# *Wavelet spectral analysis of the free surface of turbulent flows by Giulio Dolcetti and Héctor García Nava*

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Discussion

# Comment on "Wavelet spectral analysis of the free surface of turbulent flows", by Dolcetti and García Nava

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In an interesting recent study on turbulent structures in open-channel flow and their surface signatures, Dolcetti & García Nava (2018) (hereafter DGN18) present laboratory measurements of stationary free surface waves propagating against a stream with mean vertical shear induced by the boundary layer formed at the bottom of the channel. To a first approximation, the wavelength of those waves is determined by a dispersion relation modified by the shear. To evaluate this effect, DGN18 assume that the current profile takes the form

$$U(z) = U_0 \left(\frac{z}{d}\right)^n, \tag{1}$$

where $U_0$ is the surface current speed, $d$ is the mean water depth, and $z$ is the height within the water, with $z = 0$ corresponding to the bottom of the stream and $z = d$ to the mean level of the free surface. The exponent in (1) is estimated as $n = 1/3$.

DGN18 calculate the wavelength of the stationary waves both for the experiments of Horoshenkov, Nichols, Tait & Maximov (2013) and Nichols, Tait, Horoshenkov & Shepherd (2016), and for their own experiments (where they do not measure this quantity directly) using a formula involving modified Bessel functions (their Eq. (16)), which takes into account both surface tension and finite water depth effects.

A formula that is much simpler but gives results of comparable accuracy is proposed here, based on a constant-shear, deep-water gravity-wave approximation. The corresponding dispersion relation for non-stationary waves was first derived by Craik (1968) (according to Ellingsen & Li (2017)), but was re-derived by Shrira (1993) and Teixeira (2000). For waves aligned in the flow direction, Eq. (4.30) of Teixeira (2000) reduces to:

$$\sigma^2 + \Gamma\sigma - \sigma_0^2 = 0, \tag{2}$$

where $\sigma$ is the angular frequency, $\Gamma = (\mathrm{d}U/\mathrm{d}z)(z = d)$ is the shear rate at the surface, and $\sigma_0$ is the intrinsic angular frequency of free waves unaffected by shear. For waves that propagate upstream against the flow so as to be steady in a fixed frame of reference, $\sigma$ must be replaced by $-U_0k_0$, where $k_0$ is the corresponding wavenumber. For pure deep-water gravity waves, $\sigma_0^2 = gk_0$, whence Eq. (2) yields

$$k_0 = \frac{g + \Gamma U_0}{U_0^2} \quad \Rightarrow \quad \lambda_0 = \frac{2\pi}{k_0} = \frac{2\pi U_0^2}{g}\frac{1}{1 + \Gamma U_0/g}. \tag{3}$$

$k_0$ or the wavelength $\lambda_0$ are therefore obtainable in a very concise closed form. The term involving $\Gamma$ is the correction due to shear. From Eq. (1), $\Gamma = (\mathrm{d}U/\mathrm{d}z)(z = d) = nU_0/d$, which makes Eq. (3)

become:

$$\lambda_0 = \frac{2\pi U_0^2}{g}\frac{1}{1+nU_0^2/(gd)} \quad \Rightarrow \quad \frac{\lambda_0}{d} = \frac{2\pi \mathsf{F}^2}{1+n\mathsf{F}^2}, \tag{4}$$

where in the latter dimensionless relation $\mathsf{F} = U_0/(gd)^{1/2}$ is the Froude number. The wavelength normalized by $d$ therefore only depends on $\mathsf{F}$, for a given $n$.

