) with very good signal-to-noise (SNR > 40 dB).
 175 In drizzle, \bar{L} is constant since the drops are spherical, and therefore any variation σ_L is
 176 due to the finite N_{IQ} . Pulse-to-pulse H and V powers were recorded, and time series of
 177 various lengths between 0.2—30 s were constructed from these data and used to compute

178 the corresponding N_{IQ} and L values. Data was binned by N_{IQ} , and the standard deviation,
 179 σ_L , were computed for each bin. Figure 4 shows how σ_L decreases as N_{IQ} is increased over
 180 more than two orders of magnitude. σ_L is slightly overestimated for $N_{IQ} \approx 10$, and the data
 181 is in excellent agreement to that predicted by Equation 9 for $N_{IQ} > 30$.

182 4. Comparison with Existing Error Statistics

183 We now compare these new error statistics with existing methods in the literature. From
 184 observations of ρ_{hv} in rain, the bright-band and ice, Illingworth and Caylor (1991) derived
 185 empirically the relationship between their mean $\hat{\rho}_{hv}$ estimates and their standard deviation:

$$\sigma_{\rho_{hv}}^{IC} \simeq \frac{1.25(1 - \hat{\rho}_{hv})}{\sqrt{n}} \quad (13)$$

186 where n is the number of 0.2 s time-series they used to estimate the mean ρ_{hv} . Using a linear
 187 perturbation technique, Torlaschi and Gingras (2003) derive the following equation for the
 188 standard deviation on a ρ_{hv} measurement:

$$\sigma_{\rho_{hv}}^{TG} = \frac{1 - \bar{\rho}_{hv}^2}{\sqrt{2N_I}} \quad (14)$$

189 where N_I is the number of independent radar reflectivity samples used in its estimation. Note
 190 that $\bar{\rho}_{hv}$ in Equation 14 is the “true” correlation coefficient one is attempting to measure
 191 (rather than the measured value, $\hat{\rho}_{hv}$). This equation represents the standard deviation
 192 for infinite SNR conditions, and is valid for simultaneous or accurately interpolated H and
 193 V sampling. Neither of these techniques are ideal, relying on either knowing a-priori the
 194 true correlation coefficient one is attempting to measure (Torlaschi and Gingras 2003), or a
 195 number of time-series (Illingworth and Caylor 1991), not the number of independent pulses.
 196 It is not possible to compare the method of Illingworth and Caylor (1991) with our proposed
 197 method as σ_v for their data is unknown, and therefore the number of independent pulses in
 198 their time-series cannot be quantified. Figure 5a shows the errors on $\hat{\rho}_{hv}$ calculated using our

199 new method compared to those calculated using the linear perturbation method of Torlaschi
 200 and Gingras (2003) as a function of N_{IQ} in rain ($\overline{\rho_{hv}} = 0.98$). The magnitudes of the upper
 201 confidence bounds are largely similar, however, for all N_{IQ} the lower confidence interval is
 202 higher for Torlaschi and Gingras (2003) (i.e smaller deviations from $\overline{\rho_{hv}}$ are predicted), due
 203 to the asymmetric nature of the new confidence intervals on $\rho_{\hat{h}v}$. The largest difference is for
 204 small N_{IQ} . As N_{IQ} increases, both the upper and lower confidence intervals for each method
 205 converge. Although Figure 5a serves as a useful illustration of the difference between the
 206 methods, they are not strictly comparable in practice: the error calculation of Torlaschi and
 207 Gingras (2003) relies on knowledge of $\overline{\rho_{hv}}$ which in reality is unknown. Conversely, the new
 208 method requires no a-priori knowledge of $\overline{\rho_{hv}}$, and so is of much greater practical use.

209 Figure 5b illustrates the theoretical bias introduced by averaging many short samples of
 210 $\rho_{\hat{h}v}$, rather than \hat{L} , in rain ($\overline{\rho_{hv}} = 0.98$). This bias is significant for small N_{IQ} . For example,
 211 when $N_{IQ} = 10$, the bias on \hat{L} is 0.1, which is significant for the purpose of estimating μ in
 212 rainfall; this bias in L could lead to an underestimate of μ of ≈ 8 at $Z_{DR} = 2$ dB (see figure
 213 8). It is not important whether spatial or temporal averaging is used to increase the number
 214 of independent I and Q samples, as long as $\overline{\rho_{hv}}$ does not vary substantially over the scales
 215 considered.

216 In summary, confidence intervals that rely on the linear perturbation method overestimate
 217 the precision of ρ_{hv} measurements, and require knowledge of the “true” ρ_{hv} one is attempting
 218 to measure. Fundamentally, failure to use the transform L when averaging short time-
 219 series will lead to significant biases in correlation coefficient estimates. This is particularly
 220 important for operational ρ_{hv} applications that typically use very short dwell times (discussed
 221 in Section 8a), and would lead to a significant bias in retrievals of μ in rain.

222 5. Practical measurement of ρ_{hv}

223 To fully exploit our new error estimates, and retrieve rain DSDs, some practical consid-
224 erations for the measurement of ρ_{hv} must first be considered.

225 a. *Effect of alternate sampling*

226 When estimating the correlation coefficient, the non-simultaneous transmission and re-
227 ception of H and V pulses must be accounted for. Assuming a Gaussian autocorrelation
228 function to correct for this staggered sampling (Sachidananda and Zrnic 1989) can lead to
229 unphysical samples where $\hat{\rho}_{hv} > 1$ (Illingworth and Caylor 1991). In our analysis, we employ
230 a cubic polynomial interpolation to obtain H and V power estimates at the intermediate
231 sampling intervals (Caylor 1989), which is very effective. We find that the interpolation
232 scheme works well: for drizzle with $\bar{L} = 2.4$, we observe that average values of \hat{L} , binned
233 by σ_v , are constant to within ± 0.02 as σ_v varies between 0.1—2 ms⁻¹. This is evidence
234 of successful interpolation, since there is no systematic trend to lower L values at higher
235 spectral widths.

236 b. *Signal-to-noise ratio*

237 The addition of noise to the received signals acts to reduce the correlation between H
238 and V time-series. The reduction factor, f , has been shown (Bringi et al. 1983) to vary
239 predictably as:

$$f = \frac{1}{\left(1 + \frac{1}{SNR_H}\right)^{\frac{1}{2}} \left(1 + \frac{1}{SNR_V}\right)^{\frac{1}{2}}} \quad (15)$$

240 for simultaneous (or accurately interpolated) H and V sampling, where SNR_H and SNR_V
241 are the signal-to-noise ratios for the H and V polarisations respectively. This was verified by

242 Illingworth and Caylor (1991) with measurements of ρ_{hv} in drizzle. Whilst it is in principle
 243 possible to correct for the presence of noise using this equation, due to the high degree of
 244 precision required in this work, only data with SNR > 34 dB are included in our analysis,
 245 which corresponds to a maximum achievable ρ_{hv} measurement of 0.9996. However, instru-
 246 mental effects (described in Section d below) will have the same effect of adding uncorrelated
 247 noise, and so in practice this maximum value is never reached.

248 *c. Effect of phase error*

249 To avoid a bias in $\hat{\rho}_{hv}$ due to random phase error from our magnetron system (Liu et al.
 250 1994), we cross correlate the power of the received echoes as opposed to the complex I and
 251 Q signals, and take the square root, following Illingworth and Caylor (1991).

252 *d. Instrumental effects*

253 Even in drizzle with very high SNR, antenna imperfections and other effects such as
 254 irregular magnetron pulse timing and pulse shape reproducibility will cause measured ρ_{hv} to
 255 always be < 1 (Illingworth and Caylor 1991; Liu et al. 1994) as effectively they cause the H
 256 and V pulses to sample slightly different volumes. Here, we propose a method to quantify
 257 and account for this bias, analogous to the SNR factor (Equation 15) suggested by Bringi
 258 et al. (1983). We consider the H and V echoes to consist of two parts: a common sample
 259 volume, and parts of each sample volume which are unique to a particular polarisation. By
 260 treating the former as “signal” and the latter as unwanted “noise”, we obtain an equation
 261 similar to Equation 15. Full details are provided in the appendix. The practical upshot
 262 is that the measured ρ_{hv} is the “true” ρ_{hv} multiplied by some dimensionless factor, f_{hv}^{max} ,
 263 relating to how well matched the H and V sample volumes are. For spherical drops, $\overline{\rho_{hv}}$
 264 should be unity. The estimates of $\overline{\rho_{hv}}$ for all such data should therefore be equal to f_{hv}^{max} .
 265 When comparing observations with simulated ρ_{hv} , we multiply each of the predicted values

266 by f_{hv}^{max} so that they are directly comparable to the observations. ρ_{hv} has been measured
 267 in drizzle ($Z_{DR} < 0.1$ dB) for a large number of samples on several days. Typically, f_{hv}^{max} is
 268 ≈ 0.996 , but varies by ± 0.001 from day to day, which we suggest is the result of slightly
 269 irregular magnetron pulse timing and shape reproducibility for the CAMRa system, which
 270 may be temperature dependent. For this reason, f_{hv}^{max} has been determined individually for
 271 each case.

272 **6. Using L and Z_{DR} to Estimate μ in Rainfall**

273 We now attempt to use our high-precision measurements of L to retrieve μ estimates
 274 in rainfall. The independence of (D_0, μ) and (L, Z_{DR}) on the drop number concentration
 275 means that a single L and Z_{DR} observation pair corresponds to a unique D_0 and μ value. In
 276 order to forward model L and Z_{DR} for various gamma distributions, we must first assume
 277 an appropriate drop shape model.

