

FinalGen revisited: new discoveries

Article

Supplemental Material

Full position analyses

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FINALGEN revisited: new discoveries – detailed analyses

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Romero's FINALGEN of 2012 creates designer endgame tables for specific positions that feature no more than one non-pawn piece per side. Larger hard discs and faster solid-state discs have extended the reach of this software and encouraged its greater use. Some new discoveries illustrate here what is now feasible and how FINALGEN may be combined with other tools to reach definitive and likely truths.

Pedro Pérez Romero (2012) published the software FINALGEN to create position-specific 'EGT' endgame tables for chess positions meeting one criterion. Neither side must have more than one piece other than the king: the remaining men must be pawns. The software uses the fact that pawns' future positions are highly restricted. It is greatly helped if pawns are advanced, facing each other and in the extreme, blocking each other.

Since 2012, computer hardware and software have improved. 'HDD' hard discs have increased capacity and offer better GiB/\$. 'SSD' solid-state discs are replacing HDDs for operational purposes: they too are providing greater capacity and their life expectancy is improving. SSDs also fail more gracefully than HDDs that can crash in dramatic fashion. USB3.2 has superseded USB2.0 and 16-lane PCIe 5.0 will supersede today's commonly available 4-lane PCIe 3.0. These advances greatly facilitate and encourage the greater use of FINALGEN, increasing the number and complexity of positions that it may feasibly address on a specific computer.

Section 1 here surveys current performance figures using Romero's own FINALGEN tutorial examples and others. Sections 2-7 investigate specific chess positions which have been addressed by the authors. Our full chess analyses cannot be accommodated here but are available within a pgn file and, more fully illustrated, analysed and annotated, in the supplementary document with the repository version of this note (Müller and Haworth, 2019), along with the full statistics of the FINALGEN computations.

1 Key concepts and notation

Notation is needed to refer to various position depth metrics, expressible in moves or plies. Strategies for choosing moves should defend the theoretical value of the position and this is assumed. Here it should be noted that FINALGEN's verdict 'Win or draw' introduces a notional value of $\frac{3}{4}$ points! In a decisive position, and sometimes in a drawn position, move-optimality may further be determined by a strategy successively minimising (or maximising) depth using a set of depth metrics. Moves may be the only legal move or the only value-preserving move. In the context of a defined strategy, moves may be uniquely optimal, equi-optimal or sub-optimal. The amount by which a move is optimal (sub-optimal) indicates the positional-strength preserved (lost). It should be noted that depth-optimisation is not the only fruit. In chess studies and in games, the defender should be making the move that makes it hardest for the attacker to find the winning move. This is not necessarily the depth-maximising move which, in any case, can vary with metric and sometimes makes no chessic sense at all. Further, a winning strategy will be easier to understand if it does not take advantage of every helpful fortuity.

Depth metric notation:

- DTF: 'Depth according to FINALGEN', usually to a conversion before a clear win,
- DTM, DTC: 'Depth to Mate', 'Depth to Conversion' (i.e., to capture, conversion and/or mate),
- DTZ: 'Depth to Zeroing (of the *ply count*)', i.e., to pawn-push, capture and/or mate,

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DTZ₅₀¹: ‘Depth to Zeroing of *ply count* in the context of the FIDE 50-move draw-claim rule,
dtc/f/m/z 9p(m): an actual position depth of 9 plies (moves) in the DTC/F/M/Z metric.

Strategies for choosing optimal moves:

$S(\text{FMZ})^- \equiv \text{SF}^-\text{M}^-\text{Z}$: a strategy preserving value² and minimising *dtf*, *dtm* and *dtz* in that order,

$S(\text{F}\&\text{M})^+ \equiv S(\text{F}^+\&\text{M}^+)$: a strategy preserving value and maximising both *dtf* and *dtm*,

$S\omega/S\beta$: defined strategies $S\omega$ for White and $S\beta$ for Black,

$S(\text{FMZ})^\mp \equiv S(\text{FMZ})^-/S(\text{FMZ})^+$, and similarly, $S(\text{FMZ})^\pm \equiv S(\text{FMZ})^+/S(\text{FMZ})^-$.

Move optimality, sub-optimality and evaluation:

' ≡ a value-preserving, equi-optimal move (not noted in our pgn files),

" ≡ a value-preserving, uniquely-optimal move ('!' in our pgn files),

"" ≡ a uniquely value-preserving move ('!!' in our pgn files),

[±4pF] ≡ move conceding 4 ply in terms of the DTF metric, '+(-)' for attacker (defender),

° ≡ the only legal move,

? ≡ a move ‘dropping ¼ point’, e.g., FINALGEN finesses ‘win’ ↔ ‘win or draw’,

?? ≡ a move dropping ½ point, i.e., ‘win’ → ‘draw’ or ‘draw’ → ‘loss’, and

Eds+0.97 ≡ an engine (FRITZ14) evaluation of +0.97 at nominal ply-depth *s* in its search.

Further, it is worth noting that our use of ‘!’ and ‘!!’ in the pgn files is unusual, and in particular, that these symbols are not emoticons marking notably perceptible and/or surprising moves as is the custom. Indeed, many of the moves denoted in this way are entirely obvious.

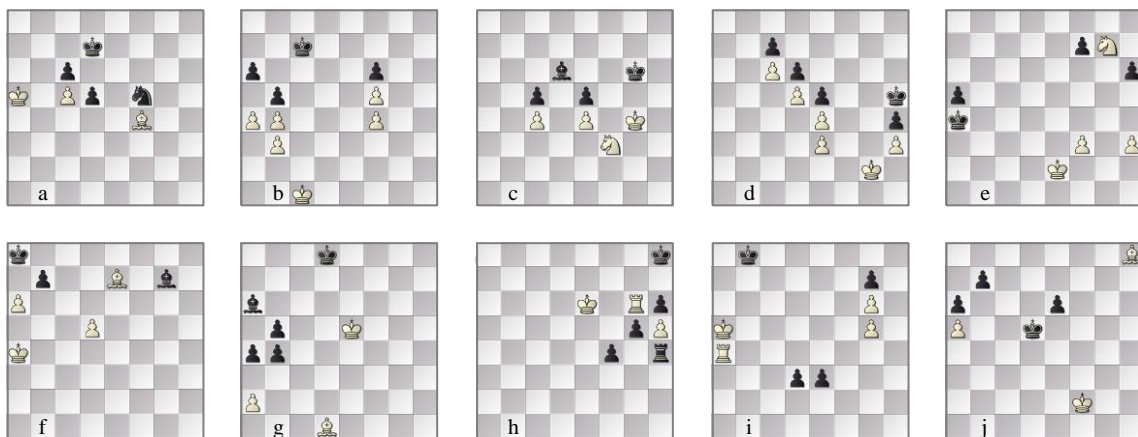


Fig. 1. Romero’s examples: (a-i) R01-R09 and (j) R10b.

2 Positions 01-10b: Romero’s original FINALGEN demonstrations

Romero (2012) provides eleven endgame positions. Four of these - R01/06/10a/10b - are admittedly directly evaluable using online ‘EGT’ endgame tables (de Man et al, 2018; Guo, 2018; Lomonosov, 2012). However, these examples are a convenient introduction to FINALGEN and show where SF/M/Z strategies diverge. A further five – R03/05/07/08/09 – feature eight men and are arguably in range for albeit fallible, deep-searching, multi-threaded chess engines with 7- or 6-man EGTs.

FINALGEN creates EGTs in two modes, ‘Normal’ (‘N’ below) and ‘Search for Draw’ (‘SfD’). ‘SfD’ is more efficient in both space and time than ‘N’ and only returns a ‘Draw’ verdict if this is the case.

The second author reproduced all the Romero (2012) results apart from those for R02 and R10b where the currently downloadable FINALGEN v1.4 gives an indecisive ‘Win or Draw’ evaluation. Rusz (2018) confirms that evidently a previous version created the results on the website. The supplementary document investigates why FINALGEN does not see the win in these two examples.

² ‘Preserving value’ is done in the context of five values rather than three when using ‘szygy’ EGTs. De Man added ‘frustrated win’ and ‘saved loss’ values in between ‘win’, ‘draw’ and ‘loss’.

EGT creation was done on two computers^{3,4} in both ‘N’ and ‘SfD’ modes and with EGT files NTFS-compressed or not. The computational experiments were not run under strict benchmark conditions, e.g., with defined clock-rate and controlled temperatures. The space figures are accurate to five figures⁵ but the time figures are reasonably indicative rather than exactly reproducible. Space and time constraints did not allow all eight options to be pursued for every position of interest.

Fig. 2 includes Romero’s 2012 evaluations of the positions with FINALGEN v1. The appendix notes some detailed logistics but the headlines are:

- SfD mode elapsed times are no more than 43% of ‘N’ elapsed times,
- SfD-EGTs are no more than 8% of the size of N-EGTs,
- SfD mode indicates ‘won or drawn’ on all won and some drawn positions,
- SfD is therefore a good ‘first try’ at definitively evaluating a drawn endgame,
- elapsed times with SSDs can be as little as 6% of previous times with HDDs,
- N-EGTs compress reliably to ~29%, and SfD-EGTs to ~39% of their intrinsic size,
- On PC B, FINALGEN took ~40% longer with compressed N-EGTs, and
- On PC B, FINALGEN took ~7% longer with compressed SfD-EGTs.

