An Empirical Study of Asset Value and Volatility in Structural Credit Models

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by

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Abstract

Asset value volatility is at the heart of the capital structure optimisation theory as proposed by Leland (1994). Asset value and volatility are two key unobservable inputs required for pricing equity and debt using structural models that are based on option pricing theory. These inputs are normally calculated using equity prices, book value of debt and leverage. However, when tested empirically, the prediction errors of structural models suggest that such calculations of asset value and volatility may be inaccurate. Furthermore, in all structural models it is assumed that asset value volatility is constant over time, which may also lead to inaccurate credit spread predictions.

This is the first empirical study into bond-implied asset values and volatility. This study uses three-year time series of daily bond– and equity prices for 36 Western-European companies to derive bond-implied asset values and asset value volatilities. Bond-implied values were calculated using a structural credit model developed by Leland and Toft (1996). The values were obtained by simultaneously solving a system of two equations and two unknowns. The two equations were the value of a bond and the value of equity; the two unknowns were asset value and asset value volatility. An equity-based method for calculating asset value and volatility was used to generate an alternative data set for the same daily observations. The two sets of data for each firm were compared to determine whether there are statistically significant differences between bond-implied and equity-based asset values and volatility, and whether bond-implied volatility is stable over time. Additionally, the correlation between bond-implied volatility and the bond's time to maturity was tested to determine whether there is a constant relationship between these two variables.

This study shows that there are significant differences between asset values and asset value volatilities derived from bond prices and those derived using equity values and leverage. Additionally, it is shown that bond-implied asset value volatility is not constant over time. The relationship between bond-implied asset value volatility and a bond's time to maturity as measured by their correlation coefficient was not uniform across the sample, in some cases demonstrating strong positive correlation, in other cases strong negative correlation and in some cases no strong relationship at all.

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1. Introduction

The theory of capital structure optimisation evolved from the seminal work by Modigliani and Miller (1958), which assumed debt is risk free and no market imperfections, through the introduction of agency costs and the risk of asset substitution by Jensen and Meckling (1976), to the inclusion of risky debt by Leland (1994). Leland's approach to capital structure optimisation is based on option pricing theory originally developed by Black and Scholes (1973) and Merton (1974), and it helps highlight the importance of debt in the capital structure optimisation decision. Quantifying the level of debt that optimises a firm's capital structure is part of the process of credit risk measurement and management.

Credit risk measurement and management have been gaining importance in recent years for a variety of reasons, including the increased level of debt issued by companies, disintermediation in capital markets, increased computing power, developments in the field of finance and changes to the regulatory framework of international banks and financial institutions.

The advances in option pricing theory by Black and Scholes (1973) and Merton (1974) led to the development of a new family of credit pricing models called 'structural credit models' or 'contingent claims models' which use quantitative methods to estimate the fair price of credit risk. Such models use several observable inputs taken from bond/loan indentures and from a company's balance sheet, but they also require two unobservable inputs: the firm's asset value and its volatility. Several methods have been used in order to estimate these inputs, but empirical research by Jones, Mason and Rosenfeld (1984), Wei and Guo (1997), Eom, Helwege and Huang (2004) and others suggest that structural models do not estimate credit spreads accurately.

The primary objective of this study is to derive and analyse asset value and asset value volatility as implied by bond prices and to compare the implied values to those obtained using an estimation method first proposed by Jones, *et al.* (1984).

In order to conduct the analysis, two methods were employed to calculate asset value and asset value volatility: using equity volatility and leverage, and implied from bond prices using a structural model. The methodology resulted in data sets for 36 firms, each set of data comprising a three-year daily time series of equity-derived and bond-implied asset values and asset value volatilities. Statistical tests were then used to determine whether there are statistically significant differences between the equity-derived and bond-implied data sets, whether bond-implied asset value volatility is constant over time, and whether there is correlation between a bond's time to maturity and its implied asset value volatility.

1.1. Capital Structure and Volatility

The search for an optimal capital structure has been an important discussion topic amongst researchers and practitioners for many years, and has given rise to several different approaches. A firm's capital structure affects the valuation of its equity, debt, and its ability to undertake and finance new projects. As such, the factors associated with capital structure optimisation permeate all areas of modern corporate finance and investment decisions.