278 *a. Mean Drop Shapes*

279 There are numerous drop shape parameterisations in the literature. Here, we examine
 280 drop axis ratios and diameters from the recent experiments of Thurai and Bringi (2005),
 281 Szakáll et al. (2008) and the 4th order polynomial fit to many experiments given by Brandes
 282 et al. (2002). Figure 6a shows the mean axis ratio as a function of drop diameter, for each
 283 of these models. The Thurai and Bringi (2005) data suggests that mean drop shapes are
 284 slightly prolate for $D < 1$ mm, although it is in the margin of measurement error that the
 285 drops are spherical (Beard et al. 2010). Since it is known that drops become spherical as
 286 their diameter tends to 0 mm due to surface tension, our fit to the data is adapted so that
 287 drops < 1 mm are precisely spherical.

288 To choose the best mean drop shape model, a 5 hour dwell was made with CAMRa at a
 289 1.5° elevation angle over a nearby Joss-Waldvogel RD-80 impact disdrometer (approximately

290 7 km away) in a frontal rain band on 25 April 2014. The disdrometer measures drop sizes in
 291 127 size bins from 0.3 to 5.0 mm. The instrument is regularly calibrated by the manufacturer
 292 and rain rate estimated with this instrument agrees very well with that from a co-located
 293 rain gauge. Radar measurements of Z_{DR} are calibrated regularly (to within ± 0.1 dB) by
 294 making observations of drizzle (low Z), which we know to have a Z_{DR} value of 0 dB. The
 295 range resolution of the radar measurements is 75 m, and averaged to 30 s to match the
 296 integration time used by the disdrometer to estimate the DSD parameters. At this elevation
 297 angle, the radar was sampling rain at a height of 183m above the disdrometer. Figures
 298 6b—d show the observed radar measurement from the closest gate to the disdrometer, and
 299 the corresponding disdrometer Z_{DR} values calculated using the Thurai and Bringi (2005),
 300 Szakáll et al. (2008) and Brandes et al. (2002) drop shape models respectively. The Szakáll
 301 et al. (2008) axis ratios are systematically smaller compared to both of the other models for
 302 almost all D . Using this model makes the disdrometer estimates of Z_{DR} always larger than
 303 the radar estimates. Thurai and Bringi (2005) and Brandes et al. (2002) agree for $D = 2$ — 7
 304 mm, after which the axis ratios of Thurai and Bringi (2005) are closer to those of Szakáll
 305 et al. (2008). Therefore, radar and disdrometer Z_{DR} for the Thurai and Bringi (2005) and
 306 Brandes et al. (2002) models largely agree, apart from $Z_{DR} \lesssim 0.4$ dB. The largest differences
 307 between these models occurs for $D < 2$ mm. Here, Szakáll et al. (2008) and Brandes et al.
 308 (2002) predict more oblate drops than Thurai and Bringi (2005).

309 The Szakáll et al. (2008) model produces the largest radar-disdrometer overall bias of
 310 ≈ 0.23 dB. The biases from Brandes et al. (2002) for Z_{DR} bins of 0.2, 0.4 and 0.6 dB (\pm
 311 0.1 dB bin width) are 0.09, 0.16 and 0.13 dB respectively. For Thurai and Bringi (2005),
 312 they are only 0.04, 0.08 and 0.09 dB respectively, and are very similar to Brandes et al.
 313 (2002) at higher Z_{DR} . These reduced biases at low Z_{DR} suggest that the experimental
 314 results of Thurai and Bringi (2005) best represent natural raindrop shapes. We therefore
 315 chose this model in our analysis. It is unclear why the very small residual difference between
 316 radar and disdrometer estimates of Z_{DR} using the Thurai and Bringi (2005) shape model is

317 observed. Some possible explanations are that the radar calibration is slightly out causing a
318 systematic underestimation, the small sampling volume of the disdrometer could be biasing
319 Z_{DR} , or there could be residual error in the mean drop shape model. However, this very
320 small difference is unimportant for retrievals that follow.

321 *b. Drop Oscillations*

322 Drop oscillations increase the variety of shapes within a radar pulse volume at any given
323 time. This means that the \bar{L} we are attempting to estimate will be lower than that predicted
324 by modelling only the mean drop axis ratios for drops of a given size. In order to account
325 for this, we must parameterise these drop oscillations. In the Thurai and Bringi (2005)
326 experiment, artificial rain drops were created from a hose and allowed to fall 80 m from
327 a bridge before drop axis ratio and counts were measured with a 2D video disdrometer
328 (2DVD) on the valley floor. This fall distance is more than sufficient to allow the drops to
329 achieve steady state oscillations, and so the standard deviations of axis ratios measured in
330 this experiment are interpreted as drop oscillation amplitudes. However, the large standard
331 deviations of the axis ratios for $D < 2$ mm are likely artificial, caused by the finite resolution
332 of the 2DVD instrument (Beard et al. 2010). Since drop oscillations are thought to originate
333 from vortex shedding (Beard et al. 2010) which increases as a function of drop size, the
334 magnitude of oscillations should decrease eventually to zero as the drop diameter tends to
335 0 mm. Beard and Kubesh (1991) suggest that resonant drop oscillations occur for drop
336 sizes between 1.1 and 1.6 mm, however more recent measurements from the Mainz wind
337 tunnel show that amplitudes of the axis ratios for these drop sizes were less than 0.025
338 (Szakáll et al. 2010). For this reason, the polynomial fit to oscillation amplitude data from
339 the Mainz wind tunnel (Szakáll et al. 2010) is used for $D < 2$ mm, which has the desired
340 reduction in oscillation amplitude for small drops ¹. For $D > 2$ mm, we revert to the more

¹Equation 1 in Szakáll et al. (2010) does not agree with the fit in Figure 3 (black line). By digitising the Mainz wind tunnel data, we calculate that Equation 1 should in fact be $1.8 \times 10^{-3} D_0^2 + 1.07 \times 10^{-2} D_0$

341 statistically robust drop oscillations from Thurai and Bringi (2005). Since the oscillations
 342 are aerodynamically induced, with an amplitude only a function of the drop size, they should
 343 not vary with environmental conditions. In our analysis, the oscillations were included by
 344 integrating over Gaussian PDFs of axis ratios (Thurai and Bringi 2005) in our Gans theory
 345 computations. Figure 7 shows the effect of oscillations on computed L and Z_{DR} for values
 346 of $\mu = -1$ (black lines), $\mu = 16$ (grey lines). Including drop oscillations for the purpose of
 347 estimating μ becomes increasingly important with increasing Z_{DR} ; the difference between L
 348 at $\mu = 16$ computed with and without oscillations is as large as an equivalent change in μ of
 349 ≈ 8 . We find that the modification of the oscillation magnitudes for drop diameters < 2 mm
 350 has a relatively small impact (< 0.01) on predicted L for Z_{DR} larger than 0.8 dB where we
 351 attempt retrievals of μ . However, we find that the use of Szakáll et al. (2010) oscillations for
 352 all drop diameters has a large impact on predicted L values (for $\mu = -1$, L is ≈ 0.1 lower).
 353 This is potentially important for retrievals of μ .

354 Comparatively large amplitude (but short lived, lasting less than ≈ 0.4 s) collision in-
 355 duced oscillations can also occur (Szakáll et al. 2014). Rogers (1989) estimate that the
 356 collision rate for an average rain drop in a 55 dBZ rain column is $\approx 1 \text{ min}^{-1}$. This would
 357 imply that rain drops (even in very heavy rainfall) spend an almost negligible fraction of
 358 time ($\approx 0.5\%$) affected by collision-induced oscillations. Rain drop clustering increases the
 359 likelihood of these collisions (Jameson and Kostinski 1998). For rain rates of around 10
 360 mm hr^{-1} (comparable to those presented in the following case studies), McFarquhar (2004)
 361 estimate the collision rate to be $\approx 5 \text{ min}^{-1}$, implying drops are affected only 3% of the time.
 362 For very large rain rates (100 mm hr^{-1}), this fraction increases to 6% as the collision rate
 363 approximately doubles to 10 min^{-1} . Consequently, their impact on L measurements is likely
 364 to be small and can be ignored, other than for exceptional rain rates (Thurai et al. 2013).

365 Figure 8 shows how L varies as a function of Z_{DR} for gamma distributions with $\mu = -1$,
 366 0, 2, 4, 8, 12 and 16 computed using Gans theory with the drop shape and oscillation model
 367 discussed above. Note that lines of constant μ diverge with increasing Z_{DR} . For $Z_{DR} \gtrsim$

368 0.5 dB, it becomes possible to distinguish μ , given the typical error on an L measurement
369 (shown in Figure 9).