Tag	Endgame	FEN	PPR (2012) val.	F'Gen 1.4			'N' ≡ 'Normal' mode						
				val.	plies	files folders	Comp.		Uncompressed		c/unc.		
							GiB	Time	GiB	Time	GiB	Time	
R01	KBPKNPP	8/3k4/2p5/K1Pp1n2/5B2/8/8/8 b	0-1	0-1	97	56	19	3.4044	7' 12"	12.2464	4' 09"	0.2780	1.73
R02	KPPPPKPPPP	8/2k5/p4p2/1p3P2/PP3P2/1P6/8/2K5 w	1-0	1-0=	(43)	129	44	1.4253	117' 54"	5.0917	89' 3"	0.2799	1.32
R03	KNPPKBPP	8/8/3b2k1/2p1p3/2P1P1K1/5N2/8/8 w	=	=	—	74	24	8.6579	23' 49"	31.0491	14' 19"	0.2788	1.66
R04	KPPPPKPPPP	8/2p5/2Pp4/3Pp2k/4P2p/4P2P/6K1/8 w	1-0	1-0	37	141	48	3.0465	253' 33"	10.6806	207' 19"	0.2852	1.22
R05	KNPPKPPPP	8/5pN1/7p/p7k7/5P1P/4K3/8 b	=	=	—	98	33	2.6227	34' 15"	8.7636	21' 43"	0.2993	1.58
R06	KBPPKBPP	k7/1p2B1b1/P7/3P4/K7/8/8/8 w	1-0	1-0	17	55	18	8.7770	15' 53"	31.6531	10' 26"	0.2773	1.52
R07	KBPKBPPPP	3k4/8/b7/1p2K3/pp6/8/P7/3B4 w	=	=	—	78	24	17.4253	36' 03"	62.6432	19' 54"	0.2782	1.81
R08	KRPKRPPPP	7k/8/4K1Rp/6pP/5p1r/8/8/8 w	1-0	1-0	43	78	23	19.1697	50' 0"	65.6880	31' 33"	0.2918	1.58
R09	KRPPKPPPP	1k6/6p1/6P1/K5P1/R7/3pp3/8/8 w	=	=	—	68	23	0.5674	6' 18"	1.8119	3' 46"	0.3132	1.67
R10a	KBPKPP	8/1p1k4/pB6/P7/8/5K2/8/8 w	1-0	1-0	29	59	20	0.0749	0' 19"	0.2542	0' 17"	0.2945	1.12
R10b	KBPKBPPPP	7B/1p6/p3p3/P2k4/8/8/5K2/8 w	1-0	1-0=	63	77	26	0.6281	5' 43"	2.0809	3' 08"	0.3018	1.82
Nav1	KRPKRPPPP	8/8/6p1/5p1p/4kr1P/8/2R3K1/8 w 0 65	—	0-1	132	92	25	31.7637	83' 21"	107.446	51' 52"	0.2956	1.61
Nav2	KRPKRPPPP	8/8/6p1/3k1p1p/R4P1r/8/5K2/8 w 0 66	—	0-1	82	96	25	37.4639	104' 33"	127.101	60' 44"	0.2948	1.72
Nav3	KRPPKPPPP	positions 11a and then 11b	—	0-1	—	106	25	48.6171	128' 27"	164.685	79' 06"	0.2952	1.62
VG	KQPPPKQP	7k/6p1/2q5/4Q1KP/5PP1/8/8/8 w 0 60	—	1-0	65	103	26	49.7734	191' 37"	159.6078	141' 47"	0.3118	1.35
Tiv	KRPPKBPP	8/1k6/p7/1pb2K1R/8/P7/1P6/8 w 0 45	—	1-0	143	103	28	35.3708	87' 50"	118.6495	53' 23"	0.2981	1.65
LC0	KQPPPKQP	8/6pk/1q6/5pP1/1P3P2/1Q6/1K6/8 w 0 94	—	1-0	159	159	30	97.3764	445' 08"	323.4801	349' 12"	0.3010	1.27
Kpv	KNPPKBPP	8/4N3/8/1p6/1PkP4/P3K3/6b1/8 b 37 66	—	=	—	95	25	34.8860	81' 15"	126.8626	41' 25"	0.2750	1.96

Fig. 2. Positions analysed here with FINALGEN on PC B.

2.01 Position R01: 7-man, 8/3k4/2p5/K1Pp1n2/5B2/8/8/8 b, 0-1, *dtf/m/z* = 97/113/55p:

S(F&M)[±]: 1... Nd4" 2. Bh6" Kc7" 3. Bg7' Nb3+" 4. Kb4" Nd2" 5. Ka5" Kb7" 6. Bc3' Nc4+" 7. Kb4" Kc8" 8. Be1" Kd7" 9. Kc3' Ne5" 10. Bg3" Ke6" 11. Kd4" Nf3+" 12. Kd3" Ng5" 13. Bf4" Kf6" 14. Bc7" Ne4" 15. Bd8+" Ke6" 16. Kd4" Kd7" 17. Bh4" Kc7" 18. Be7' Kb7" 19. Bh4" Ka6" 20. Bd8" Ng3' 21. Bc7" Nf5+' 22. Ke5" Ng7' 23. Bd6" Ne8" 24. Be7' Kb5" 25. Kd4" Ng7" 26. Kd3" Ne6" 27. Kc3" Nxc5" {6-man, *dtf/m/z* = 44/60/16p} 28. Kd4" Nb3+" 29. Kc3" Nc1" 30. Bg5" Ne2+" 31. Kd3" Ng1" 32. Be3" Nf3"" 33. Ke2" Nh4" {S(FM)[±]:} 34. Bf4'

{S(MF)[±]:} 34. Bf2")

34. ... Kc4" 35. Bc7' c5" 36. Bb6' d4" 37. Ba5' Ng2' 38. Bb6' Kd5' 39. Ba5' Ke4' 40. Kd1" c4' 41. Bb4' d3' 42. Bc3' Nf4" 43. Ba1' Ne2" 44. Bg7" c3" 45. Bxc3" Nxc3+" 46. Kd2" Kd4" 47. Kc1" Ke3" 48. Kb2° d2" 49. Kxc3" d1=Q" 1-0.

Romero notes that after 35 minutes, FRITZ11 chose 1... d4?? {FINALGEN, ‘draw’} 2. Kb4"" Ne3' 3. Be5"" d3' 4. Bc3"" Nf1' 5. Kb3"" Ke6' 6. Bg7"" Kd5' 7. Kc3"" d2' 8. Kc2"" Kxc5' 9. Bc3' Kd5' 10. Bf6' Ng3' 11. Ba1' Ne4' 12. Kd1' Kc4' 13. Kc2"" c5' 14. Bg7' Kb4' 15. Ba1' c4' 16. Bh8' Kc5' 17. Bg7' Kd5' 18. Bh8' Ke6' 19. Ba1' Kf5' 20. Kd1' Kf4' 21. Ke2"".

FRITZ14 at depth 28 prefers 1. ... Ke6 and does not differentiate between 1. ... d4?? and 1. ... Nd4".

³ PC A: Toshiba Portege Z30-A (2015), x64, 1.7GHz Core i5-4210U/4GiB, 2TiB HDD via USB2.

⁴ PC B: Razer Blade 15 (2018), x64, 2.2GHz Core i7-8750H/16GiB, 0.5TiB PM961 SSD via M.2 PCIe-3 x4.

⁵ Windows7/HDD and Windows10/SSD agree on file ‘size’ but vary in ‘size on disc’ by about 1 in 1,000,000.

2.02 Position R02: endgame study (N. Grigoriev)

10-man position 8/2k5/p4p2/1p3P2/PP3P2/1P6/8/2K5 w, 1-0. FINALGEN v1.4 in fact says "1-0/=" here:

1. Kd1 Kd7 2. Ke1 Ke7 3. Kd2 Kd8 4. Ke2 Ke8 5. Kd3 Kd7 6. Ke3 Kd6 7. Ke4 Kc6 {White has the opposition} **8. Kf3 Kd5 9. Kg3** (9. Kg4?? Ke4=) **9. ... Kd4 10. Kh4 Kc3 11. axb5 axb5 12. Kh5 Kxb4**

{7-man: dtm/z = 39/3p: S(MZ)[±]}

13. Kg6" Kxb3" 14. Kxf6" Kc3" 15. Ke5" b4" 16. f6"" b3" 17. f7" b2" 18. f8=Q" b1=Q" {and now White uses the power of the first check to force the exchange of queens} 19. Qc5+" Kd2 20. Qf2+ {[+4pM]: Qd4+!}

(20. Qd4+" Ke2 21. Qe4+" Qxe4+" 22. Kxe4"")

20... Kc3 {[-4pZ: Kd3!]}

(20... Kd3" 21. Qd4+" Ke2 22. Qe4+" Qxe4+" 23. Kxe4"")

21. Qd4+" Kb3 22. Qb6+" Kc2 23. Qxb1+" Kxb1" 1-0

2.03 Position R03: A. Nimzovitsch study – "White to win"

8-man position p1w: 8/8/3b2k1/2p1p3/2P1P1K1/5N2/8/8 w. FINALGEN says 'draw'.

The main line proposed by AN, as informed by FINALGEN, 'Tdn' (FRITZ14 at search-depth *n*) and 7-man EGTs:

1. Nh4+' Kf6"" 2. Kh5' Bc7' 3. Nf3' Ba5' 4. Nh2' Bc3? {FINALGEN 1-0/=, Td32 0.15} **5. Ng4+"" Ke6' 6. Kg6' Bd4' 7. Nf6' Be3' 8. Nh7' Bd4' 9. Ng5+' Kd6' 10. Kf5' Bc3?** {1-0, dtf = 41p} **11. Nf7+" Kc6** [-4pF] **12. Ke6" Bb2** [-4pF] **13. Nxe5+" {7-man, dtf/m/z = 28/44/4p} Kc7' 14. Ng6** [+6pF] **Kd8' 15. Kf7' 1-0.**

8/8/5k2/2p1p2K/2P1P3/2b5/7N/8 w - - 8 4 after 4. ... Bc3? Is this 1-0 or ==?

8/8/3k4/2p1pKN1/2PbP3/8/8/8 b - - 0 10 after 10. Kf5. Does 10. ... Ke7"" draw or lose?

After some moves (1. Nh4+ Kf6"" 2. Kh5 Bc7 3. Nf3 Ba5 4. Nh2) FINALGEN still considers the result to be a draw. However, it pronounces 4... Bc3?, 'Win or draw' and 10. ... Bc3? definitely leads to a loss. 10. ... Ke7"" was needed here to maintain 'White win or draw'.

Black should have continued with 4... Bd2 (or Bd8) 5. Ng4+ Kf7 6. Nxe5+ {==} Kf6 7. Nd3 Ke6 8. Kg4 Be3 9. Kf3 Bg1" 10. Kf4 Bh2+" 11. Kf3 Bg1 12. e5 Kf5" 13. Kg3 Bd4 ½-½.

8/5k2/8/2p1N2K/2P1P3/8/3b4/8 b - - 0 6 after 6. Nxe5+

After 10. Kf5 in the author's solution, Black should continue 10. ... Ke7"" 11. Nf3 Kd6"" {Td30 +0.15} so it is not clear that White wins here.