A pioneering approach to the assessment of a firm's capital structure was developed by Modigliani and Miller (1958) who wrote a seminal paper on the cost of capital, corporate valuation and capital structure using the following assumptions:

- Capital markets are frictionless
- Individuals can borrow and lend at the risk-free rate
- There are no costs associated with bankruptcy
- Firms' capital structure comprises risk free debt and risky equity
- All firms are in the same risk class
- All cash flows are perpetuities (not growth)
- Corporate insiders and outsiders have the same information
- No agency costs (i.e. management always seeks to maximise shareholders' value).

Using this framework, Modigliani and Miller showed that in the absence of any market imperfections the value of the firm is independent of the type of financing it uses. This assertion is also known as the "Modigliani-Miller Proposition I". A subsequent paper by Modigliani and Miller (1963) showed that when corporate taxes are introduced, the firm's value is maximised by maximising the debt component of its capital structure, because of the tax shield created by debt, also known as the gain from leverage. Miller (1977) modified the earlier results of Modigliani and Miller (1958) and Modigliani and Miller (1963) by introducing personal taxes as well as corporate taxes. Miller (1977) showed that if personal income on dividend income is lower than personal tax on interest income, then the pre-tax return on holding bonds should be high enough to offset this disadvantage, otherwise investors would not invest in bonds. In such cases, the advantage of holding shares in a levered company could be offset by the higher interest payments that need to be made to bondholders.

Jensen and Meckling (1976) developed an alternative approach to capital structure optimisation, based on agency theory. They argued that the choice of projects undertaken by a firm is affected by its ownership structure, and this helps to explain firms' choice of capital structure. On the one hand, lenders face the risk of asset substitution, whereby firms borrow money for projects with a given risk profile but then choose riskier projects once funds have been lent. Lenders seek to mitigate this risk by including various types of covenants and monitoring mechanisms in their bond indentures and loan agreements. Additionally, lenders may charge higher rates of interest to compensate for this potential risk, and the costs of borrowing may increase with the proportion of debt financing used by the firm. External equity also has associated agency costs, as the greater the proportion of external equity used to finance the firm, the greater the costs of monitoring the firm's management to ensure that it seeks to maximise shareholders' wealth. Jensen and Meckling (1976) argue that given the increasing agency costs associated with external debt and with external equity, there is a point at which the capital structure of the firm minimises the total agency costs, thus providing an optimal capital structure.

Developments in the field of options pricing theory by Black and Scholes (1973) and Merton (1974) led to a more quantitative approach to capital structure modelling proposed by Leland (1994). By valuing the equity of a levered firm as a call option on the firm's value, and the bonds issued by the firm as risk-free bonds less a put option on the firm's assets, Black and Scholes (1973) and Merton (1974) provided the basis for an alternative approach to capital structure optimisation, whilst still assuming that the Modigliani-Miller Proposition I holds. Using option pricing theory, the value of a call option increases with the increase in volatility of the reference asset. The value of a put option also increases with volatility, all other things being equal. Therefore, using the option pricing approach, it is possible to show that if asset value volatility increases, the value of a firm's equity will increase, and at the same time the value of its bonds will decrease (because the put option subtracted from the value of a risk-free bond has increased). This transfer of value from bondholders to equity holders is another representation of the agency costs identified by Jensen and Meckling (1976) and is referred to as the "bondholder wealth expropriation hypothesis" by Copeland and Weston (1988). Leland (1994) and Leland and Toft (1996) used option pricing theory to develop an approach for optimising the capital structure of firms that choose the amount and maturity of their debt, incorporating corporate taxes, bankruptcy costs and endogenously determined bankruptcy triggers. Their approach provides a more precise and quantitative methodology for determining an optimal capital structure not only in terms of its composition of debt and equity, but also in terms of the debt maturity profile, at the same time incorporating the concepts developed by Modigliani and Miller (1963) and Jensen and Meckling (1976).

Using option pricing theory to model optimal capital structure brings to the fore two important inputs that are needed in order to value the firm's equity (call option) and debt (risk-free debt less a put option): asset value and asset value volatility. The volatility of asset values is particularly important, as it represents the risk of the assets. Volatility is present in stock price valuation, risky debt valuation and in determining the bankruptcy barrier in structural models. As such it permeates all the concepts underlying the theory of optimal capital structure and investment decisions which use option pricing theory to explain the relationship between shareholders and bondholders.