370 7. μ Retrieval Case Studies

371 We now estimate μ using measurements of L and Z_{DR} for stratiform rain case studies on
372 31 January, 25 April and 25 November 2014, and a convective case study on 22 May 2014.
373 Typical rain rates for each of these case studies can be found in Table 1. Dwells were made
374 at an elevation angle of 1.5° . Strict data quality filters were applied: $\text{SNR} > 34$ dB, linear
375 depolarisation ratio (LDR) < -27 dB (close to the limit of cross-polar isolation) to ensure
376 no melting particle contamination or ground clutter and range > 5 km to avoid near-field
377 effects. Theoretical L and Z_{DR} were computed using Gans theory using the drop shape and
378 oscillation model discussed in Section 6 (see Figure 8). Observations were averaged from
379 10 to 30 s and from range gates of 75 to 300 m to increase the measurement precision of
380 L . At each gate, the most likely pair of μ and D_0 given the observed L and Z_{DR} values
381 was obtained by selecting the closest point in a look-up table of Gamma DSD calculations.
382 Figure 9a shows the observed L binned every 0.02 and Z_{DR} binned every 0.05 dB for the
383 example of 25 November 2015. Overlaid are lines of constant $\mu = -1, 0, 2, 4, 8, 12$ and
384 16. Figure 9b is the same distribution normalised to sum to 1 for each Z_{DR} bin. The f_{hv}^{max}
385 on this day was calculated to be 0.9963 (see Section 5d). The observations of L and Z_{DR}
386 are generally well contained within the expected range. The median error on \hat{L} is $\sigma_L \approx$
387 0.025, and is shown as a representative error bar in Figure 9. A comparison of these data
388 with disdrometer measurements from Williams et al. (2014) is included. In this experiment,
389 the mass spectrum mean diameter (D_m) and mass spectrum standard deviation (σ_m) were
390 measured using a 2DVD. A $\sigma_m - D_m$ fit was derived from 18969 1-minute drop spectra
391 (which can readily be converted to a $\mu - D_0$ fit). This was in turn used to predict a $L - Z_{DR}$
392 relationship, shown by the grey dashed line. L and Z_{DR} were also predicted using the

393 proposed $\mu - \Lambda$ relationship of Cao et al. (2008), also derived from a 2DVD, where:

$$\Lambda = \frac{3.67 + \mu}{D_0} \quad (16)$$

394 This is shown by the black dashed line.

395 The median and inter-quartile range of retrieved μ per Z_{DR} bin for this day is shown in
396 Figure 10. The median retrieved μ is 5 at $Z_{DR} = 0.8$ dB, increasing to 8 for $Z_{DR} = 1.6$ dB.
397 There is significant spread in retrieved μ values, containing contributions from measurement
398 uncertainty on L , as well as “true ” microphysical variability. The impact of changes in L
399 on retrieved μ is non-linearly related to μ ; σ_L contributes more to retrieved μ variability
400 for more monodisperse (higher μ) DSDs, compared to more polydisperse (lower μ) DSDs.
401 Conversely, the contribution of σ_L to retrieved μ variability decreases as Z_{DR} increases, as
402 the dual polarisation signature is larger and μ is more easily distinguishable (see Figure 8).
403 To estimate the contribution that the uncertainty on L measurements makes to this observed
404 variability, μ was retrieved using the median $L \pm$ the representative uncertainty depicted
405 in Figure 9. This was then compared to the inter-quartile range of the retrieved μ for each
406 Z_{DR} bin. For Z_{DR} bins of 0.8, 1, 1.2, 1.4 and 1.6 dB, we estimate that 88%, 66% 32%, 31%
407 and 27% of the variability respectively can be attributed to σ_L . For $Z_{DR} > 1$ dB, most of
408 the variability seen in Figure 10 can be attributed to “true” microphysical variability.

409 Figure 11 shows a comparison with retrieved μ for all of the case studies collected. Each
410 of the dwells in January, April and November were made in stratiform rain, whereas the
411 May case study contains dwells from convective rain. Overlaid are predicted mean μ values
412 (solid grey) and upper and lower bounds that contain 55% of the measurements (dashed
413 grey) of Williams et al. (2014) as a function of Z_{DR} from the disdrometer measurements.
414 The solid black line shows the predicted $\mu - Z_{DR}$ using the $\mu - \Lambda$ relationship of Cao et al.
415 (2008). There is a large spread in the radar retrieved median μ values from case to case.
416 Each median μ estimate is from a very large number of retrieved μ estimates, such that the
417 standard error is smaller than the markers themselves, and so is not shown. The values of

418 retrieved μ in January are ≈ 0 , close to an exponential DSD for all Z_{DR} smaller than 1.1 dB.
419 This is below that predicted by Williams et al. (2014), but agrees well with μ predicted by
420 Cao et al. (2008). Interestingly, the case studies of April and November show μ increasing
421 with Z_{DR} between 0.5 dB and 1.5 dB, compared to the trend seen by Williams et al. (2014)
422 and Cao et al. (2008) towards an exponential DSD. The retrieved median μ values from
423 the May case study, although agreeing with the decreasing trend with Z_{DR} , are significantly
424 above the Cao et al. (2008) predictions and the upper bound of μ from Williams et al. (2014).
425 Our retrieval suggests that in this case, the rain rate would be overestimated by almost 2
426 dB if an exponential DSD or the fit of Cao et al. (2008) is assumed. Whereas the μ values
427 are not outside the full range of data measured by Williams et al. (2014), the use of the
428 proposed $\mu - D_m$ relationship would cause an overestimate of ≈ 1 dB (see Figure 1).

429 8. Discussion

430 Our retrievals of μ made using ρ_{hv} and Z_{DR} are typically larger than the radar estimates
431 of μ of between 1—3 by Thurai et al. (2008) and 0—2 of Illingworth and Caylor (1991).
432 Perhaps this is not surprising, given that the imperfect co-location of the H and V sample
433 volumes was unaccounted for, and their $\hat{\rho}_{hv}$ would have been biased low due to averaging ρ_{hv}
434 rather than L , both of which are accounted for in our data. Furthermore, Illingworth and
435 Caylor (1991) do not include drop oscillations in their retrievals, which will have led to a
436 significant underestimate of μ . Whereas there is some agreement of the magnitudes of μ for
437 $Z_{DR} < 1$ dB with predicted Williams et al. (2014) and Cao et al. (2008) values, the apparent
438 opposite trend towards more monodisperse distributions is consistent among 3 of the 4 case
439 studies. For the retrieved μ to agree with the trend predicted by Williams et al. (2014) or
440 Cao et al. (2008), a reduction in the drop oscillation amplitudes for smaller drops would be
441 required so that predicted L values are higher. However, this would not explain the difference
442 between the May retrieval results and the predicted μ from disdrometer measurements; we

443 estimate that it would require oscillations that are at least an order of magnitude *larger*
444 to bring these median μ estimates into agreement with Williams et al. (2014) or Cao et al.
445 (2008). An incorrect parameterisation of the drop oscillations alone is unlikely to be able to
446 account for the disagreement with Williams et al. (2014) and Cao et al. (2008), however, to
447 better establish the accuracy of the technique, a better quantification of raindrop oscillations
448 is desirable.

449 μ estimates derived using radar are sensitive to higher moments of the DSD, whereas
450 disdrometer estimates tend to use lower moments of the DSD (Cao and Zhang 2009). This
451 could be partly responsible for the differences between the radar and disdrometer estimated
452 μ values. If the DSD shape is not perfectly described by Equation 1, the “effective” μ which
453 is derived may be different even if the underlying DSD shape is the same. It is also possible
454 that what we have captured is simply natural variability of the DSD in different types of
455 rainfall (i.e convective and stratiform), and there is not a universal $\mu - D_0$ relationship. More
456 case studies are needed to gather a statistical understanding of the behaviour of μ using this
457 retrieval method.

458 *a. Implications for Operational Use of L*

459 Operational radar networks favour the use of rapid scan rates to maximise sample fre-
460 quency and total sample volume. For UK Met Office radars observing rain with 1 ms^{-1}
461 Doppler spectral width, each gate contains $N_{IQ} \approx 11$ ($\sigma_L \approx 0.3$). Clearly, many more
462 N_{IQ} are needed than are available for individual gate estimates of μ . Greater measurement
463 precision can be achieved by averaging (with the confidence interval computed using the
464 aggregated number of independent I and Q samples), and assuming μ is spatially conserved
465 over the chosen averaging area. To obtain a μ estimate over approximately 1 km^2 , for ex-
466 ample, would require the averaging of 2 rays and 10 gates (at a range from the radar of 30
467 km); this L estimate would be calculated using $N_{IQ} = 220$ ($\sigma_L \approx 0.058$). Whereas this may
468 not be sufficient to distinguish μ to as high a resolution as our retrieval (which uses long

dwells and $N_{IQ} > 1000$), this will at least be able to decipher whether μ is ‘high’ or ‘low’. Practically, as illustrated in Figure 1, this may be all that is necessary to offer improved rain rate estimates; it is relatively unimportant whether μ is 8 or 16, but it is very important to know if it is 0 or 4. Therefore, this method could (with sufficient care to ensure only rain echoes and good SNR) allow for improved rain rates using Z , Z_{DR} and L compared to only Z and Z_{DR} .

For the typical σ_L used in these calculations, we can approximate the error on the retrieved rain rate by considering the contribution of σ_L to the uncertainty in μ . For a ‘typical’ μ of 6, the range of retrieved μ is $\approx \pm 4$. By referring to Figure 1, we can see that this corresponds to a difference in rain rate of ± 0.5 dB, or $\pm 12.5\%$. The impact of uncertainty in μ on rain rate is almost constant for all Z_{DR} (each of the μ lines are approximately parallel in Figure 1 for $Z_{DR} \gtrsim 0.5$ dB). Therefore, this error will decrease for higher rain rates as the contribution of σ_L to uncertainty in μ decreases as a function of Z_{DR} .

9. Conclusions

In this paper, a new variable $L = -\log_{10}(1 - \rho_{hv})$ is defined that is Gaussian distributed with a width predictable by the number of independent I and Q samples, which in turn can be estimated using the Doppler spectral width. This allows, for the first time, the construction of rigorous confidence intervals on each ρ_{hv} measurement. The predicted errors using this new method were verified using high quality measurements in drizzle from the Chilbolton Advanced Meteorological Radar.

Importantly, the proposed method is of much greater practical use than the linear perturbation error estimation method, as it does not require knowledge of the unknown ‘true’ ρ_{hv} that one is trying to estimate. The method works for both simultaneous or accurately interpolated alternate sampling. However, it does not work for alternate estimators which rely on the Gaussian autocorrelation function to estimate the zero-lag correlation between

494 H and V pulses (Sachidananda and Zrnica 1989), where ρ_{hv} estimates can be > 1 .

495 A new technique to account for the imperfect co-location of H and V sampling vol-
496 umes on ρ_{hv} measurements is presented. The impact of drop oscillations on the observed L
497 measurements was shown to be significant; omitting oscillations from our Gans simulations
498 leads to an underestimate of retrieved μ of ≈ 8 . We further show that failure to use L over
499 ρ_{hv} measurements when averaging can lead to a significant bias low in ρ_{hv} estimates (and
500 consequently μ), particularly for very short dwell times such as those used operationally.