2.04 Position R04:

11-man position 8/2p5/2Pp4/3Pp2k/4P2p/4P2P/6K1/8 w, dtf = 39p:

SV-/SV+: **1. Kf1" Kg5"**

(1... Kg6 [-2pF] 2. Kf2" {p2b} Kf6" 3. Ke2" {p3b} Ke7' (3... Kf7') 4. Kf3" Kf6' 5. Kg4")

2. Ke1" Kg6" (2... Kf6 [-2pF] 3. Ke2" {p3b})

3. Kf2" {p2b} Kf6" 4. Ke2" Ke7' 5. Kf3" Kf6' 6. Kg4" {and, adding to PPR's line} Kg7' 7. Kxh4" Kg6" 8. Kg4" Kf6" 9. h4" Kg6" 10. h5+" Kf6" 11. h6" Kg6" 12. h7"" (12. Kf3? FINALGEN 'win or draw') 12. ... Kxh7" 13. Kf5"" Kg8' 14. Ke6" Kf8' 15. Kd7" Kf7' 16. Kxc7" Ke7' 17. Kb8' Kd8' 18. c7+" Ke7' 19. c8=Q" 1-0.

2.05 Position R05:

8-man position 8/5pN1/7p/p7/k7/5P1P/4K3/8 b, drawn:

{With wtm, 1-0 in 22m/43p}

1. ... Kb3"" 2. Kd2' a4"" 3. Kc1' Kc3' 4. f4' Kd4' 5. Nh5' Ke4' 6. Kb2' Kf3' 7. Ka3' Kg2' 8. Kxa4'

{7-man, 'draw'} **Kxb3'**

(SZ/SZ: 8. ... f5" 9. h4" Kh3" 10. Kb4' Kxh4" 11. Nf6' h5" 12. Kc4' Kg3"" 13. Nxb5+"" Kg4"" 14. Ng7' Kxf4"" 15. Nxf5")

9. Kb4' Kg4' 10. Nf6+"" Kxf4' 11. Kc4' Kf5' 12. Ng8' h5' 13. Kd3' Kf4' 14. Ke2' Kg3' 15. Kf1' ½-½.

2.06 Position R06: Kasparian study (1928)

7-man position k7/1p2B1b1/P7/3P4/K7/8/8/8 w, $dtf/m/z = 17/45/1p$:

SF/SF[±]: **1. d6^{'''} Be5' 2. d7^{'''} Bc7' 3. Kb5^{'''} bxa6+**

(3. ... Ka7' is easier for White 4. Bc5+')

4. Kc6^{'''} (4. Kxa6?? Kb8^{'''} 5. Bd6' Ka8^{'''} 6. Bxc7 {stalemate ==}) **4. ... Ba5^{'''} 5. Bf6' Ka7'** (5. ... Kb8' 6. Be5+' Ka8' 7. Bc7^{'''})
6. Bd4+' Ka8' 7. Bb6^{''} and adding to PPR's line Bxb6^{''} 8. Kxb6^{''} Kb8' 9. d8=(Q/R)#^{''} 1-0.

2.07 Position R07: Kasparian study

1. Kd4^{'''}

(1. Bc2?? { $dtf = 33p$ } Kc7^{'''} 2. Kd4^{''} Kd6' 3. Bd3' a3^{''} 4. Bc2 {[-2pF: Bb1!]} Bc8^{''} 5. Bb1' (5. Ke3' Be6') 5... Be6^{''})

(1. Kd6?? { $dtf = 47p$ } b3^{'''} 2. a3 {[-36pF: Bxb3!]} b4^{''} ({Kasparian's} 2. ... b2?? 3. Bc2^{'''} b4' 4. Kc5' bxa3' 5. Bb1^{'''})
3. axb4' b2 { [+4pF: B(d3/e2)'']} 4. Bc2^{''} a3')

1... b3'

(1... Bc8' 2. Bc2^{'''} Be6' 3. Kc5')

(1... Kc7' 2. Kc5^{'''} b3' 3. a3^{'''} Bc8' 4. Kb4^{'''} Be6' 5. Kc3' Kd6' 6. Bh5' Bc4' 7. Be8' Kd5' 8. Kb2' Kd4' 9. Bd7')

2. Kc3^{'''}

(2. axb3?? a3^{'''} 3. Kc3^{''} b4+^{'''} 4. Kc2^{''} Bd3+^{''} 5. Kc1^{''} Kc7' 0-1)

(2. a3?? b4^{'''} 3. axb4^{''} b2^{'''} 4. Bc2^{''} a3^{'''} 5. Bb1^{''} (5. Kc3 {[-2pF]} Bc4^{'''} 6. Bb1' Be6 { [+2pF: Bd5!]}) 5. ... Bc8^{'''} 6. Kc3^{''} Be6^{'''} 7. Bd3' Kc7' 8. Bb1' Kd6' 9. Bc2' Ke5' 10. Bb1^{''} Bd5^{''} 11. b5^{''} Kd6^{'''} 0-1)

2... bxa2'

(2... b4+['] 3. Kb2^{'''} bxa2' (3... Bc4' 4. a3^{'''}) 4. Bxa4^{'''} {and not the -4.30 of FRITZ14 d34})

3. Kb2^{'''} {threatening 4. Bxa4} **a3+** (3... a1=Q+? 4. Kxa1^{'''} a3' 5. Be2^{'''} Kc7' 6. Bxb5^{''} =)

4. Ka1^{'''}

(4. Kxa2?? b4^{'''} 5. Bb3' Kc7' 6. Bg8 {[-2pF: B(a4/c2/d1)]} Kb6' 7. Kb3 {[-2pF: Bb3]} Kc5^{''} 0-1)

4... b4' 5. Bb3' Kc7' 6. Ba4' Kd6' 7. Bb3' Kc5' 8. Ba4' Kd4' 9. Bb3' Kc3' 10. Ba4' Bf1' 11. Bb3' Bd3' 12. Ba4^{'''} Be2' 13. Bb3^{'''} 1/2-1/2.

2.08 Position R08: Selesniev study

1. Kf7^{'''} Rxh5 {7-man: $dtf/m/z = 41/41/11p$ } **2. Rg8+^{'''} Kh7° 3. Rg7+^{'''} Kh8° 4. Kg6^{''} g4^{''} 5. Ra7** {Selesniev's line but [+2pF]}

({S(FZM)[±]} 5. Rd7^{''} Rg5+^{''} 6. Kxh6^{'''} Rd5' 7. Rxd5^{'''} Kg8^{''} 8. Kg5^{''} Kf7^{''} 9. Kxg4' Ke6^{''} 10. Ra5')

5. ... Rg5+^{''} 6. Kxh6^{''} g3 [-4pF] **7. Kxg5' g2^{''} 8. Ra1^{'''} f3** [-2pF] **9. Kg6^{'''}** { $dtf/m/z = 22/22/2p$ } 1-0

2.09 Position R09:

1. Kb6^{'''}

(1. Rd4?? $dtf = 17p$ e2^{'''} (1. ... d2?? 2. Kb6^{'''}) 2. Kb6^{''} Kc8^{''} 0-1)

(1. Re4?? $dtf = 17p$ d2^{'''} (1. ... e2?? 2. Kb6^{'''} Kc8^{'''} 3. Kc6^{'''} =) 2. Kb6^{''} Kc8^{''} 0-1)

1. ... d2' 2. Rd4^{'''} Kc8^{'''} 3. Kc6^{'''} e2' 4. Ra4^{'''} Kb8' 5. Rb4+^{'''} Ka7' 6. Ra4+^{'''} Kb8° 7. Rb4+^{'''} Kc8' 8. Ra4^{'''} Kd8' 9. Rd4+^{'''} Ke7' 10. Re4+^{'''} Kf8' 11. Rf4+^{'''} Kg8' 12. Ra4' Kf8^{'''} 13. Rf4+^{'''} {p11b == } 1/2-1/2.

2.10 Position R10a: source, J. Walker

6-man position 8/1p1k4/pB6/P7/8/5K2/8/8 w, $dtf/m/z = 29/43/17p$:

Presumably J. Walker's line: **1. Ke4^{''} Ke6** {[-2pF]}

(S(FZM)[±]: 1. ... Kd6^{''} 2. Kd4^{''} Kc6^{''} 3. Ke5^{''} Kd7^{''} 4. Kd5^{''} Ke7^{''} 5. Bg1' Kd7^{''} 6. Kc5' Kc7^{''} 7. Bh2+^{''} Kd7^{''} 8. Kb6^{''} Kc8^{''} 9. Bc7^{''} Kd7^{''} 10. Kxb7^{''} Ke6' 11. Kxa6^{''} Kf5' 12. Kb5' Kg4' 13. a6^{''} Kf5' 14. a7^{''} Kg6' 15. a8=Q^{''})

2. Kd4 [+4pF] **Kd6^{''} 3. Kc4' Kc6' 4. Bc5^{''} Kc7' 5. Kd5^{''} Kb8** [-2pF] **6. Kd6' Kc8' 7. Bb6' Kb8° 8. Kd7^{''} Ka8° 9. Bc7** [+8pF] **b5 10. axb6^{'''} Kb7^{''} 11. Kd6'** and continuing with strategy S(FZM)[±]: a5^{''} 12. Kc5^{''} a4^{''} 13. Kb5^{''} a3^{''} 14. Be5^{''} a2^{''} 15. Ba1^{''} Kc8^{''} 16. Kc6^{''} Kb8^{''} 17. b7^{''} Ka7' 18. Be5^{''} Ka6^{''} 19. b8=Q^{''} 1-0.

2.11 Position R10b: Kasparian study

7-man position 7B/1p6/p3p3/P2k4/8/8/5K2/8 w, 1-0, *dtf/m/z* = '63'/75/29p. FINALGEN v1.4 says "1-0/=":

1. Bf6'''

(1. Ke3? e5''' 2. Bf6' b6' 3. axb6' Kc6''' 4. Bd8' a5'''=)

1. ... Kc5'

(1... Kc6' 2. Bd4''' Kb5' 3. Bb6''' Kc6' 4. Ke3' Kd6' 5. Kd4' Kc6' 6. Ke5' Kd7''' 7. Kf6' Kd6''' 8. Kf7' Kd7''' 9. Kf8' Kd6''' 10. Ke8' e5''' 11. Kf7''' e4''' 12. Kf6' e3''' 13. Bxe3''' b6' 14. axb6''' a5''' 15. Bc5+' Kc6''' 16. Ke5' a4''' 17. Kd4' a3''' 18. Kc3' a2' 19. Kb2''' 1-0)

2. Bd8''' b6? {cited by PPR, loses immediately, *dtf* = 39p}

(2. ... Kc4')

3. axb6''' Kc6'' 4. Ke3''' a5 [-2pF]

(4... e5 [-2pF] 5. Ke4''' a5'' 6. Kxe5''')

5. Kd4''' a4 [-6pF] 6. Kc4' e5 [-2pF] 7. Kb4''' e4 [-2pF] 8. Kxa4''' e3'' 9. Ka5''' e2'' 10. Bh4''' 1-0

3 Positions 11a-11b: Navrotescu–Shah, Pays de Charleroi Open, 2017, round 4

p63w: 8/8/6p1/5p1p/4kP1P/6r1/2R5/4K3 w - - 0 63, 0-1

This game featured in the Chessbase (2018) *Endgame Magic 110* as the finale of a rich and fascinating discussion between the first author and IM Sagar Shah. Navrotescu lost in Fig. 3a with **56. Rd3??** as 56. Bc4!! draws. Even so, the win was not simple and White need not have resigned on move 65: the endgame is a big fish which still has to be landed. The position after **64. ... Rxf4** attracted the sharp minds and serious attention of GMs Tigran Gharamian, Sandipan Chanda and Jacob Aagaard. For months, the win was not at all clear until Jacob attacked it with FINALGEN. It was then clear why the win was not clear: it is exceptionally deep. Sandipan then commented on the themes that emerged, noting that Black must prevent White from establishing its king on f4 and its rook on a4. Perhaps, looking past the sometimes-distracting detail of the computer's lines, there is perhaps a principle here that, given a choice of taking the f- or h-pawn, one should take the h-pawn.