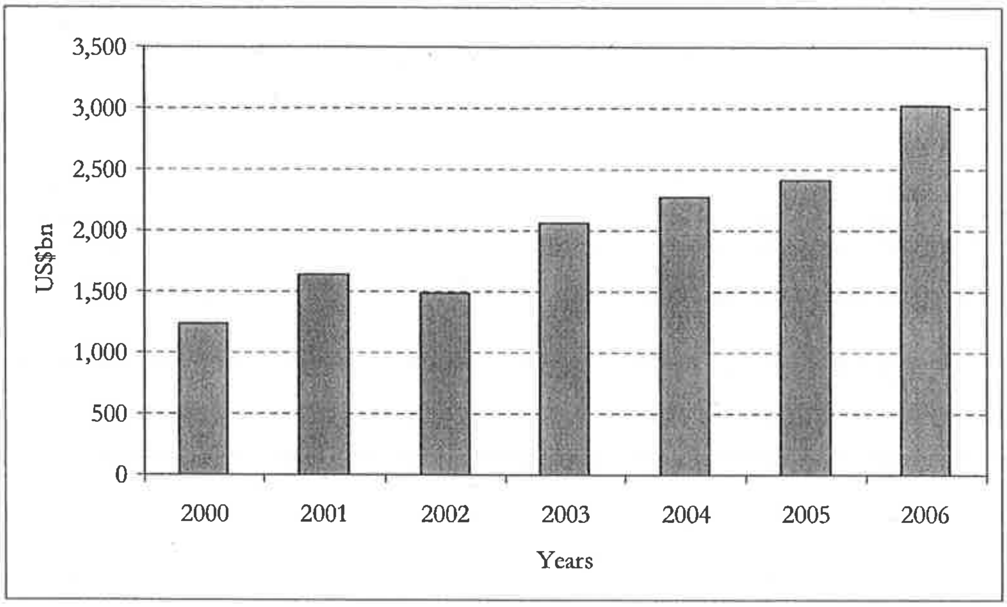
1.2. The Importance of Credit Risk

The concepts of equity valuation and shareholder value creation are familiar to all students and researchers of finance and management and to many members of the public at large. Sharp swings in equity prices are reported in the news and in the popular press in many countries. Credit risk and its estimation have been left largely out of the limelight for a variety of reasons, among them:

- Whilst shares are relatively liquid instruments and can be bought and sold in small amounts, bonds are not as liquid and they trade in amounts that are out of reach for most retail investors.
- Management teams in many companies have explicit incentives to maximise shareholders' returns, not in keeping their company's credit risk at acceptable levels.
- Credit risk is not as tangible a concept as a share price – it is more difficult to comprehend and measure.

On the other hand, the level of debt issuance by companies is significant and rising, as the table below shows:

Figure 1.1: Total US and European corporate bonds issued



Source: Bloomberg League Tables

In 2007 the Securities Industry and Financial Markets Association estimated that the total amount of debt instruments outstanding in the United States at the end of 2006 was equal to US$26 trillion (www.sifma.org). This figure dwarfed the total market capitalisation of all the S&P100 companies which was around US$7 trillion.

The importance of understanding credit risk stems not only from the size of the debt markets or the increasing amount of issuance, but also due to the aspect of default and losses incurred by creditors when debts cannot be repaid. Although the majority of debts outstanding are repaid or refinanced when they fall due, a small proportion of borrowers default on their debts. In order to reflect the risk of default creditors vary the rate of interest they charge according the perceived risk entailed by the borrower – a high risk borrower will normally be charged more than a low risk borrower. However, when defaults occur, the losses incurred by creditors can be significant. Zazzarelli, Bodard de, Cantor, Hamilton and Emery (2007) calculated the average recovery rate for defaulted senior secured bonds in Europe between 1982 and 2006 to be 44.46%, which means creditors of these defaulted companies lost 55.54% of the amounts owed to them. Holders of subordinated bonds recovered on average only 30.76% of the amounts outstanding, entailing an average loss of principal of 69.24%. Clearly the ability to estimate the probability of default and the magnitude of the potential loss are crucial if investors seek to maximise their returns from investing in bonds and other forms of defaultable securities.