501 High-precision measurements of L and Z_{DR} in rainfall are then used to estimate μ in
502 the gamma DSD for four case studies. We find that our estimates of μ in stratiform rain
503 somewhat agree in magnitude with those from disdrometer studies for small Z_{DR} , but there
504 appears to be a tendency to more monodisperse DSDs between $Z_{DR} = 0.8$ and 1.5 dB, unlike
505 the trend towards an exponential distribution suggested by disdrometer measurements. The
506 convective case study does display this trend toward lower μ as Z_{DR} increases, but the
507 magnitude of μ remains much larger than predicted by disdrometer measurements. If true,
508 this would lead to overestimates of retrieved rain rate by ≈ 1 dB if the $\mu - D_m$ relationship of
509 Williams et al. (2014) is used, or 2 dB if an exponential distribution or the $\mu - \Lambda$ relationship
510 of Cao et al. (2008) is used. We find that the μ retrieval exhibits sensitivity to the choice of
511 drop oscillation model. A better understanding of raindrop oscillations would be useful to
512 fully establish the accuracy of our retrieval technique.

513 The variability in our radar retrieved μ could simply be natural variability of the DSD
514 between convective and stratiform rainfall; there may not be a universal $\mu - D_0$ relationship.
515 More case studies are desirable to investigate this further.

516 The μ retrieval technique employed here offers improvements over the radar estimates
517 of Illingworth and Caylor (1991) and Thurai et al. (2008). Illingworth and Caylor (1991)
518 did not take into account the imperfect co-location of the H and V sample volumes on
519 measurements of ρ_{hv} , the effect of drop oscillations, or the fact their ρ_{hv} estimates would
520 be biased low by averaging short time-series. Each of these effects would cause μ to be

521 underestimated. The same is true of Thurai et al. (2008), however drop shapes measured by
 522 2DVD measurements include oscillations, and so are included in their μ estimates.

523 The new error statistics of ρ_{hv} presented here could aid operational applications that
 524 require uncertainty on $\hat{\rho}_{hv}$ to be quantified, or use averages of $\hat{\rho}_{hv}$. The use of L operationally
 525 to retrieve μ is limited by use of rapid scan rates and the corresponding few independent
 526 I and Q samples. However, assuming that μ is a smoothly varying parameter, averaging
 527 L could help improve rain rate retrievals; the uncertainty on operationally retrieved rain
 528 rates using the retrieval technique presented here is estimated to be approximately $\pm 12.5\%$.
 529 Practically, retrieved rain rates are less affected by changes in higher values of μ compared
 530 to changes in lower values. Therefore, operationally, simply being able to distinguish regions
 531 of ‘high’ and ‘low’ μ with L could be sufficient to provide an improvement over existing
 532 $Z - Z_{DR}$ retrieval techniques.

533 APPENDIX

534 **The effect of imperfectly co-located H and V samples on ρ_{hv}**

535 Consider two measurements of the (complex) amplitudes at horizontal and vertical polar-
 536 isation A_H and A_V . If the two polarisations do not have perfectly matched sample volumes,
 537 then each amplitude is the sum of (i) a component which is common to both polarisations
 538 C_H, C_V , (ii) a component which is different for each polarisation D_H, D_V :

$$A_H = C_H + D_H \tag{A1}$$

539 (and similarly $A_V = C_V + D_V$). The co-polar correlation coefficient is:

$$\rho_{hv} = \frac{\sum A_H A_V^*}{\sqrt{\sum |A_H|^2 \sum |A_V|^2}} \tag{A2}$$

540 where the sums \sum are taken over many reshufflings of the raindrops. Substituting in the
 541 expressions for A_H and A_V leads to:

$$\rho_{hv} = \frac{\sum C_H C_V^* + \sum D_H C_V^* + \sum C_H D_V^* + \sum D_H D_V^*}{\sqrt{\sum |C_H + D_H|^2 \sum |C_V + D_V|^2}} \quad (\text{A3})$$

542 The first term in the numerator dominates as the number of pulses is increased. This is
 543 because D_H, D_V , are uncorrelated ith C_V, C_H (because the reshuffling of particles in the
 544 different sample volumes is not connected or organised in any way), while C_H and C_V are
 545 highly correlated (because the true ρ_{hv} is close to 1). The final term is small because $D_H,$
 546 D_V , are not correlated (by the same argument), and this term is small in any case since
 547 $|D| \ll |C|$

548 This leaves us with:

$$\rho_{hv} = \frac{\sum C_H C_V^*}{\sqrt{\sum |C_H + D_H|^2 \sum |C_V + D_V|^2}} \quad (\text{A4})$$

549 In the case of a perfect radar with perfect co-location of the H and V samples, then D_H, D_V
 550 are zero and we get a correlation coefficient which is the true ρ_{hv} which we are trying to
 551 obtain (ie setting $A = C$ in equation A2).

552 In general, for an imperfect radar, we have $D_H, D_V > 0$ and from the results above we
 553 see that:

$$\rho_{hv} = \rho_{hv}^{\text{true}} \times f_{hv}^{\text{max}} \quad (\text{A5})$$

554 where

$$f_{hv}^{\text{max}} = \left(\frac{\sum |C_H|^2}{\sum |C_H + D_H|^2} \times \frac{\sum |C_V|^2}{\sum |C_V + D_V|^2} \right)^{1/2} \quad (\text{A6})$$

555 This result is directly analogous to the results of Brangi et al. (1983) on ρ_{hv} in the presence
 556 of noise. If we identify C as our “signal” and D as our “noise” this equation is identical to
 557 Equation A1.

558 Crucially, the relationship between the true ρ_{hv} (ρ_{hv}^{true}) and the one which is actually
 559 observed is determined simply by how much power (on average over many pulses) comes

560 from the particles which are different for the H and V sample volumes, relative to how much
561 power comes from the particles which are common to the H and V sample volumes, and that
562 this factor should be constant for different microphysical situations. Thus if we can measure
563 ρ_{hv} in drizzle where we know $\rho_{hv}^{\text{true}} = 1$, then the measured ρ_{hv} is simply equal to f_{hv}^{max} . This
564 scaling factor can then be applied to data from all other situations.

565 **Acknowledgements**

566 The authors would like to thank the staff at the Chilbolton Facility of Atmospheric and
567 Radio Research for operation and maintenance of CAMRa, and Chris Walden in particular
568 for his help with time-series data acquisition. The first author was funded by a National
569 Environmental Research Council Studentship.

REFERENCES

- 572 Andrić, J., M. R. Kumjian, D. S. Zrnić, J. M. Straka, and V. M. Melnikov, 2013: Polarimetric
573 signatures above the melting layer in winter storms: An observational and modeling study.
574 *Journal of Applied Meteorology and Climatology*, **52 (3)**, 682–700.
- 575 Balakrishnan, N., and D. Zrnic, 1990: Use of polarization to characterize precipitation and
576 discriminate large hail. *Journal of the atmospheric sciences*, **47 (13)**, 1525–1540.
- 577 Beard, K. V., V. Bringi, and M. Thurai, 2010: A new understanding of raindrop shape.
578 *Atmospheric Research*, **97 (4)**, 396–415.
- 579 Beard, K. V., and R. J. Kubesh, 1991: Laboratory measurements of small raindrop dis-
580 tortion. Part 2: Oscillation frequencies and modes. *Journal of the atmospheric sciences*,
581 **48 (20)**, 2245–2264.
- 582 Brandes, E. A., and K. Ikeda, 2004: Freezing-level estimation with polarimetric radar. *Jour-
583 nal of Applied Meteorology*, **43 (11)**, 1541–1553.
- 584 Brandes, E. A., G. Zhang, and J. Vivekanandan, 2002: Experiments in rainfall estimation
585 with a polarimetric radar in a subtropical environment. *Journal of Applied Meteorology*,
586 **41 (6)**, 674–685.
- 587 Bringi, V. N., and V. Chandrasekar, 2001: *Polarimetric Doppler Weather Radar, Principles
588 and Applications*. Cambridge University Press, Cambridge, UK.
- 589 Bringi, V. N., T. A. Seliga, and S. M. Cherry, 1983: Statistical properties of the dual-
590 polarization differential reflectivity (Z_{DR}) radar signal. *IEEE Trans. Geosci. Rem Sens.*,
591 **GE-21 (2)**, 215 –220, doi:10.1109/TGRS.1983.350491.

- 592 Cao, Q., and G. Zhang, 2009: Errors in estimating raindrop size distribution parameters
593 employing disdrometer and simulated raindrop spectra. *Journal of Applied Meteorology*
594 *and Climatology*, **48** (2), 406–425.
- 595 Cao, Q., G. Zhang, E. Brandes, T. Schuur, A. Ryzhkov, and K. Ikeda, 2008: Analysis of video
596 disdrometer and polarimetric radar data to characterize rain microphysics in Oklahoma.
597 *Journal of Applied Meteorology and Climatology*, **47** (8), 2238–2255.
- 598 Caylor, I. J., 1989: Radar observations of maritime clouds using dual linear polarisation.
599 Ph.D. thesis, University of Manchester, Manchester, UK.
- 600 Caylor, J., and A. J. Illingworth, 1989: Identification of the bright band and hydromete-
601 ors using co-polar dual polarization radar. *Preprints, 24th Conf. on Radar Meteorology,*
602 *Florida, USA, Amer. Meteor. Soc.* 352–357.
- 603 Doviak, R. J., and Zrnica, 2006: *Doppler Radar and Weather Observations*. Dover Publica-
604 tions, Inc., Mineola, New York, USA.
- 605 Fisher, R. A., 1915: Frequency distribution of the values of the correlation coefficient in
606 samples from an indefinitely large population. *Biometrika*, **10** (4), 507–521.
- 607 Giangrande, S. E., J. M. Krause, and A. V. Ryzhkov, 2008: Automatic designation of the
608 melting layer with a polarimetric prototype of the WSR-88D radar. *Journal of Applied*
609 *Meteorology and Climatology*, **47** (5), 1354–1364.
- 610 Goddard, J., J. D. Eastment, and M. Thurai, 1994: The chilbolton advanced meteorological
611 radar: A tool for multidisciplinary atmospheric research. *Electronics & communication*
612 *engineering journal*, **6** (2), 77–86.
- 613 Illingworth, A. J., and I. J. Caylor, 1991: Co-polar correlation measurements of precipitation.
614 *Preprints, 25th Int. Conf. on Radar Meteorology, Paris, France, Amer. Meteor. Soc.* 650–
615 653.