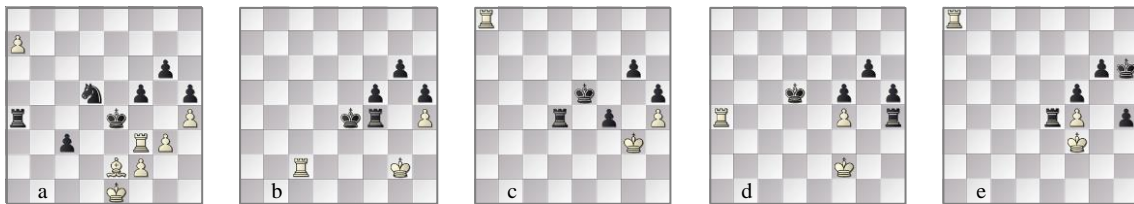


Fig. 3. Navrotescu-Shah: as played (a) 56w and (b) 65w, hypothetically leading with SF[±] to (c) 103w after f4; the easier win (d) 66w after Rxf4 leading with SF[±] to (e) 75w after h4.

After **56. Rd3??** [Ed24-3.54], the play was **56. ... Rxa7 57. f3+ Ke5 58. Bd1 Ra3 59. f4+ Ke4 60. Bc2 Ne3 61. Rxc3+ Nxc2+ 62. Rxc2 Rxc3 63. Kf2 Rf3+** (63. ... Rg4 64. Rc4+ Kd5 65. Ra4 Rxh4, Fig. 3d) **64. Kg2 Rxf4**, Fig. 3b. Our FINALGEN computations ran from figures 3b and 3d. 65. ... Rxh4 would have left a win with *dtf* = 44 moves and a first phase of nine moves leading to Fig. 3e. However, the chosen **64. ... Rxf4** left a much more difficult win with *dtf* = 66 moves and a first phase in the line we followed of 37 moves leading to Fig. 3c. In both cases and particularly as played, White could have fought on for a while, hoping to be defending the usually drawn KRKR(f)P(h) endgame (Müller and Konoval, 2016; Müller and Lamprecht, 2001) and/or hoping for the 50-move-draw.

63. Kf2 Rf3+?!

(63... Rg4 64. Rc4+ Kd5 65. Ra4 Rxh4 {GH, SF⁺/SF⁻, *dtf* = 82p} 66. Kf3"

(66. Kg3 [-4pF] Rg4+ " 67. Kf3" Ke6 [+6pF] 68. Ra1 [-6pF] Kf6 [+6pF] 69. Rb1 [-8pF] h4' 70. Ra1 [-6pF] Kg7 [+6pF] 71. Ra8 [-6pF] Rg3+ " 72. Kf2 " Rb3' 73. Kg2' h3+ " 74. Kf2 [-18pF] (74. Kh2" Rf3") 74... h2" 75. Kg2 [-32pF] h1=Q+ " 76. Kxh1 " Rb4 ""

66... Ke6" 67. Ra6+ " Kf7 "" 68. Ra7+ " Kf8' 69. Ra8+ " Kg7" 70. Ra7+ " Kh6" 71. Ra8" Rh3+ ' 72. Kg2" Re3" 73. Kf2' Re4" 74. Kf3' h4" 75. Rb8'

(75. Rh8+ [-2pF] Kg7° 76. Rxh4 [-20pF] {7-man, *dtm* = 57p} g5 "" 77. Rh5 " Rxf4+ ")

75... Rc4' 76. Rb3" Kh5' 77. Ra3" Rc2" 78. Rb3" Rh2" 79. Rb6" Rh3+ " 80. Kg2" Rc3' 81. Rd6' h3+ " 82. Kh2" Rf3" 83. Rd4" Kg4' 84. Rd6" Rf2+ " 85. Kh1" Kh5" 86. Rd4" h2" 87. Ra4' Kg4" 88. Ra2! Rxf4' (88... Rxa2?? stalemate) {7-man, *dtf/m/z* = 42/52/4p. S(FMZ)[±]} 89. Rg2+ " Kh5 "" 90. Rd2' Rf1+ " 91. Kxh2" {6-man, URE §2.1} g5" 92. Kg2" Ra1' 93. Rd3" f4" 94. Rh3+ " Kg4" 95. Rg3+ " Kf5" (95. ... fxg3 {stalemate}) 96. Rb3' Ra2+ " 97. Kg1' Kg4" 98. Rb5' f3" 99. Re5' Kf4" 100. Rd5" g4" 101. Rd4+ " Kg3" 102. Rd1" Rc2" 103. Kf1" Rh2" 104. Ke1" f2+ " 105. Kf1" Rh1+ " 106. Ke2° f1=Q+ " 107. Rxf1" Rxf1" 108. Kxf1" Kh2")

64. Kg2 Rxf4 65. Resigns 0-1

(64. ... Rxf4 {SF⁺/SF⁻, *dtf* = 130p} 65. Ra2" Kd5"

(65. ... Rg4+ [+6pF] 66. Kh3 " f4??

(66... Rg1" 67. Ra4+ " Kd5" 68. Ra5+ " Kc6" 69. Kh2" Kb6 [+8pF] 70. Ra3 [-4pF] Rg4 [+2pF] 71. Kh3" f4?? 72. Rc3'=)

67. Ra4+ "" Kf5' 68. Ra5+ "" Kf6' 69. Ra6+ "" Kg7' 70. Ra4' f3' 71. Ra7+ '=)

66. Ra5+ " Kc6" 67. Kh3" Rb4" 68. Ra1' Rb6" 69. Kg3" Kd5' 70. Kf3" Ke6" 71. Re1+ " Kd7" 72. Ra1" Rb3+ " 73. Kf2" Rd3" 74. Rg1" Rd6" 75. Kf3" Ke7" 76. Ra1" Rd3+ ' 77. Kf2" Ke6' 78. Ra4" Ke5' 79. Ra5+ " Rd5" 80. Ra6" Rd6" 81. Ra5+ " Ke4" 82. Ra4+ " Kd3" 83. Ra3+ " Kc2" 84. Rg3" Kd2" 85. Rg2" Kd3" 86. Kf3" Kd4" 87. Ra2" Kc3" 88. Ra3+ "

Kb4" 89. Ra1' Rd4" 90. Kg3" Kc4" 91. Kh3' Rd6" 92. Kg2' Kc3" 93. Ra3+" Kd2" 94. Ra2+" Ke1' 95. Ra1+' Ke2"
96. Ra4' Rd2" 97. Ra6" Rd4" 98. Ra2+" Kd3" 99. Kg3" Ke4' 100. Re2+" Kd5" 101. Ra2' Ke5" 102. Ra8' f4+" 103.
Kf2" Rd2+' 104. Kf3' Rd3+" 105. Kg2" Kf5' 106. Rg8" Rg3+" 107. Kh2" Kg4" 108. R_{xg}6+" K_{xh}4"
{6-man, URE 2.2.2: S(FMZ)[±], *dff/m/z* = 44/64/14p} 109. Ra6' Kg4" 110. Rd6" Ra3' 111. Kg2" Ra2+" 112. Kg1" h4"
113. Rg6+" Kf3" 114. Rb6" Rd2" 115. Rf6" Rd1+" 116. Kh2° Ke3" 117. Kg2" Rd4" 118. Re6+" Re4+" 119. Ra6"
f3+" 120. Kg1" h3" 121. Ra8" Kf4" 122. Kh2" f2" 123. Rf8+" Ke3+" 124. Kg3" Rg4+" 125. K_{xg}4" h2+" 126. Re8+"
Kd3" 127. Rd8+" Ke2" 128. Re8+" Kf1" 129. Rh8" Kg1" 130. Rh3' f1=Q')

4 Position 12: Vidit Gujrathi–L’Ami, NED–IND, 43rd Olympiad r10, Batumi 2018, 60w

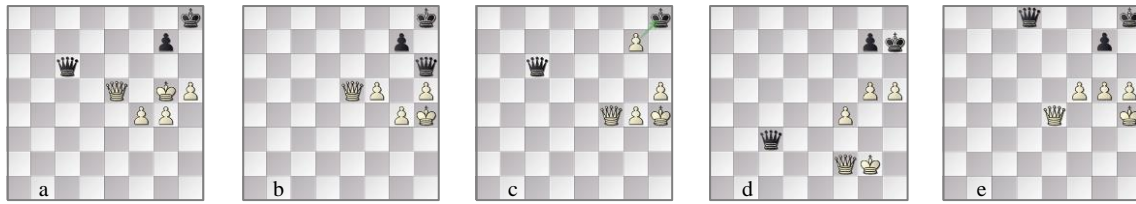


Fig. 4. Vidit Gujrathi–L’Ami: as played, (a) ‘won’ 60w, (b) drawn 61b and (c) 7m-draw 69b. In the chosen winning line by FINALGEN, (d) 66. g5 and (e) 76. f5 are wiser and later advances than the played line.