In recent times, with the increase in debt issuance, credit risk measurement began gaining importance among banks, regulators, and a growing community of investors. Saunders and Allen (2002) highlight several reasons for the increased profile of credit risk, its measurement and management:

- Structural increase in bankruptcies during recent recessions;
- Disintermediation as many large and medium-size borrowers issue bonds to a range of investors rather than borrow from banks using loans;
- Increased competition among lenders and margin erosion;
- Growth of derivatives that facilitate trading credit risk;
- Technological advances such as increased computing power and more comprehensive data bases of loans and their default history;
- Regulatory pressures, most notably the introduction of 'Basle II' which is a new approach to calculating regulatory capital held by banks using internal models to calculate Value-At-Risk ('VAR') and correlation among credit-risky assets.

The importance of credit risk and management and the advances made in this field were highlighted in the remarks made by the Chairman of Board of Governors of the Federal Reserve System of the USA, Mr. Bernanke (2006):

> "Risk-management principles are now ingrained in banks' day-to-day credit allocation activities. The most sophisticated banking organizations use risk-rating systems that characterize credits by both the probability of default and the expected loss given default. Consistent with the principles of the Basel II accord, the largest banks evaluate credit decisions by augmenting expert judgment with quantitative, model-based techniques.
>
> ... new analytical tools and techniques have made lending to corporate borrowers highly quantitative. Among these tools are models that estimate the risk-adjusted return on capital and thus allow lenders to price relevant risks before loan origination. Other tools include proprietary internal debt-

rating models and third-party programs that use market data to analyze the risk of exposures to corporate borrowers that issue stock."

Remarks by Chairman Ben S. Bernanke at the Stonier Graduate School of Banking, Washington D.C., 12th June 2006.

The growing importance of managing credit risk and the developments in the field of finance, mathematics and computing facilitate increasingly sophisticated approaches to credit risk measurement.

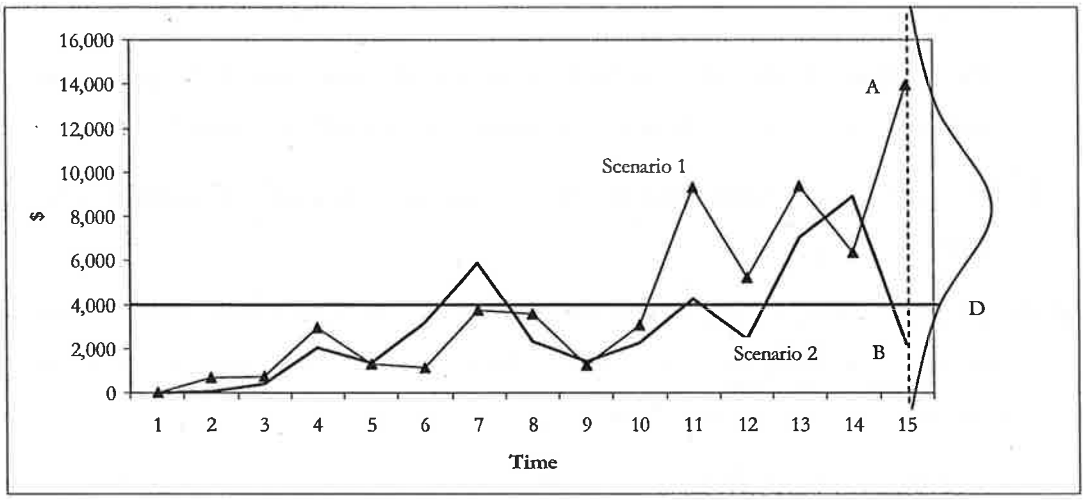
1.3. Developments in Credit Risk Measurement

The aim of detecting company difficulties and predicting failure has been the topic of research for many years. Before quantitative methods were developed, specialised agencies were established to assess credit worthiness using qualitative measures. One of the first was incorporated in Cincinnati, Ohio in 1849, later growing into Dun & Bradstreet, Inc. In the 1930's, following the great depression, aggregate studies such as Smith and Winakor (1935), Merwin (1942) and Hickman (1958) looked at the financial ratios of firms that experienced financial difficulties and failures. Altman (1966) undertook multiple discriminant analysis using a set of 66 firms and developed the Z-score. The initial set of variables was classified into five categories: liquidity, profitability, leverage, solvency and activity.