616 Jameson, A., 1987: Relations among linear and circular polarization parameters measured
617 in canted hydrometeors. *Journal of Atmospheric and Oceanic Technology*, **4** (4), 634–646.

618 Jameson, A., and A. Kostinski, 1998: Fluctuation properties of precipitation. Part ii: Re-
619 consideration of the meaning and measurement of raindrop size distributions. *Journal of*
620 *the atmospheric sciences*, **55** (2), 283–294.

621 Johnson, R. W., D. V. Kliche, and P. L. Smith, 2014: Maximum likelihood estimation
622 of gamma parameters for coarsely binned and truncated raindrop size data. *Quarterly*
623 *Journal of the Royal Meteorological Society*, **140** (681), 1245–1256.

624 Liu, L., V. N. Bringi, V. Chandrasekar, E. A. Mueller, and A. Mudukutore, 1994: Analysis of
625 the co-polar correlation coefficient between horizontal and vertical polarizations. *Journal*
626 *of Atmospheric and Oceanic Technology*, **11** (4), 950–963.

627 McFarquhar, G. M., 2004: The effect of raindrop clustering on collision-induced break-up of
628 raindrops. *Quarterly Journal of the Royal Meteorological Society*, **130** (601), 2169–2190.

629 Papoulis, A., 1965: Probability, random variables, and stochastic processes.

630 Park, H. S., A. V. Ryzhkov, D. Zrnica, and K.-E. Kim, 2009: The hydrometeor classification
631 algorithm for the polarimetric WSR-88D: Description and application to an mcs. *Weather*
632 *and Forecasting*, **24** (3), 730–748.

633 Rogers, R., 1989: Raindrop collision rates. *Journal of the Atmospheric Sciences*, **46** (15),
634 2469–2472.

635 Sachidananda, M., and D. Zrnica, 1989: Efficient processing of alternately polarized radar
636 signals. *Journal of Atmospheric and Oceanic Technology*, **6** (1), 173–181.

637 Schafer, R., S. Avery, P. May, D. Rajopadhyaya, and C. Williams, 2002: Estimation of
638 rainfall drop size distributions from dual-frequency wind profiler spectra using deconvolu-

639 tion and a nonlinear least squares fitting technique. *Journal of Atmospheric and Oceanic*
640 *Technology*, **19** (6), 864–874.

641 Seliga, T., and V. Bringi, 1976: Potential use of radar differential reflectivity measurements
642 at orthogonal polarizations for measuring precipitation. *Journal of Applied Meteorology*,
643 **15** (1), 69–76.

644 Szakáll, M., K. Diehl, S. K. Mitra, and S. Borrmann, 2008: A wind tunnel study on the
645 oscillation of freely falling raindrops. *Proc. Fifth European Conf. on Radar in Meteorology*
646 *and Hydrology (ERAD 2008)*.

647 Szakáll, M., S. Kessler, K. Diehl, S. K. Mitra, and S. Borrmann, 2014: A wind tunnel study
648 of the effects of collision processes on the shape and oscillation for moderate-size raindrops.
649 *Atmospheric Research*, **142**, 67–78.

650 Szakáll, M., S. K. Mitra, K. Diehl, and S. Borrmann, 2010: Shapes and oscillations of falling
651 raindrops — a review. *Atmospheric Research*, **97** (4), 416–425.

652 Tabary, P., A. Le Henaff, G. Vulpiani, J. Parent-du Châtelet, and J. Gourley, 2006: Melting
653 layer characterization and identification with a C-band dual-polarization radar: A long-
654 term analysis. *Proc. Fourth European Radar Conf*, 17–20.

655 Tang, L., J. Zhang, C. Langston, J. Krause, K. Howard, and V. Lakshmanan, 2014: A
656 physically based precipitation–nonprecipitation radar echo classifier using polarimetric
657 and environmental data in a real-time national system. *Weather and Forecasting*, **29** (5),
658 1106–1119.

659 Thurai, M., and V. Bringi, 2005: Drop axis ratios from a 2D video disdrometer. *Journal of*
660 *Atmospheric & Oceanic Technology*, **22** (7).

661 Thurai, M., D. Hudak, and V. Bringi, 2008: On the possible use of co-polar correlation coeffi-

662 cient for improving the drop size distribution estimates at C band. *Journal of Atmospheric*
663 *and Oceanic Technology*, **25 (10)**, 1873–1880.

664 Thurai, M., M. Szakáll, V. N. Bringi, and S. K. Mitra, 2013: Collision-induced drop oscilla-
665 tions from wind-tunnel experiments. *Proc. 36th Conference on Radar Meteorology*.

666 Tokay, A., A. Kruger, and W. F. Krajewski, 2001: Comparison of drop size distribution mea-
667 surements by impact and optical disdrometers. *Journal of Applied Meteorology*, **40 (11)**,
668 2083–2097.

669 Torlaschi, E., and Y. Gingras, 2003: Standard deviation of the co-polar correlation coefficient
670 for simultaneous transmission and reception of vertical and horizontal polarized weather
671 radar signals. *Journal of Atmospheric and Oceanic Technology*, **20 (5)**, 760–766.

672 Ulbrich, C. W., 1983: Natural variations in the analytical form of the raindrop size distri-
673 bution. *Journal of Climate and Applied Meteorology*, **22 (10)**, 1764–1775.

674 Unal, C., 2015: High resolution raindrop size distribution retrieval based on the doppler spec-
675 trum in the case of slant profiling radar. *Journal of Atmospheric and Oceanic Technology*,
676 **(2015)**.

677 Williams, C. R., 2002: Simultaneous ambient air motion and raindrop size distributions
678 retrieved from UHF vertical incident profiler observations. *Radio Science*, **37 (2)**, 8–1.

679 Williams, C. R., and Coauthors, 2014: Describing the shape of raindrop size distributions
680 using uncorrelated raindrop mass spectrum parameters. *Journal of Applied Meteorology*
681 *and Climatology*, **53 (5)**, 1282–1296.

682 Wilson, D. R., A. J. Illingworth, and T. M. Blackman, 1997: Differential doppler velocity:
683 A radar parameter for characterizing hydrometeor size distributions. *Journal of Applied*
684 *Meteorology*, **36 (6)**, 649–663.

685 **List of Tables**

686 1 Typical rain rates (R) for each of the case studies, calculated from disdrometer
687 measurements (April) and radar retrieved N_0 , D_0 and μ values (January, May
688 and November) 32

TABLE 1. Typical rain rates (R) for each of the case studies, calculated from disdrometer measurements (April) and radar retrieved N_0 , D_0 and μ values (January, May and November)

Month	Typical R (mm hr ⁻¹)	Peak R (mm hr ⁻¹)
31 January 2014	1–3	8
25 April 2014	2–3	7
22 May 2014	2–7	>30
25 November 2014	2–5	10

List of Figures

- 689 **List of Figures**
- 690 1 Rain rate (in dB referenced to 1 mm hr⁻¹) per unit radar reflectivity as a
691 function of Z_{DR} computed using Gans theory for gamma distributions of μ
692 = -1, 0, 2, 4, 8, 12 and 16. The rain rate can vary by as much as 2.5 dB for
693 a given pair of Z and Z_{DR} observations as a result of drop spectrum shape
694 variability. 36
- 695 2 Example time-series (0.5 s) for single 75 m gates from 1.5° elevation dwells in
696 (a) drizzle ($Z_{DR} = 0$ dB) at 1203 UTC on 6 February 2014, and (b) heavier
697 rainfall ($Z_{DR} = 1.1$ dB) at 1706 UTC on 31 January 2014. For both examples,
698 SNR > 40 dB. For drizzle, the H and V echo time-series vary in unison as the
699 drops are all spherical. In heavier rainfall, the broader axis ratio distribution
700 causes the H and V time-series to be less correlated. The rate of fluctuation
701 of the signals is determined by the Doppler spectral width. 37
- 702 3 The frequency distribution of (a) $\hat{\rho}_{hv}$ calculated from 1159 time-series (0.21 s,
703 75 m gates) in drizzle ($Z_{DR} < 0.1$ dB) and (b) $\hat{L} = -\log_{10}(1-\hat{\rho}_{hv})$. The data
704 was collected at 1203 UTC on 6 February 2014 during a 1.5° elevation dwell
705 and has very high SNR (> 40 dB). σ_v for these data ranges between 0.9—1.3
706 ms⁻¹. Overplotted on \hat{L} is a Gaussian curve with same mean and standard
707 deviation as the measured distribution. 38
- 708 4 σ_L as a function of the number of independent I and Q samples used to
709 estimate L for high SNR measurements in drizzle ($Z_{DR} < 0.1$ dB, SNR > 40
710 dB) at 1203 UTC on 6 February. Different markers correspond to different
711 Doppler spectral widths. 39