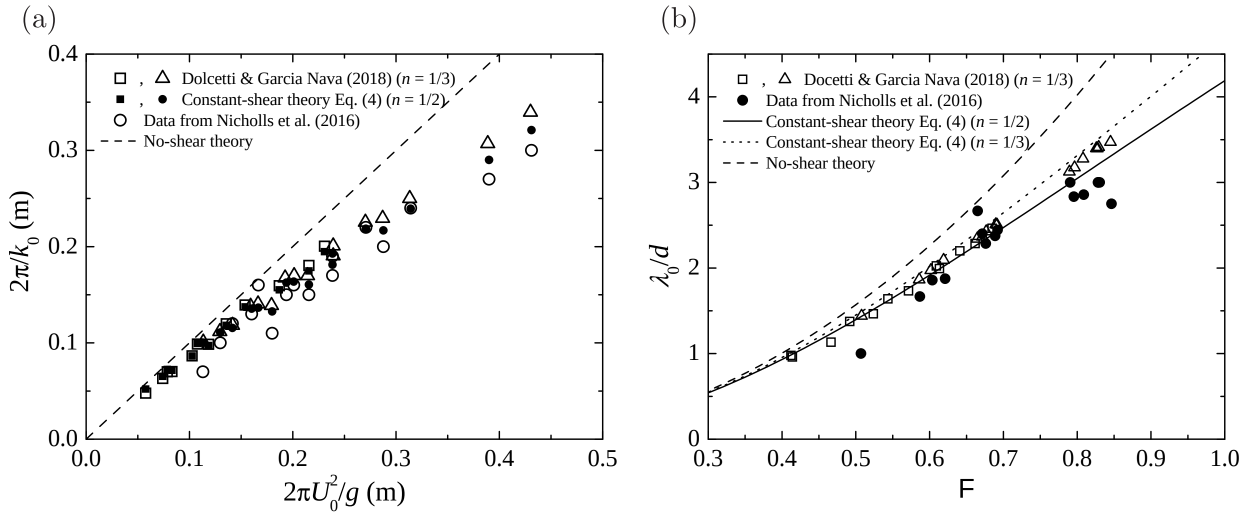


Figure 1 Wavelength of stationary surface waves propagating against a sheared stream. See legend for meaning of lines and symbols. (a) $\lambda_0$ as a function of $2\pi U_0^2/g$. Note that, from theory, $\lambda_0$ should also depend on $\mathsf{F}$. (b) $\lambda_0/d$ as a function of $\mathsf{F}$.

Figure 1a shows $\lambda_0$ as a function of $2\pi U_0^2 g$, as in Fig. 8 of DGN18. The filled squares and circles were calculated using Eq. (4), whereas the open symbols are either data from Nichols et al. (2016) (circles) or values calculated by DGN18 using their Eq. (16) with $n = 1/3$ (squares and triangles). The agreement of Eq. (4) with both datasets is quite good, although $n = 1/2$ was assumed instead. This is consistent with the fact that in Fig. 8 of DGN18 the values calculated from their Eq. (16) with $n = 1/2$, corresponding to the lower error bars (not shown here), produce the best agreement with the data of Horoshenkov et al. (2013).

Figure 1b shows the same data normalized by $d$. Since Eq. (4) predicts that $\lambda_0/d$ is an exclusive function of $\mathsf{F}$, the corresponding variation is denoted by lines. This means that, if the model is accurate, each dataset should collapse to a single curve in this representation (this does not happen in Fig. 1a, because $\lambda_0$ is expected to depend on $\mathsf{F}$ in addition to $2\pi U_0^2/g$ – see first equation of Eq. (4)). The experimental data of Nichols et al. (2016) naturally have considerable scatter, since it is technically challenging to measure $\lambda_0$ experimentally, but they show a trend compatible with Eq. (4). The data from DGN18, on the other hand, follow almost perfectly a single line (because they also result from an analytical calculation, namely Eq. (16) of DGN18), and by comparison with (4) for $n = 1/3$ corroborate the good accuracy of the approximation proposed here.

It is worth mentioning that both shallow water and surface tension effects appear to be negligible in these datasets. Based on the values of $k_0$ estimated by DGN18 (which differ little from those given by (4)), the lowest value taken by the wavelength is $\lambda_0 = 4.8\,\text{cm}$. This is still substantially larger than the value $\lambda_0 = 1.7\,\text{cm}$ at which the transition to waves dominated by surface tension occurs (if relevant, this effect would show on the left limit of the graphs in Fig. 1, but is actually undetectable). On the other hand, the lowest value of $k_0 d$ provided by the dataset of Horoshenkov et al. (2013) and Nichols et al. (2016) is 2.1, which yields only a 1.6% departure of the phase speed magnitude relative to the deep-water approximation. This inaccuracy should be largest towards the right end of the graphs in Fig. 1 but, again, is undetectable.

The overall conclusion is that, as long as the gravity-wave and deep-water approximations remain valid, Eq. (4) seems accurate enough for the purposes envisaged by DGN18. This may happen because the shear generated by bottom friction in open-channel flow varies slowly with depth near the free surface, as assumed in (2).

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