At position 60w of Fig. 4a, FINALGEN says White has a win in 33 moves. However, Vidit Gujrathi played **60. f5??** and the position is now drawn, a fact which totally escaped top engine STOCKFISH, kibitzing at the time (Chessbomb, 2018). Position 61b of Fig. 4b required 61. ... Qd2^{'''} but **61. ... Qc1?** was played, and now FINALGEN says ‘win or draw’. In fact, all lines seem to lead to a draw with pawns and evaluations stalled at 0.00, or with a 7-man draw as at position 69b as played, see Fig. 4c. An analysis engine recognising position-repetition and interfaced to FINALGEN and 7-man EGTs would speed searching here. The winning SF⁺ optimal line from Fig. 4a is a lesson in *festina lente*, showing much later advances of the g-pawn as in Fig. 4d and the f-pawn as in Fig. 4e. Both had been providing some shelter from a rainstorm of Black checks until it abated.

We are not dependant on FINALGEN for the next observations but it is worth noting how difficult this 6-man endgame is. On thirteen occasions, the most challenging play leaves value preserved by a unique move. The half-point separating Vidit Gujrathi’s win from L’Ami’s draw in fact transferred six times, on moves **71. ... Kh8**, **72. Qe5+**, **76. ... Qa3+**, **82. Qe3**, **85. ... Kh8** and **87. h6**. After **89. g5** and another 50 moves, the draw was claimed by L’Ami under FIDE Article 9.3.1.

60. f5?? {FINALGEN =}

(SF[±]: 60. Qd4^{''} Qe6^{''})

(60. ... Kh7 [-2pF] 61. Kh4^{''} Qh1+^{''} 62. Kg3^{''} Qe1+^{''} 63. Qf2^{''} Qc3+^{''} 64. Kh4 [+2pF] Qf6+ [-6pF] 65. g5^{''} Qe6 [-4pF] 66. Qc2+^{''} Kh8 [-10pF] 67. Qc3^{''} Kg8^{''} 68. Qe5^{''} Qc6 [-4pF] 69. Qb8+^{''} Kf7^{''} 70. g6+^{''} Ke7^{''} 71. Qa7+^{''} Kd8^{''} 72. Qxg7^{''} Qh1+^{''} 73. Kg5^{''} Qg2+^{''} 74. Kh6^{''} 1-0)

61. Kh4^{''} Qe1+^{''}

(61... Qe7+ [-2pF] 62. g5^{''} Qe1+^{''} 63. Kg4^{''} Qe2+^{''} 64. Kg3^{''} Qxh5 [-8pF], 7-man, \$\$) 65. Qd8+^{''} Kh7^{''} 66. Qd3+^{''} g6 [-12pF] 67. Qf3^{''} 1-0)

62. Kh3^{''} Qe2^{''} 63. Kg3^{''} Qe1+^{''} 64. Qf2^{''} Qc3+^{''} 65. Kg2^{''} Kh7^{''} 66. g5^{''} Qc1^{''} 67. Qd4^{''} Qc2+^{''} 68. Kf3^{''} Qb3+^{''} 69. Qe3^{''} Qd1+^{''} 70. Kg3^{''} Qb1^{''} 71. Kh2^{''} Qf5^{''} 72. Qe5^{''} Qc2+^{''} 73. Kg3^{''} Qd3+^{''} 74. Kh4^{''} Qd8^{''} 75. Qe4+^{''} Kh8^{''} 76. f5^{''} Qb8^{''} 77. Kg4^{''} Qd8^{''} 78. g6^{''} Qd1+^{''} 79. Kh4^{''} Qd8+^{''} 80. Kh3^{''} Kg8^{''} 81. h6^{''} Kh8^{''} 82. Qf3^{''} Qc8^{''} 83. Qd5^{''} Qe8^{''} 84. Qd4^{''} Qe7^{''} 85. Qxg7+^{''} Qxg7^{''} 86. hxg7+^{''} Kxg7^{''} {4-man: S(FM)[±]} 87. Kg4^{''} Kf6^{''} 88. Kh5^{''} Kg7^{''} 89. Kg5^{''} Kf8^{''} 90. f6^{''} Kg8^{''} 91. g7^{''} Kf7^{''} 92. Kf5^{''} Kg8^{''} 93. Ke6^{''} Kh7^{''} 94. Kf7^{''} Kh6^{''} 95. g8=Q^{''} Kh5^{''} 96. Qg3^{''} Kh6^{''} 97. Qh4^{''})

60. ... Qh6+ **61. Kh4^{''} Qc1?** FINALGEN "1-0/=", we think ‘draw’; Qd2^{'''} draws. **62. f6^{'''} Qh1+^{'''}** **63. Kg5^{''} Qc1+^{'''}** **64. Kg6^{''} Qc2+^{'''}** **65. Qf5^{''}**

(65. Kf7^{''} Qc4+ 66. Ke7^{''} Qxg4^{''} 67. f7?= Qb4+^{''} 68. Qd6^{''} Qb7+^{''} 69. Qd7^{''} Qe4+^{''} =)

65. ... Qc6^{'''} **66. Qf4^{''}**

(66. h6? Qe8+^{'''} 67. Kg5^{'''} Qe3+^{'''} 68. Kh4^{''} Qe1+^{'''} 69. Kh5^{''} Qe8+^{'''} 70. f7^{''} Qxf7+^{'''} = 71. Qxf7^{''} g6+^{'''} 72. Kh4^{''} g5+^{''} 73. Kxg5^{''} {stalemate})

66. ... Qc2+ **67. Kg5^{''} Qc5+^{'''}** **68. Kh4^{''} Qc6^{''}** **69. f7g7+?** {FINALGEN, “=”} **Kxg7^{'''}** **70. Qe5+^{''} Kh7^{'''}** **71. Qe7+^{''} Kh8?** {FINALGEN “1-0/=", 1-0, dtm = 36m}

(71... Kg8^{'''} =)

72. Qe5+?? {=}

(72. Qf8+^{''} Kh7^{''} 73. Kg5^{''} Qc7^{''} 74. Qh6+^{''} Kg8^{''} 75. Qf6^{''} Kh7^{''} 76. Qf8^{''} Qa7? {1-0, dtf = 63p} 77. Qh6+^{''} Kg8^{''} 78. Qg6+ [+4pF] Kh8^{''} 79. Qf6+^{''} Kh7^{''} 80. Qe5? FINALGEN “1-0/=” Qb7^{'''} 81. h6^{''} Kg8^{'''} 82. Kh5^{''} Qh1+? {1-0, dtf = 33p} 83. Kg6^{''} Qc6+^{''} 84. Qf6^{''} Qe8+^{''} 85. Kg5^{''} Qb5+^{''} 86. Kh4^{''} Qd7^{''} 87. Qg7+^{''} Qxg7^{''} 88. hxg7^{''} Kxg7 [-2pF] 89. Kg5^{''} 1-0)

72. ... Kh7^{'''} **73. Kg5^{''} Qh6+^{'''}** **74. Kf5^{''} Qf8+^{'''}** **75. Ke4^{''} Qb4+^{''}** **76. Kf3^{''} Qa3+?** {FINALGEN “1-0/=", 1-0 dtm=48m}

(76. ... Qb3+' 77. Kf4' Qf7+' 78. Qf5+' Kh8'' =)

77. Kf4''' Qc1+? { 1-0, *df/m/z* = 59/71/21p} **78. Kf5''' Qc8+' 79. Qe6'' Qf8+'** {[-30pM], *dtm* = 19m} **80. Ke4''** [+34pF]

(80. Ke5'' Qc5+' 81. Kf6'' Qd4+' 82. Ke7'' Qc5+ [-2pF] 83. Ke8'' Kh8 [-2pF] 84. Qf6+' Kg8'' 85. Qf7+' Kh8° 86. Qf8+' 1-0)

80. ... Qb4+ [-14pF] **81. Kf3** [+24pF] **Qc3+** [-4pF] **82. Qe3??=**

({GH: *dtm*-optimal} 82. Kf4'')

82. ... Qf6+ **83. Kg3 Qd6+** **84. Kh3 Qc6** **85. Qe7+ Kh8??** {*dtm* = 75m}

(85... Kh6' =)

(85... Kg8' =)

86. Kg3''' Qc4' but [-44pM]

(86... Qh6' 87. Qe5+' **1-0**, *dtm* = 146p)

87. h6? =

(87. Qf6+' Kh7' 88. h6'' Qc7+' 89. Kh3''' Qb7' 90. Kh4''' Qh1+' 91. Kg5''' Qb7''' 92. Qe5' Kg8''' 93. Kh5' Qf7+' 94. Kh4' Qf2+' **1-0**, *df* = 31p 95. Kg5''' Qf7' 96. h7+' Kxh7'' 97. Qf5+' Kh8'' 98. Qf6+' 1-0)

87. ... Qf7''' 88. Qe5+' Kh7''' 89. g5' Kg6'''

(89... Qd7?? **1-0**, *df* = 91p 90. Kf4'')

90. Qf4' Qe6' **91. Kf2' Qa2+' 92. Kf3' Qb3+' 93. Qe3' Qd1+' 94. Kf4' Qd6+' 95. Kg4' Qd7+' 96. Kh4' Qa4+' 97. Kg3' Qc6'** **98. Qe5' Qb6'** **99. Kf3' Qb3+' 100. Kf4' Qf7+' 101. Ke3' Qb3+' 102. Kd2' Qa2+' 103. Kd3' Qb3+' 104. Ke4' Qc4+' 105. Ke3' Qb3+' 106. Kf4' Qf7+' 107. Kg3' Qb3+' 108. Kg4' Qc4+' 109. Qf4' Qe6+' 110. Kf3' Qb3+' 111. Qe3' Qf7+' 112. Ke2' Qa2+' 113. Qd2' Qe6+' 114. Kd1' Qg4+' 115. Kc2' Qa4+' 116. Kb1' Qb3+' 117. Kc1' Qa3+' 118. Kd1' Qf3+' 119. Kc2' Qc6+' 120. Qc3' Qa4+' 121. Kd3' Qb5+' 122. Kc2' Qe2+' 123. Kc1' Qf1+' 124. Kc2' Qe2+' 125. Qd2' Qc4+' 126. Kb1' Qb3+' 127. Ka1' Qa3+' 128. Qa2' Qc3+' 129. Qb2' Qa5+' 130. Kb1' Qe1+' 131. Kc2' Qe2+' 132. Kb1' Qd1+' 133. Ka2' Qa4+' 134. Qa3' Qc2+' 135. Ka1' Qd1+' 136. Kb2' Qd2+' 137. Kb3' Qd3+' 138. Kb2' Qd2+' 139. Kb1' Qd1+' 140. Qc1' ^{1/2}/_{1/2}**

5 Tiviakov-Korsunsky, 1989 ...

Now finally comes a real revolution. Human theory has thought that Figure 6a's position 45w from Tiviakov-Korsunsky (Redhotpawn.com, 2014) is a fortress: the first author had also claimed this many times including (Müller, 2007). But White can win, as first pointed out by Jonathan Hawkins (2012) in his excellent book on page 105. Either White invades with his king to c6, this winning aim being known to human theory, or amazingly, White exchanges pawns with a3-a4 at the right time.