- 712 5 (a) A comparison of the confidence intervals calculated using the new method
713 and that of Torlaschi and Gingras (2003) in rain ($\overline{\rho_{hv}} = 0.98$) and (b) the
714 bias introduced by averaging $\rho_{\hat{h}v}$ instead of \hat{L} , as a function of N_{IQ} . For
715 all N_{IQ} , the lower confidence interval is higher for the Torlaschi and Gingras
716 (2003) method, particularly for lower N_{IQ} , due to the asymmetric nature of
717 the confidence intervals on ρ_{hv} using the new method. Averaging $\rho_{\hat{h}v}$ and not
718 \hat{L} for small N_{IQ} can lead to a large bias. 40
- 719 6 (a) Comparison of mean drop axis ratios as a function of equivalent drop
720 diameter (D) from recent experiments of Thurai and Bringi (2005), Szakáll
721 et al. (2008) and the 4th order polynomial fit of older experimental data con-
722 structed by Brandes et al. (2002). The model of Thurai and Bringi (2005) has
723 been adapted so that drops are spherical for $D < 1$ mm. Panels (b)—(d) show
724 radar and disdrometer Z_{DR} comparisons calculated using Thurai and Bringi
725 (2005), Szakáll et al. (2008) and Brandes et al. (2002) from a 5 hour dwell
726 over a nearby Joss-Waldvogel RD-80 impact disdrometer (approximately 7
727 km away) in a frontal rain band on 25 April 2014. The time resolution of the
728 radar measurements was decreased to 30 s to match the integration time of
729 the disdrometer. At a 1.5° elevation angle, the radar was sampling rain at
730 a height of ≈ 183 m above the disdrometer. The dashed line is a 1:1 line.
731 The smallest biases are achieved with the Thurai and Bringi (2005) model,
732 especially for smaller Z_{DR} , suggesting that these shapes best represent those
733 of natural rain drops. Therefore, this model is chosen for the analysis. 41
- 734 7 Predicted L and Z_{DR} values for gamma distributions of $\mu = -1$ (solid) and
735 16 (dashed) with no oscillations (grey), and including oscillations (black).
736 The inclusion of drop oscillations are crucial to interpretation of L and Z_{DR}
737 measurements. The f_{hv}^{max} is assumed to be 0.9963 to match the case study in
738 Section 7. 42

- 739 8 Theoretical L and Z_{DR} computed using Gans theory for gamma distributions
740 with $\mu = -1, 0, 2, 4, 8, 12$ and 16 , using Thurai and Bringi (2005) mean drop
741 axis ratios and oscillation model described in Section 6b. The precision of L
742 required to estimate μ decreases as Z_{DR} increases. The f_{hv}^{max} is assumed to
743 be 0.9963 to match the case study in Section 7. 43
- 744 9 (a) 2D PDF of L and Z_{DR} observations, and (b) normalised 2D PDF such
745 that the distribution equals 1 for each Z_{DR} bin for observations of L and Z_{DR}
746 collected from dwells on 25 November 2014. L is binned over 0.02 , and Z_{DR}
747 every 0.05 dB. Overplotted are theoretical L and Z_{DR} computed using Gans
748 theory for gamma distributions of $\mu = -1, 0, 2, 4, 8, 12$ and 16 . Typical
749 errors on L and Z_{DR} are shown as error bars; the error on Z_{DR} is very small.
750 The grey dashed line is the predicted L and Z_{DR} observations using DSD
751 parameters from the power-law fit to disdrometer measurements in Williams
752 et al. (2014). The black dashed line is the predicted L and Z_{DR} observations
753 using the $\mu - \Lambda$ relationship of Cao et al. (2008). The f_{hv}^{max} for this day is
754 measured to be 0.9963 . 44
- 755 10 Box plot of retrieved μ as a function of Z_{DR} for Z_{DR} bins of 0.2 dB on 25
756 November 2014, showing the median and inter-quartile range of the data. 45
- 757 11 Median retrieved μ as a function of Z_{DR} for Z_{DR} bins of 0.1 dB for case
758 studies of 31 January, 25 April, 22 May and 25 November 2014. The solid line
759 is the predicted μ as a function of Z_{DR} from the power law fit to disdrometer
760 measurements of Williams et al. (2014), and σ_μ corresponds to the upper and
761 lower bounds that contain 55% of the data. The solid black line shows the
762 predicted $\mu - Z_{DR}$ using the $\mu - \Lambda$ relationship of Cao et al. (2008). 46

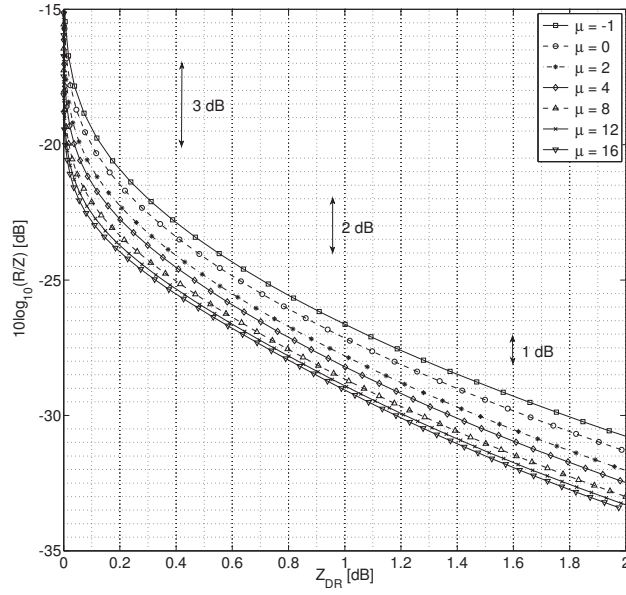


FIG. 1. Rain rate (in dB referenced to 1 mm hr^{-1}) per unit radar reflectivity as a function of Z_{DR} computed using Gans theory for gamma distributions of $\mu = -1, 0, 2, 4, 8, 12$ and 16 . The rain rate can vary by as much as 2.5 dB for a given pair of Z and Z_{DR} observations as a result of drop spectrum shape variability.

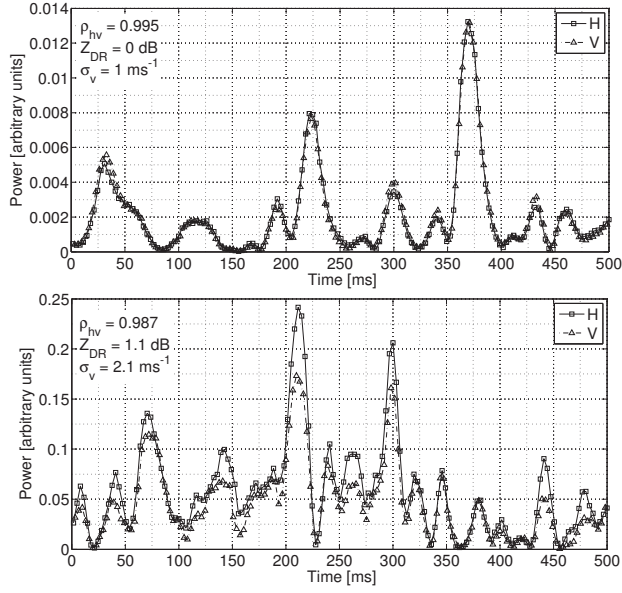


FIG. 2. Example time-series (0.5 s) for single 75 m gates from 1.5° elevation dwells in (a) drizzle ($Z_{DR} = 0 \text{ dB}$) at 1203 UTC on 6 February 2014, and (b) heavier rainfall ($Z_{DR} = 1.1 \text{ dB}$) at 1706 UTC on 31 January 2014. For both examples, $\text{SNR} > 40 \text{ dB}$. For drizzle, the H and V echo time-series vary in unison as the drops are all spherical. In heavier rainfall, the broader axis ratio distribution causes the H and V time-series to be less correlated. The rate of fluctuation of the signals is determined by the Doppler spectral width.

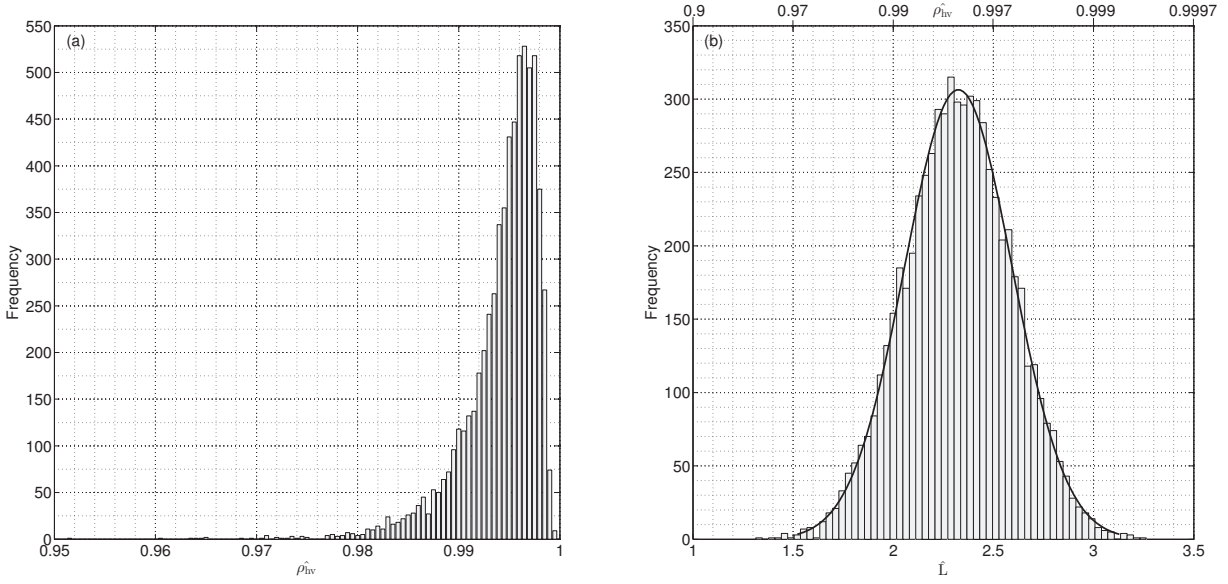


FIG. 3. The frequency distribution of (a) $\hat{\rho}_{hv}$ calculated from 1159 time-series (0.21 s, 75 m gates) in drizzle ($Z_{DR} < 0.1$ dB) and (b) $\hat{L} = -\log_{10}(1-\hat{\rho}_{hv})$. The data was collected at 1203 UTC on 6 February 2014 during a 1.5° elevation dwell and has very high SNR (> 40 dB). σ_v for these data ranges between $0.9\text{--}1.3$ ms^{-1} . Overplotted on \hat{L} is a Gaussian curve with same mean and standard deviation as the measured distribution.