As the theory of finance developed, so did the proposed approaches to measuring credit risk. A quantitative approach that is grounded in financial theory and became increasingly popular was initially proposed by Black and Scholes (1973) who noted in their seminal paper on pricing options that their methodology can be used to value credit risky instruments. The Black and Scholes (1973) approach is based on the view that in a limited liability company, shareholders have an option to pay the company's debt and retain their control over its assets, or they can choose not to pay the company's debt and lose their control over its assets. Viewed in this way, a company's share is a call option on the company's assets, whose exercise price is the level of the company's debt. The company's debt can also be viewed as an option – by lending money to a company, creditors provide the company with an option: to pay back the debt when its due and retain control over the assets or to default on payments and lose control of the assets. In other words, creditors sell shareholders a put option referenced on the company's asset value with an exercise price equal to the level of debt outstanding. The approach proposed by Black and Scholes (1973) is known as 'structural credit model' because it is based on the firm's capital structure, or as the 'contingent claims model' due to the interplay between the interests of creditors and shareholders. Merton (1974) developed the Black and Scholes (1973) approach into a closed form solution for pricing zero coupon bonds. Merton's approach assumes that the firm's asset value is normally distributed, that it has a simple capital structure comprising one class of shares and one class of zero coupon bonds, no dividends or refinancing is allowed and risk-free interest rates are known and constant. In Merton's approach, at the bonds' maturity date the firm's assets are sold and bondholders receive their repayment; any residual value is distributed to shareholders.

The Merton (1974) model can be summarised diagrammatically as shown in figure 1.2 below:

Figure 1.2: the Merton model



The lines denoted 'A' and 'B' represent two possible paths that the firm's asset value might follow over time. As the firm's asset value is assumed to be normally distributed, all possible values are represented by the bell-shaped curve. The line denoted 'D' is the firm's outstanding debt. At maturity, if the firm's asset value is at level 'B', the firm defaults because the asset value is lower than the debt outstanding – bondholders will suffer a loss and equity holders will receive nothing. If the firm's value is at level 'A', bondholders will be repaid in full and equity holders will receive a positive residual.

Merton's model contains some restrictive assumptions that limit its applicability to practical implementation. Other researchers, among them Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), Collin-Dufresne and Goldstein (2001) and others proposed different versions of the structural model seeking to address some of the limitations of the original Merton model.

1.4. The Research Problem

The Merton (1974) model and all the structural models that were developed since require five key inputs (the more sophisticated the model, the more additional inputs are needed):

1. Risk free interest rate, denoted by r, which in many models is assumed to be constant over time.
2. Time to maturity, denoted by T, which is the time at which debt is due to be repaid.
3. The exercise price of the option, denoted by D, which is the level of the company's debt.
4. The firm's asset value, denoted by V, is equal to the fair value of its equity value and the fair value of its debt and is assumed to be normally distributed.
5. The asset value volatility, denoted by σ_V, which in most models is assumed to be constant over time.

Of the five key inputs, V and σ_V are unobservable. The firm's asset value is only observable when the company is sold, and as this is not a regular occurrence, it is not possible to observe the firm value's distribution over time.

Several empirical studies and practical applications of the structural models propose methods for estimating the values of the firm's assets and asset value volatility. However, empirical studies of the accuracy of structural models (e.g. Jones, *et al.* (1984), Eom, *et al.* (2004) and others) conclude that there is an accuracy problem with structural models – they tend to under estimate short term credit spreads and to under- or over estimate long term credit spreads.

This study aims to compare time series of asset values and volatility implied by bond prices with calculations of asset values and volatility based on a method proposed by Jones, *et al.* (1984) and to determine whether there are significant differences between the two sets of values, and whether bond implied asset value volatility is constant over time (a key assumption of structural models).

1.5. Research Methodology

In order to derive bond implied asset value and volatility, this study utilises a version of the structural model developed by Leland and Toft (1996), which provides closed form solutions for valuing a firm's equity, an individual bond and the firm's total debt outstanding. Furthermore, the Leland and Toft (1996) model calculates an endogenously-determined default threshold rather than assuming it is exogenously determined, thus the default threshold changes with the firm's capital structure.

In order to derive bond-implied asset values and volatility, this study uses a system of two equations and two unknowns. The two equations are the formulae for valuing a single bond and for valuing a company's equity. The two unknowns are V and σ_V. In order to solve such a system of equations, the market value of the bond and equity are needed, and then the two equations are solved simultaneously in order to derive values of V and σ_V that satisfy the market-observed values of the bond and equity.