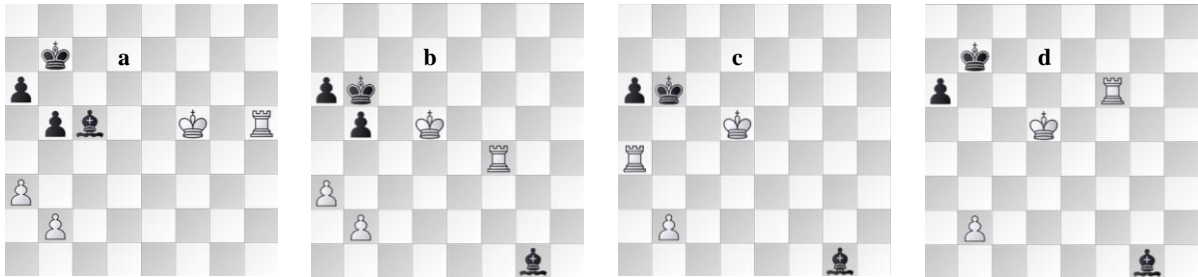


Fig. 6. Tiviakov-Korsunsky: (a) 45w, and after sideline (b) 52. ... Kb6, (c) 54. Rxa4 and (d) 56. ... Kb7.

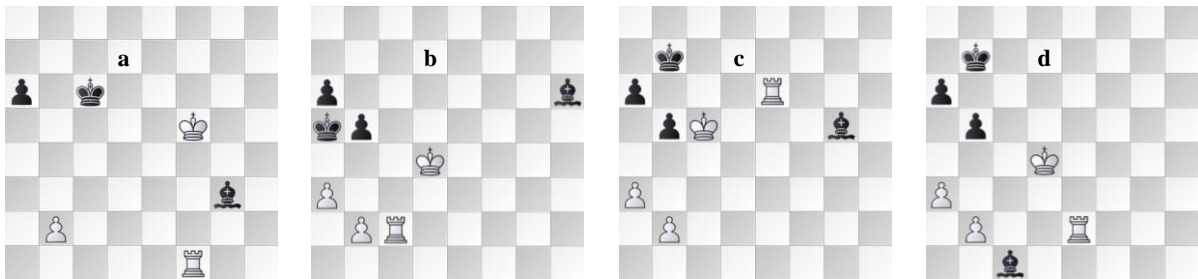


Fig. 7. Tiviakov-Korsunsky after sideline (a) 62. Kf5, (b) 61. ... Bh6, (c) 66. Kc5 and (d) 65. Re2.

45. Ke4" Bf2

((S(FMZ)[±];) 45... Kc6" 46. Rh6+ K7" 47. Kd5' Bf2' 48. Re6' Bg1' 49. Re4" Kb6' 50. a4" bxa4" 51. Rxa4" **KRPKB**, *dtf/m/z* = **130/146/82p**. Be3' 52. Ra1" Kb7' 53. Re1' Bf2" 54. Rf1" Be3" 55. Rf3" Bg1" 56. Kd6" Bh2+" 57. Ke6" Kc6" 58. Rf1' Bg3" 59. Kf5" Bd6" 60. Rc1+ K6" 61. Ke4" Bc5" 62. Kd3" Kb5" 63. Ra1" Kb6" 64. Kc4" Be3" 65. Re1' Bf2" 66. Rf1" Be3" 67. Rf3" Bg1" 68. Kb4" Bd4" 69. Rb3" Be5" 70. Ka4+" Ka7" 71. Ka5" Bf6" 72. Kb4" Kb6" 73. Ka4+" Ka7" 74. Rb4" Be5" 75. Kb3" Bd6" 76. Rg4" Be5" 77. Rg2" Kb7" 78. Kc4" Bf4" 79. Rg7+" Kb6" 80. Rg4" Bb8" 81. Rg6+" Kb7" 82. Kd5" Bf4" 83. Rg4" Bb8" 84. Kc5" Ba7+" 85. Kd6" Bb8+" 86. Kd7" a5" 87. Rg8' Bf4" 88. Rf8" Bd2" 89. Rc8" Bb4" 90. Rc6" Bf8" 91. Rc2' Kb6" 92. Rc8" Bg7" 93. Kd6" a4" 94. Rb8+" Ka7" 95. Rb5" a3" 96. bxa3" **KRPKB**, *dtf/m/z* = **40/62/4p**. Bd4" 97. Kc7" Ka8" 98. Rb8+" Ka7° 99. Rb4" Be5+" 100. Kc6" Bg3" 101. Rb2" Be5" 102. Rb7+" Ka6" 103. Rb4" Ka7" 104. Rg4" Bh2" 105. Kb5" Bb8" 106. a4" Kb7" 107. Rg7+" Ka8" 108. a5" Bf4" 109. Ka6" Be5" 110. Rg8+" Bb8" 111. Kb5" Ka7" 112. a6" Be5" 113. Rd8" Bf4" 114. Rd7+" Kb8" 115. a7+" Ka8" 116. Ka6" Bc7" 117. Re7" Bb8" 118. axb8=Q+" Kxb8 {1-0})

46. Rf5 Bg1 47. Rf1 Bc5 48. Kd5 Be3 49. Rf7+ Kb6 50. Rf3 Bg1 51. Rf1

(51. Rf6+ Kb7 52. Rf4 Kb6, Figure 6b, is more direct. Now, remarkably, White should exchange pawns with 53. a4!! bxa4 54. Rxa4, Figure 6c, *dtc/m/z* = -50/-73/-41m. White's rook now wins a long domination duel as in, e.g., this initially DTC/M-minimaxing line from YK/AR: 54... Bf2' 55. Rf4' Bg1" 56. Rf6+" Kb7" Figure 6d 57. Rf1" Be3" 58. Rf3" Bg1" 59. Kd6" Bh2+" 60. Ke6" Kc6" 61. Rf1" Bg3" 62. Kf5" Figure 7a. This is really extraordinary! White's king has moved to f5 to win the domination fight. Chess really is a rich game! 62. ... Bd6" 63. Rc1+ K6" 64. Ke4" Bc5" 65. Kd3" Kb5" 66. Ra1" Kb6" 67. Kc4" Be3" 68. Re1' Bf2" 69. Rf1" Be3" 70. Rf3" Bg1" 71. Kb4" Bd4" 72. Rb3" Be5" 73. Ka4+" Ka7" 74. Ka5" Bf6" 75. Kb4" Kb6" 76. Ka4+" Ka7" 77. Rb4" SC/SC+ Bd8 (SM C/SM+ C+: 77. ... Be5" 78. Kb3" Bd6" 79. Rg4" Be5" 80. Re4" Bg3" 81. Kb4" Kb6" 82. Rg4" Bb8" 83. Kc4" Kc6' 84. Rg6+" Kb7" 85. Kd5" Bf4" 86. Rg4" Bb8" 87. Kc5" Ba7+" 88. Kd6" Bb8+" 89. Kd7" a5" 90. Rc4' Kb6" 91. Ke6" Kb5" 92. Kd5" a4" 93. Rc5+" Kb6" 94. Kc4" Bf4" 95. Rb5+" Ka6" 96. Kc5" Be3+" 97. Kc6" Bc1" 98. Rb8" Ka5" 99. Kc5" Be3+" 100. Kc4" Bd2' 101. Ra8+" Kb6° 102. Rxa4+" -) 78. Kb3" Ba5" 79. Rg4" Kb7" 80. Kc4" Bb6" 81. Kd5" Bf2" 82. Kd6" Be1" 83. Rg8" Bb4+' 84. Kd5" Be1" 85. Rf8' Kb6" 86. Rf6+" Kb5" 87. Rf4" Bd2" 88. Rf8" Kb6" 89. Rb8+" Kc7" 90. Re8" Be1" 91. Re2" Kb6" 92. Kc4" Bf4' 93. Re6+" Kb7" 94. Kc5" Bg5" 95.

Rb6+'' Ka7'' 96. Kc6'' Be3'' 97. Rb7+' Ka8° 98. b4' Bd4' 99. Rd7'' Bf2'' 100. Rd2'' Be1'' 101. Rd1'' Bf2'' 102. Ra1'' Ka7'' 103. b5'' a5' 104. Rxa5+'' +-)

51. ... Be3 52. Ke4 Bg5 53. Rf5 Bc1 54. Rf2 Bg5 55. Kd4 Bc1 56. Re2 Ka5

(56. ... Bg5 57. Re6+ Kb7 58. Kc5 Bd8 59. b4 Bh4 60. Rb6+ Ka7 61. Kc6+-)

57. Kc3 Kb6 58. Kd4 Ka5 59. Rc2 Bh6 60. Rg2 Bc1 61. Rc2 Bh6 Figure 7b **62. Rc7?! allows Black to get back in his house.**

(62. Rg2 wins more quickly, e.g., 62. ... Bc1

(62. ... Bf8 63. Kc3 Kb6 64. Rg6+ Kb7 65. b4+-) (62. ... Ka4 63. Rg6 Bc1 64. Kc3+-)

63. Re2 Kb6

(63. ... Bh6 64. Kc3 Bg7+ 65. Kb3 Bf6 66. Re6 Bd4 67. Ka2 b4 68. axb4+ Kb5 69. Kb3+-)

64. Kd5 Bg5

(64. ... a5 65. Kd4 a4 66. Kd5 b4 67. Rc2 Be3 68. axb4 Kb5 69. Rc8+-)

65. Re6+ Kb7 66. Kc5, Figure 7c, and White's king invades to c6. 66. ... Bd8 67. b3 Bg5 68. Rb6+ Ka7 69. Kc6+-)

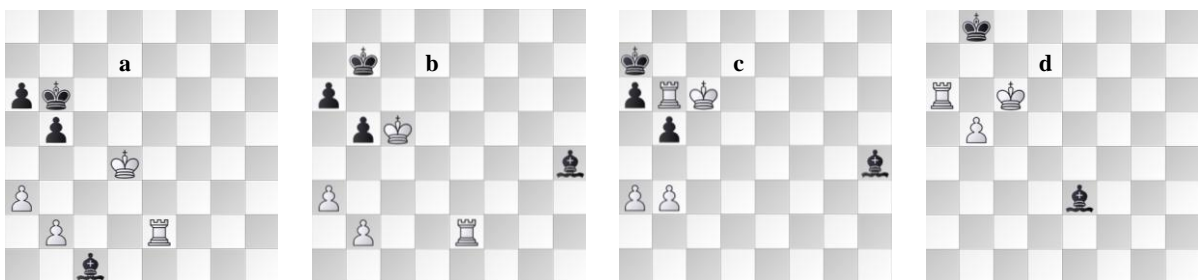


Fig. 8. Tiviakov-Korsunsky after (a) 65. ... Kb6?!, (b) 66. ... Bh4, (c) 71. Kc6, and (d) 79. Rxa6.