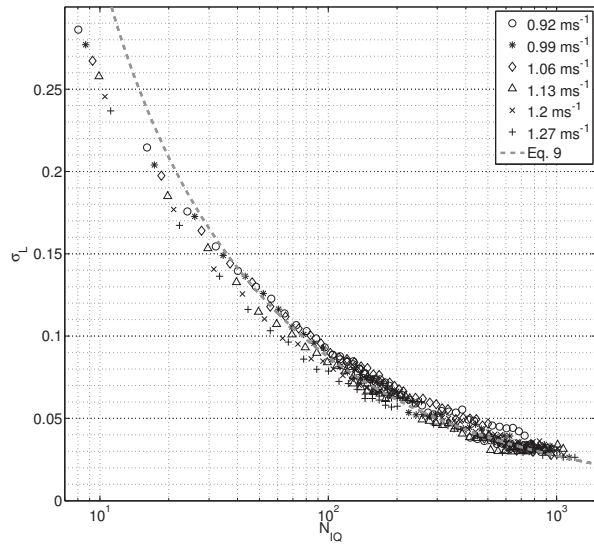


FIG. 4. σ_L as a function of the number of independent I and Q samples used to estimate L for high SNR measurements in drizzle ($Z_{DR} < 0.1$ dB, SNR > 40 dB) at 1203 UTC on 6 February. Different markers correspond to different Doppler spectral widths.

Rain, $\overline{\rho_{hv}} = 0.98$

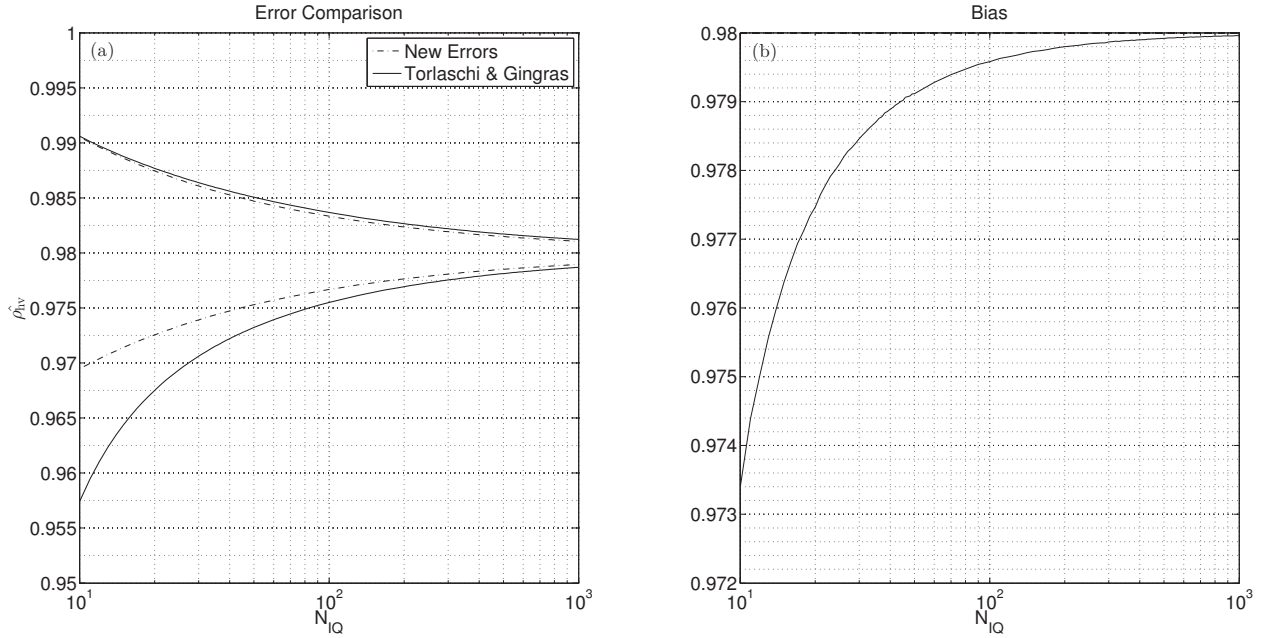


FIG. 5. (a) A comparison of the confidence intervals calculated using the new method and that of Torlaschi and Gingras (2003) in rain ($\overline{\rho_{hv}} = 0.98$) and (b) the bias introduced by averaging $\hat{\rho}_{hv}$ instead of \hat{L} , as a function of N_{IQ} . For all N_{IQ} , the lower confidence interval is higher for the Torlaschi and Gingras (2003) method, particularly for lower N_{IQ} , due to the asymmetric nature of the confidence intervals on ρ_{hv} using the new method. Averaging $\hat{\rho}_{hv}$ and not \hat{L} for small N_{IQ} can lead to a large bias.

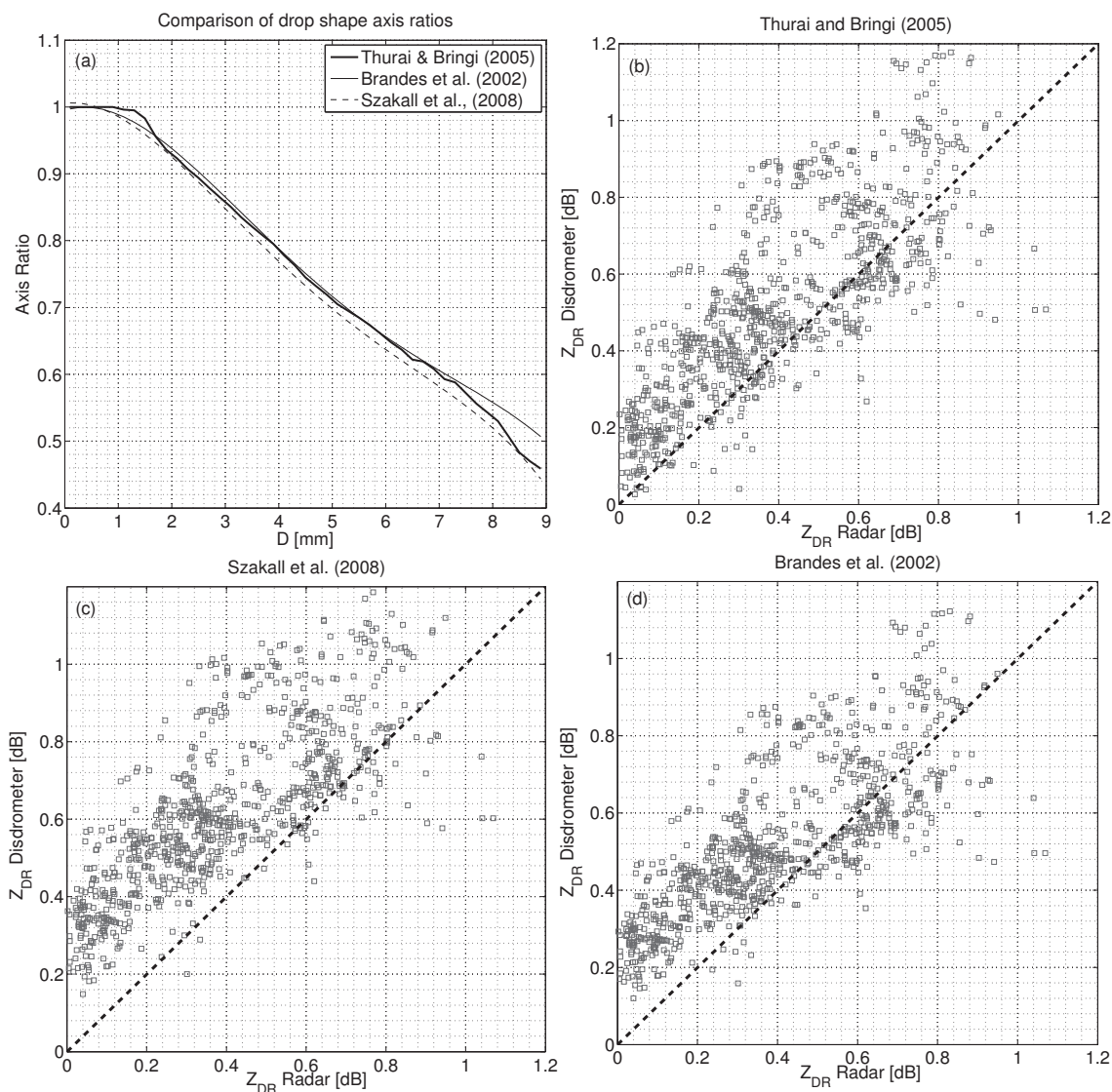


FIG. 6. (a) Comparison of mean drop axis ratios as a function of equivalent drop diameter (D) from recent experiments of Thurai and Bringi (2005), Szakáll et al. (2008) and the 4th order polynomial fit of older experimental data constructed by Brandes et al. (2002). The model of Thurai and Bringi (2005) has been adapted so that drops are spherical for $D < 1$ mm. Panels (b)—(d) show radar and disdrometer Z_{DR} comparisons calculated using Thurai and Bringi (2005), Szakáll et al. (2008) and Brandes et al. (2002) from a 5 hour dwell over a nearby Joss-Waldvogel RD-80 impact disdrometer (approximately 7 km away) in a frontal rain band on 25 April 2014. The time resolution of the radar measurements was decreased to 30 s to match the integration time of the disdrometer. At a 1.5° elevation angle, the radar was sampling rain at a height of ≈ 183 m above the disdrometer. The dashed line is a 1:1 line. The smallest biases are achieved with the Thurai and Bringi (2005) model, especially for smaller Z_{DR} , suggesting that these shapes best represent those of natural rain drops. Therefore, this model is chosen for the analysis.