An alternative method of estimating V and σ_V was used, based on methodology proposed by Jones, *et al.* (1984).

The sample data used comprised 36 bonds issued by publicly quoted Western European firms, for which daily prices of bonds and equity were available on Bloomberg for a period of three years. By solving the system of equations for each daily observation, a time series of bond-implied V and σ_V was obtained, and by utilising the methodology of Jones, *et al.* (1984) which uses equity values and leverage, an alternative time series of both values for exactly the same days were also obtained. It was then possible to conduct statistical tests to determine whether there are statistically significant differences between the values of V and σ_V that were derived from bond prices and those calculated using equity values. The time series of bond-implied σ_V also facilitated statistical tests to determine whether asset value volatility is constant over time.

1.6. Structure of the study

The remainder of this study is organised as follows:

Section 2 contains literature review of structural models, their practical applications and empirical studies, as well as a description of an alternative family of models called 'reduced form models'.

Section 3 contains the research question and hypotheses.

Section 4 describes the research methodology used, including the model, population sample, data collection, inputs and implementation.

Section 5 describes the data analysis and statistical tests used.

Section 6 provides the study's results.

Section 7 contains the conclusions from the study's results.

Section 8 summarises the study's contribution to knowledge.

Section 9 highlights the research's limitations.

Section 10 suggests some areas for further research.

Section 11 contains glossary of terms

Appendix 1 contains the results of the statistical tests conducted for each of the 32 firms used in this study.

2. Literature Review

This section provides a literature review of the main developments in structural credit models, their commercial implementation and empirical testing. By reviewing these developments, the issues investigated in this paper are also highlighted. Structural models represent one approach for pricing risky debt and estimating credit risk. Alternative methods have been proposed in response to some of the difficulties associated with implementing the structural approach. The most widely use alternative to structural model is the reduced form model and this approach is also covered in the literature review. This section is organised as follows: structural models: their theoretical evolution, practical applications and empirical tests, followed by an overview of reduced form models.

2.1. Structural Models

> "Since almost all corporate liabilities can be viewed as combinations of options, the formula and analysis that led to it are also applicable to corporate liabilities such as common stock, corporate bonds and warrants. In particular, the formula can be used to derive the discount that should be applied to a corporate bond because of the possibility of default."
>
> Source: Black and Scholes (1973)

Structural credit models originated from the work of Black and Scholes (1973) in the field of option pricing. The economic logic behind all structural models is based on the interaction between shareholders and creditors as the value of the firm develops over time. Structural models postulate that if at some point in time the ratio of the firm's assets to liabilities reaches a certain critical level, it will stop paying its liabilities as they fall due and be put into bankruptcy. At that point, the ownership of the firm transfers from its shareholders to its creditors. This situation arises in firms incorporated with limited liability, where shareholders are not liable for all of the firm's obligations. As long as the residual value of the firm's assets less its liabilities is large enough to compensate shareholders for owning the firm, it will stay solvent, but if that residual becomes too small or negative, shareholders will maximise their utility by walking away from the firm, leaving its creditors to take ownership and try to recover value in order to repay the firm's outstanding liabilities. This economic logic can be expressed in option theory terminology:

- By owning a share in the firm, shareholders own a call option on the firm's assets. The price of the call is the price of the shares (the equity value), the exercise price is the amount of outstanding liabilities (the firm's debts), and the reference security is the firm's asset value. If shareholders wish to exercise their call option, they need to pay off the firm's debts. If the firm's assets drop to some level where it makes no economic sense for shareholders to repay the firm's liabilities, the value of the call option on the assets will become worthless.
- From the creditors' perspective, lending money to a company is akin to selling a put option to its shareholders, referenced on the firm's assets. If the value of the company's assets falls below its liabilities, shareholders can put the assets to its creditors.

The economic logic underlying structural models and the elegance of option pricing theory led many academics and practitioners to try to develop a model that will facilitate the pricing of corporate bonds using the Black and Scholes (1973) framework. The evolution of such models, some examples of their practical applications, the results of empirical tests and areas that remain unresolved are described in the following sub-sections.