62. ... Kb6 63. Re7 Bc1

(63. ... Bg5 64. Re6+ Kb7 65. Kc5 Bd8 66. b3 Bh4 67. Rb6+ Ka7 68. Kc6+-)

64. Re6+ Kb7 65. Re2, Figure 7d.

Even 65. Kc5 is playable. 65. ... Bxb2 66. Re7+ Kb8 67. Re3 (67. Kb6? Bd4+ 68. Kxa6 Bc5=) 67. ... Kc7 68. Rf3 Kb7 69. Rh3 Kc7 70. Rh7+ Kb8 71. Kb6+-)

65. ... Kb6?!, Figure 8a, and now the bishop is dominated.

(65. ... Bg5! 66. Kc5 Bh4, Figure 8b, was more tenacious, e.g., 67. a4 bxa4 68. Kb4 Bg3 69. Kxa4, $d_{tc}/m/z = -53/-76/-44m$, and as in, e.g., this DTC/M-minimaxing line from YK 69... Bc7'' 70. Re6' Bd8' 71. Kb4' Bb6'' 72. Kc4'' Bg1'' 73. Rf6' Be3' 74. Kd5'' Bg1'' +- , Figure 6d once again)

66. Kd5 Bg5

(66. ... a5 67. Kd4 a4 68. Kd5 b4 69. Rc2 Be3 (69. ... bxa3 70. Rxc1 axb2 71. Rb1 a3 72. Kc4+-) 70. axb4 Kb5 71. Rc8+-)

(66. ... Ka5 67. Rc2 Be3 (67. ... Bf4 68. Rc6+-) 68. Rc6 b4 69. axb4+ Kb5 70. Rc8 Bf4 71. Rc5+ Kb6 72. Kc4+-)

67. Re6+ Kb7 68. Kc5 Bd8

(68. ... Bh4 69. Rb6+ Ka7 70. Kc6+-)

69. b3 Bh4 70. Rb6+'' Ka7 71. Kc6, Figure 8c. White's king has reached the key square c6 and it is over. **71. ... Bf2 72. Rb7+'' Ka8° 73. Rf7 Bg1 74. Rf4 Ka7 75. a4 bxa4 76. Rxa4** KRPKBP, $d_{tc}/m = -7/-12m$ **Bf2 77. b4'' Be3 78. b5'' Kb8'' 79. Rxa6''**, Figure 8d, 1-0.

6 POSITION 14: LEELA CHESS ZERO – ETHEREAL, TCEC Cup 1, game 14, pos. 90b

Position 94w: 8/6pk/1q6/5pP1/1P3P2/1Q6/1K6/8 w - - 0 94, 1-0, *dtf* = 159p

The radical ‘neural network’ and rapidly improving chess engine ‘LC0’ was of major interest to the inaugural TCEC Cup audience (Haworth and Hernandez, 2019). ETHEREAL was also a dark horse, newly risen to the Premier Division of the TCEC league. The ‘best of eight’ rapid tempo match between them passed 4-4 en route to ‘best of two’ and featured unusually long contests, largely because LC0’s evaluations frustrated the TCEC adjudication rules. Game 14 was no exception.



Fig. 6. LC0-Et: (a) 94w; FINALGEN-optimal line (b) 105w before b6^{'''}, (c) 7m 116w and (d) 6m 128b; (e) drawn 175b.

This theoretically reached FINALGEN territory at position 74w which requires 11TB for workspace so we quickly advance to position 94w of Fig. 6a. Removing the f- or g- pawns leaves a win with *dtf* = 29m/57p or 28m/55p respectively. Removing the four pawns leaves a draw. This suggests but does not guarantee that the 9-man position 94w is decisive. FINALGEN proves that White did in fact have a win: *dtf* = 159 plies: the FIDE 50-move rule, with the advancing pawn on b4 and pawns to capture, appears not to be an issue. Figs. 6(b-d) mark successive phases of the endgame. The actual game was adjudicated by the ‘TCEC draw rule’ at move 175b, Fig. 6e.

SF/SF⁺: 94. Qc4^{'''} Qf2+^{''} 95. Ka3 [+2pF]

(SF[±]: 95. Kb3^{''} Qe3+^{''} 96. Ka4^{'''} Kg6^{''} 97. b5^{'''} Kh5^{''} 98. g6^{'''} Qa7+['] 99. Kb3['] Qe3+^{''} 100. Qc3^{''} Qe2['] 101. Kb4^{''} Qe4+^{''} 102. Kc5^{''} Qe7+^{''} 103. Kb6^{''} Qd6+^{''} 104. Ka5^{'''} Qxf4^{''} **8-man**, *dtf* = 137p 105. b6^{'''} Qe4^{''} 106. Qc7^{''} Qd5+^{''} 107. Kb4['] Qd4+^{''} 108. Kb5^{''} Qd5+^{''} 109. Qc5^{''} Qf3^{''} 110. Kb4^{''} Qf4+^{''} 111. Ka5^{''} Qf3^{''} 112. Qb5^{''} Qa3+^{''} 113. Qa4[°] Qc5+^{''} 114. Ka6^{''} Qd6^{''} 115. Qa1['] Kxg6^{''} {7-man pos: *dtc/f/m/z* = 49/115/143/27p, 50-move rule irrelevant})

S(F&M&Z)[±]: 116. Qc3^{'''} Qe6^{''} 117. Ka7^{''} Qa2+^{''} 118. Kb8^{'''} Qg8+^{''} 119. Qc8^{''} Qd5^{''} 120. Qc7^{'''} Qg8+^{''} 121. Kb7^{''}

S((F&M)Z)[±]: Kh7^{''} 122. Qd7^{''} Qf8^{''} 123. Kc6^{''} Qf6+^{''} 124. Kc7^{''} Qc3+^{''} 125. Qc6^{'''} Qe5+^{''} 126. Qd6^{''} Qc3+^{''} 127. Kb8^{''} f4^{''} 128. Qxf4^{'''}

6-man, *dtf/m/z* = 90/118/6p 128. ... Qh3^{''} 129. b7^{''} Qh5^{''} 130. Qe4+^{''} Kh8^{''} 131. Qe6^{''} Qf3^{''} 132. Qd6 Qe4 133. Qh2+^{''} Kg8^{''} 134. Qa2+^{''} Kh8^{''} 135. Ka8^{''} Qh1^{''} 136. Qa5^{''} Qe4^{''} 137. Qb5^{''} Kh7^{''} 138. Qb3^{''} Qh1^{''} 139. Qc2+^{''} Kh8^{''} 140. Qc5^{''} Qg2^{''} 141. Qh5+^{''} Kg8[°] 142. Qd1^{''} {*dtf/m/z* = 62/90/56p}

S(FMZ)[±]: Qe4^{''} 143. Qb3+ Kh8 144. Qh3+^{''} Kg8^{''} 145. Qc3^{''} Kh8^{''} 146. Ka7^{''} Qe7^{''} 147. Qh3+^{''} Kg8^{''} 148. Qf5^{''} Kh8^{''} 149. Qh5+^{''} Kg8[°] 150. Qd5+^{''} Kh8^{''} 151. Ka6^{''} Qe2+ 152. Kb6^{''} Qe3+^{''} 153. Kc6^{''} Qc3+^{''} 154. Kd7^{''} Qh3+^{''} 155. Ke8^{''} Qe3+^{''} 156. Kd8^{''} Qf4^{''} 157. Qh1+^{''} Kg8^{''} 158. Qc6^{''} Kh7^{''} 159. Qc2+^{''} Kh6^{''} 160. Qc8^{''} Qh4+^{''} 161. Kd7^{''} Qh3+^{''} 162. Ke8^{''} Qe3+^{''} 163. Kd8^{''} Qd3+^{''} 164. Qd7^{''} Qg3^{''} 165. Qc6+^{''} Kh7^{''} 166. Qc2+^{''} Kh6^{''} 167. Qc1+^{''} g5^{''} 168. Qc7^{''} Qd3+^{''} 169. Qd7^{''} Qg3^{''} 170. Ke7^{''} Kg7^{''} 171. Qd6^{''} Qe1+^{''} 172. Kd7^{''} Qb1^{''} 173. b8=Q^{''}

KQQKQP, *dtm/z* = 22/8p Qxb8^{''} 174. Qxb8^{'''} Kf6^{''} 175. Qb5^{''} g4^{''} 176. Kd6^{''} g3^{''} 177. Qe5+^{''} Kf7^{''} 178. Qxg3^{''} Kf6^{''} 179. Qg4^{''} Kf7[°] 180. Qg5^{''} Kf8^{''} 181. Ke6^{''} Ke8[°] 182. Qg8[#] 1-0.)