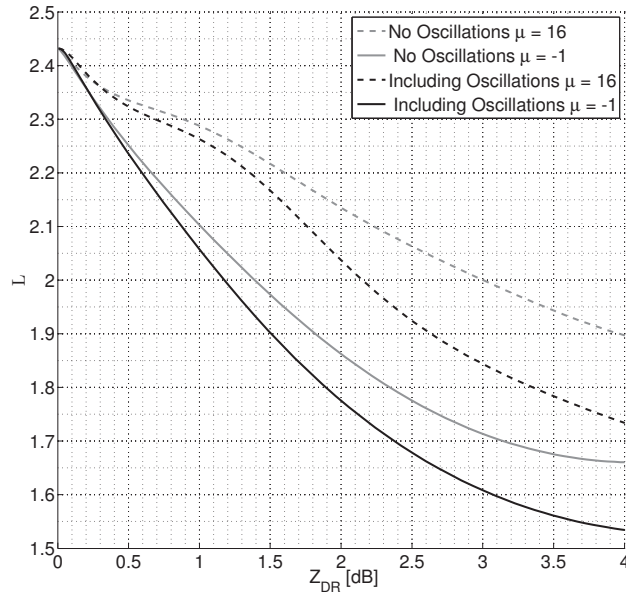


FIG. 7. Predicted L and Z_{DR} values for gamma distributions of $\mu = -1$ (solid) and 16 (dashed) with no oscillations (grey), and including oscillations (black). The inclusion of drop oscillations are crucial to interpretation of L and Z_{DR} measurements. The f_{hv}^{max} is assumed to be 0.9963 to match the case study in Section 7.

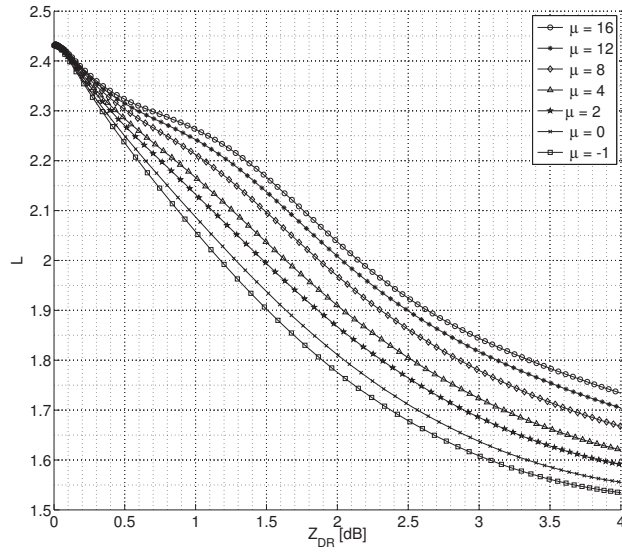


FIG. 8. Theoretical L and Z_{DR} computed using Gans theory for gamma distributions with $\mu = -1, 0, 2, 4, 8, 12$ and 16 , using Thurai and Bringi (2005) mean drop axis ratios and oscillation model described in Section 6b. The precision of L required to estimate μ decreases as Z_{DR} increases. The f_{hv}^{max} is assumed to be 0.9963 to match the case study in Section 7.

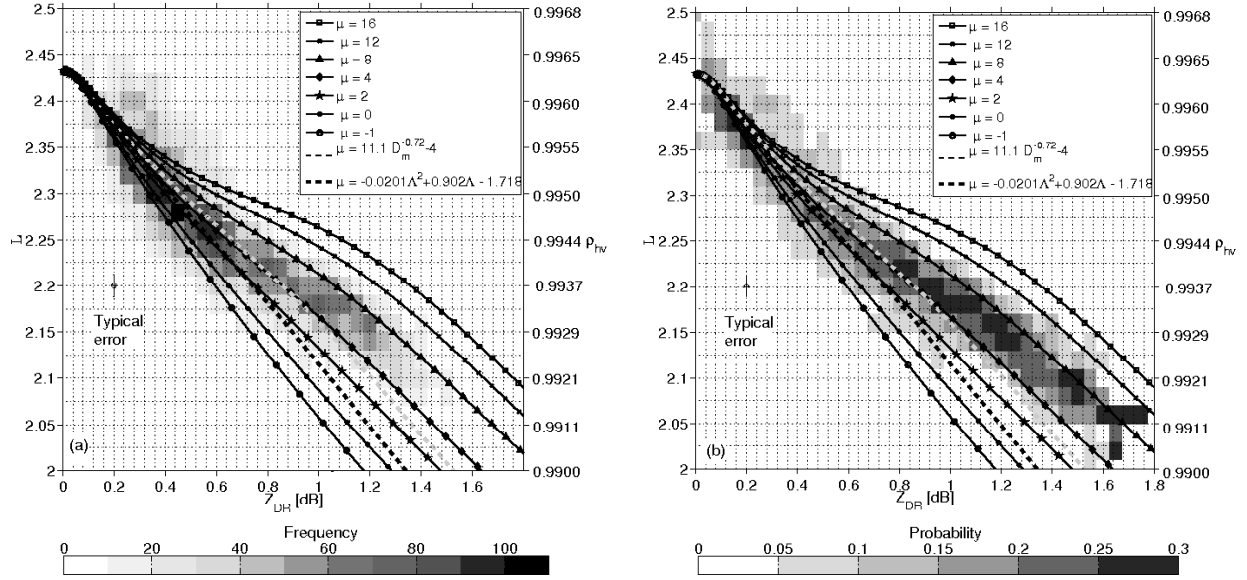


FIG. 9. (a) 2D PDF of L and Z_{DR} observations, and (b) normalised 2D PDF such that the distribution equals 1 for each Z_{DR} bin for observations of L and Z_{DR} collected from dwells on 25 November 2014. L is binned ever 0.02, and Z_{DR} every 0.05 dB. Overplotted are theoretical L and Z_{DR} computed using Gans theory for gamma distributions of $\mu = -1, 0, 2, 4, 8, 12$ and 16 . Typical errors on L and Z_{DR} are shown as error bars; the error on Z_{DR} is very small. The grey dashed line is the predicted L and Z_{DR} observations using DSD parameters from the power-law fit to disdrometer measurements in Williams et al. (2014). The black dashed line is the predicted L and Z_{DR} observations using the $\mu - \Lambda$ relationship of Cao et al. (2008). The f_{hv}^{max} for this day is measured to be 0.9963.

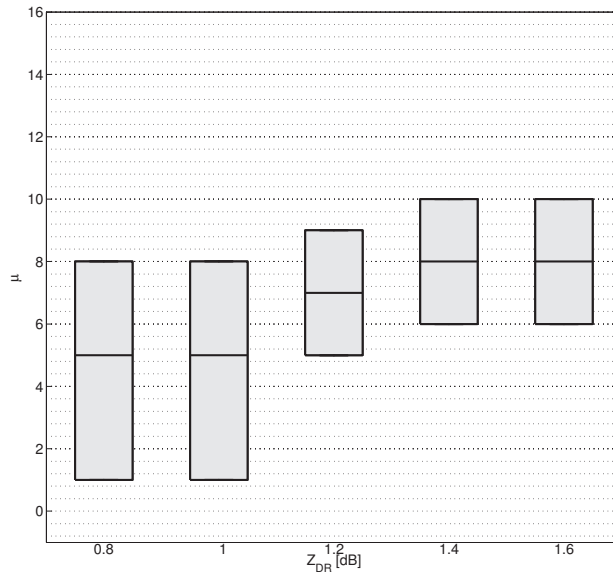


FIG. 10. Box plot of retrieved μ as a function of Z_{DR} for Z_{DR} bins of 0.2 dB on 25 November 2014, showing the median and inter-quartile range of the data.

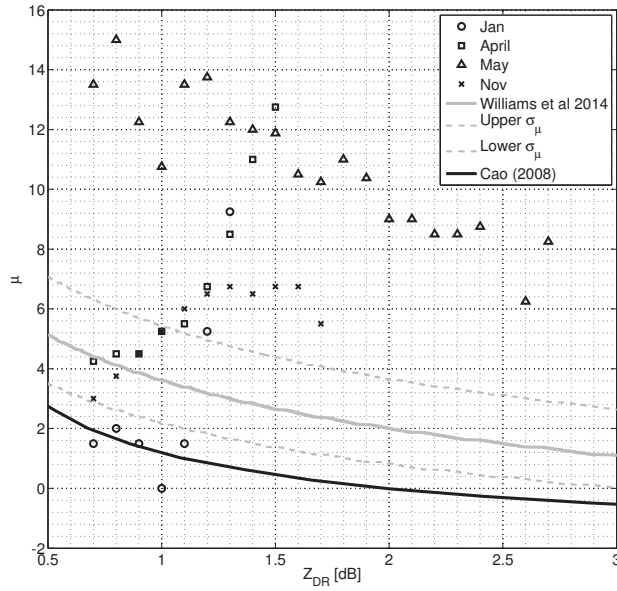


FIG. 11. Median retrieved μ as a function of Z_{DR} for Z_{DR} bins of 0.1 dB for case studies of 31 January, 25 April, 22 May and 25 November 2014. The solid line is the predicted μ as a function of Z_{DR} from the power law fit to disdrometer measurements of Williams et al. (2014), and σ_μ corresponds to the upper and lower bounds that contain 55% of the data. The solid black line shows the predicted $\mu - Z_{DR}$ using the $\mu - \Lambda$ relationship of Cao et al. (2008).