95. ... Qf3+ [-60pF] 96. Ka4^{'''} Qd1+^{''} 97. Ka5^{''} Qd8+^{''} 98. Ka6^{''} Qd6+^{''} 99. Kb5^{''} Qd8 [-2pF] 100. Kc6 [+2pF] Qc8+ [-2pF] 101. Kb5 [+4pF] Qb7+['] 102. Kc5^{''} Qc7+['] 103. Kd5^{''} Qf7+^{''} 104. Kd4^{''} Qe7^{''} 105. b5^{''} {*dtf* = 80p} Qd6+^{''} 106. Ke3? {FINALGEN “1-0/=”} Qd1^{'''} 107. Qb4['] Qg1+['] 108. Kd3['] Qf1+['] 109. Kd2['] Qf2+['] 110. Kd1['] Qf3+['] 111. Kc1['] Qe3+['] 112. Kc2['] Qe2+['] 113. Kc1['] Qf1+['] 114. Kd2['] Qf2+['] 115. Kc1['] Qe3+['] 116. Kb2['] Qe2+['] 117. Ka3['] Qd3+['] 118. Ka4['] Qd1+^{'''} 119. Ka5['] Qd8+^{'''} 120. b6? {FINALGEN “=”} Qa8+^{'''} 121. Kb5[°] Qe8+^{'''} 122. Kc4['] Qe4+^{'''} 123. Kc3['] Qf3+['] 124. Kd2['] Qf2+^{'''} 125. Kd3['] Qf3+^{'''} 126. Kc2['] Qe2+^{'''} 127. Kc1['] Qf1+['] 128. Kb2['] Qe2+^{'''} 129. Ka1['] Qf1+['] 130. Ka2['] Qe2+['] 131. Kb1['] Qd3+['] 132. Kc1['] Qf1+['] 133. Kd2['] Qf2+^{'''} 134. Kd3['] Qf3+^{'''} 135. Kc2['] Qe2+^{'''} 136. Kb1['] Qf1+['] 137. Ka2['] Qe2+['] 138. Ka3['] Qa6+['] 139. Kb3['] Qd3+^{'''} 140. Ka4['] Qa6+^{'''} 141. Qa5['] Qc4+^{'''} 142. Ka3['] Qd3+['] 143. Kb2['] Qe2+['] 144. Kc1['] Qf1+['] 145. Kd2['] Qxf4+^{'''} {8-man} 146. Kc3['] Qf3+['] 147. Kb2['] Qe4? {FINALGEN “1-0/=”} 148. Qb5? {FINALGEN “=”} Qd4+^{'''} 149. Kc1['] Qe3+['] 150. Kb1['] Qe1+^{'''} 151. Kc2['] Qe4+^{'''} 152. Kc1['] Qe3+['] 153. Kb2['] Qd2+['] 154. Kb3['] Qd1+^{'''} 155. Kc3['] Qe1+^{'''} 156. Kc2['] Qe4+^{'''} 157. Kb2['] Qd4+^{'''} 158. Kb1['] Qd1+['] 159. Ka2['] Qd2+['] 160. Ka3['] Qc3+^{'''} 161. Ka2['] Qd2+['] 162. Ka3['] Qc3+^{'''} 163. Ka4['] Qa1+['] 164. Kb3['] Qd1+['] 165. Kc3['] Qe1+^{'''} 166. Kc4['] Qe4+['] 167. Kb3['] Qb1+^{'''} 168. Kc4['] Qe4+^{'''} 169. Kc5[']

Qe5+''' 170. Kb4' Qe1+' 171. Ka4' Qa1+''' 172. Kb4' Qe1+' 173. Kc5' Qe5+''' 174. Kc6' Qe8+''' 175. Kc5''' {adjudicated 'TCEC draw'} 1/2-1/2.

7 Karpov – Kasparov, WCC 1984-10-05, Moscow, game 9

Position 66b, 8/4N3/8/1p6/1PkP4/P3K3/6b1/8 b - - 0 66, drawn:

This is the 'Fermat's Last Theorem' of endgame quests, a position that has inspired more incorrect arguments than any other, so many that the analysis of those arguments would be an interesting thesis of its own. Fortunately, this saga is now at an end as we can definitively assess both the 'ground truth' and the false proofs with FINALGEN.

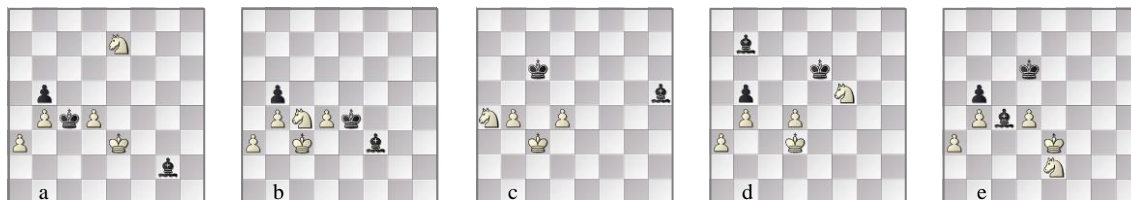


Fig. 7. Karpov-Kasparov: (a) pos. 66b; drawing line (b) 73b and (c) 77w; in a better defence d) 69w and (e) 74b.

The game is no less significant than the endgame. It saw Karpov go 4-0 up and preceded 17 draws in the controversial championship match eventually abandoned at 5-3 after five months and 48 games. The position of Fig. 7a has been an enduring mystery, visited many times over the years (Dvoretzky, 2008; Karolyi and Aplin, 2007; Marin, 2006; Müller, 2003 and 2005; Müller and Konoval, 2018; Timman, 2007). Was the draw missed (Kasparov, 2007) with the move 66. ... Bb7?

FINALGEN shows not only that the amazing defence 66. ... Bh1 draws but that it is the only drawing move. Figs. 7b-c are taken from the drawing line. It also shows that the path to the winning line after 66. ... Bb7?? is long and narrow with six 'only' moves for White. Figs. 7d-e are from a 'best defence' albeit losing line. After we had run this FINALGEN computation in 2017, Jan Kominek pointed out that Roman Jiganchine (2015), whom we salute, had already got the same result. It is intriguing to think that there must be many other famous game and study positions (van der Heijden, 2017) whose FINALGEN analyses have not been widely promulgated or even done.

66. ... Bb7?? Now Black is indeed lost.

(The amazing 66... Bh1''' is the only defence, e.g., 67. Nf5' Kd5''' 68. Ng3' Bg2''' 69. Kd3' (69. Ne2 Kc4=) 69. ... Bf3' 70. Nf1' Be4+' 71. Kc3' Bf3' 72. Ne3+' Ke4' 73. Nc4'. Karsten had thought that White wins as he achieves a3-a4 now, but Mark Dvoretzky proved him wrong with the help of the 6-man Nalimov tablebase as Kd5' 74. Nb2' Bh5' 75. a4'' bxa4''' 76. Nxa4'' Kc6' is a draw.)

67. Nf5''' Bg2?! {[-34pF]}

(67... Kd5'' is more tenacious. 68. Kd3''' Ke6'' but White wins nevertheless due to 69. Ng7+''' Kd7''

(69. ... Kd6 70. Ne8+ Ke7 71. Nc7 Bc6 72. d5'' +-)

(69... Kd5 70. Ne8'' +-)

(69... Ke7 70. Nh5'' +-)

70. Nh5'' Bg2'' 71. Nf4''' Bf1+' 72. Ke4''' Kd6'' 73. Ke3''' Bc4''

(73... Kc6 74. d5+ Kd6 75. Kd4 Bc4 76. a4'' +-)

74. Ne2''' followed by Nc3, which reaches an always winning configuration. Bf7'' 75. Nc3'' Be8'' 76. Kd2'' Bc6' 77. a4'' bxa4''

7-man: dtf/m/z = 47/59/7p. S(FMZ)±: 78. Kc2' Bd7' 79. Kb2'' Kc7'' 80. Ka3'' Kb6'' 81. Nxa4+'

6-man, dtf/m/z = 40/56/4p 81. ... Kc6' 82. Nc3'' Kb6'' 83. Kb3'' Be6+' 84. d5'' Bh3'' 85. Kc4'' Bf1+' 86. Kd4'' Bh3'' 87. b5'' Bg4'' 88. Kc4'' Bh3'' 89. d6'' Bf1+' 90. Kb4'' Bg2' 91. Na4+' Kb7'' 92. Ka5'' Kc8'' 93. Nc5'' Bh1' 94. d7+' Kd8' 95. Ka6'' Bg2' 96. b6'' Bf1+' 97. Ka7'' Bg2'' 98. b7'' Bxb7'' 99. Kxb7'' Ke7° 100. Kc7' Kf6'' 101. d8=Q+''

68. Nd6+' Kb3 {[-2pF]} 69. Nxb5''' Ka4 {[-2pF]} 70. Nd6'' Resigns, 1-0.

8 Summary

FINALGEN works beyond today's 7-man endgame table limit, bringing definitive knowledge to the table, providing benchmark standards of play and resolving questions that have remained unanswered for as long as thirty years. In the context of current technology, it has untapped potential both as a working program and as a concept.

It is currently single-threaded but the principles of concurrent programming are well known and cpu chips continue to offer more cores and processing threads. PC B's 4-lane PCIe v3.0 interface to its SSD provides over 1GB/s but PCIe v5.0 (PCI-SIG, 2019) in 16-lane mode promises to offer 16 times the bandwidth, effectively reducing disc-latency to zero. Romero (2012) provides code libraries to interface chess engines to FINALGEN EGTs: we do not believe these have been used yet. FINALGEN does not create value-only EGTs as de Man et al (2018) do. Nor does it use two ideas exploited in Bleicher's FREEZER (Müller, 2005), namely trial-constraints on the chessmen and interfacing to precomputed EGTs. The full analyses (Müller and Haworth, 2019) show how truncated FINALGEN's searches would be if 7-man EGTs could be consulted. These considerations suggest that the efficient creation of FINALGEN EGTs could be a part of chess engines' runtime calculations.

More immediately, a growing FINALGEN community, centred around the forum (Romero, 2012) could evolve and share future results. Endgame tables now give instant answers for 7-man positions but FINALGEN can address many positions with eight men or more. The examples here will hopefully inspire others to show what is possible in creating further unarguable insights into the game of chess.

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Appendix: some logistics details on working with FINALGEN and ‘syzygy’ EGTs

One caveat: FINALGEN does not reliably pick up the ‘side to move’ when importing a position.

FINALGEN can create files larger than the 4GiB limit of the venerable FAT32 file system. Media can be reformatted to support NTFS which allows files to ~8PiB.

An SSD’s lifetime is determined by the intensity of writing activity. It is therefore worth considering whether or not one should compress SSD-files from inception. Windows’ NTFS compression uses the ‘IEEE Milestone’ LZ77/LZ1 algorithm (Ziv and Lempel, 1977). Being linear and highly efficient, LZ77 requires minimal rewriting to disc. Given that NTFS is achieving 29% compression on ‘Normal mode’ (and 39% compression on ‘SdF’) files, there is an argument for compressing FINALGEN files on SSDs from inception. Further, doing so with HDDs cuts disc-latency time. On PC B, the computation overhead was about 40% in ‘Normal Mode’ and 7% in SdF mode, arguably worthwhile.

FINALGEN is not commutative. The evaluation of the Navrotescu–Shah positions 11a and 11b lead to different-sized EGTs if evaluated in the opposite order. This is presumably because the information ‘inherited’ in the first computation has different value from that inherited in the second computation. Another comparison may be instructive if artificial. The EGT created from the ‘initial’ LC0 position 94w is 9% larger than the EGT created by visiting in reverse a succession of positions along the DTF-optimal line already discovered. This does however suggest that if one is analysing a played game, a good technique is to build the EGT starting with the later positions.