2.1.1. The Evolution of Structural Models

2.1.1.1. Pricing European Options: Black and Scholes (1973)

The first step in the evolution of structural models is the work by Black and Scholes (1973) in the field of option pricing theory. Black and Scholes (1973) developed formulae for pricing European calls and European puts. European-type options can only be exercised on a predetermined date. A European call option provides its holder with the right (but not the obligation) to purchase an asset for a predetermined price. A European put option provides its holder with the right (but not the obligation) to sell an asset for a predetermined price at a predetermined date. The payout of a European call option depends on the share price at maturity:

1. If the underlying share price is greater than the option's strike price, the option has value greater than zero.

2. If the underlying stock price is less than the option's strike price, the option has negative, i.e. no value.

The payout of a European put option is the mirror image of the call option's payout. It should be noted that in addition to European options, investors can also trade American options, which can be exercised at any time up until their expiry date, but such options require more complex pricing models, which were developed after Black and Scholes' initial work.

In deriving their pricing formula for a European call option, Black and Scholes assumed the following market conditions:

1. The short term interest rate, *r*, is known and constant over time.
2. The underlying share price displays random walk characteristics in continuous time. As a result, the variance rate is proportional to the square of the price. Therefore, the stock price distribution is log normal. Black and Scholes also assumed that the variance rate of the return on the share is constant.
3. No dividends or other distributions are allowed.
4. The option is European, i.e. it can only be exercised at maturity.
5. No transaction costs for selling or buying the underlying asset or the option itself.
6. Borrowing any fraction of the share price is possible at rate *r*, the short term rate of interest.
7. Short selling is possible without penalties.

Using these assumptions, the option's value is a function of the share price, *S*, and the time to expiry, *t*, as well as variables that are treated as known constants. Black and Scholes used the approach of constructing a hedged position comprising long position in the underlying share and a short position in the call option to value the call option itself. They built on the work done by Churchill (1963) for solving the heat transfer equation in physics in order to solve for the value of a European call option C_E:

Eq. 2.1 $$C_E = SN(d_1) - Xe^{-rt}N(d_2)$$

Where:

S denotes the current value of the reference asset;

N(.) denotes the cumulative normal density function;

X denotes the option's strike price;

r denotes the risk-free interest rate;

t denotes time to maturity;

σ_S denotes the price volatility of the reference asset S;

$$d_1 = \frac{\ln\left(S/X\right) + \left(r + \sigma_S^2/2\right)t}{\sigma_S \sqrt{t}}$$

$$d_2 = d_1 - \sigma_S \sqrt{t}$$

In the Black and Scholes (1973) model, the value of the option is affected by the time to maturity and the volatility of the underlying reference share, all other things being equal:

- The longer the time to maturity, the greater the value of the option; and
- The greater the volatility the greater the value of the option.

Intuitively, the longer the time remaining until the option can be exercised, the more opportunities the underlying stock price has to exceed the exercise price. Also, the greater the volatility of the underlying stock price, the higher the probability that it will exceed the exercise price and the option could be exercised.

Nielsen (1992) interpreted the components of the Black and Scholes (1973) call pricing formula and showed that $N(d_2)$ is the risk-adjusted probability that the option will be exercised (i.e. the likelihood that at maturity the share price will exceed the strike price).

Black and Scholes (1973) also derived the pricing formula for a European put option, P_E:

Eq. 2.2 $$P_E = Xe^{-rt}N(-d_2) - SN(-d_1)$$

The inputs needed in order to price a European put option using the Black and Scholes framework are the same as those needed to price a European call option. Put option values also increase with the volatility of the underlying stock. However, as a function of time to maturity, put option prices first increase then decrease. The reason for this is that, like the European call option, as time to maturity increases, all other things being equal, there are more opportunities for the share price to dip below the exercise price and for the option to end up in the money. However, one of the assumptions underlying the Black and Scholes (1973) is that the share price follows a random walk process with a positive drift, i.e. it follows an upward trend over time. Thus, as the time horizon increases, the effect of the positive drift becomes more pronounced. At the point where the effect of the positive drift outweighs the effect of volatility, the value of the put option begins to decrease.

Black and Scholes (1973) assumed that no arbitrage opportunities exist in the markets for options and their inputs. Therefore, if the no arbitrage assumption holds, the value of a call option and a put option referenced on the same asset with the same maturity must obey the put-call parity condition:

Eq. 2.3 $$C_E - P_E = S - Xe^{-rt}$$

If the call-put parity did not hold, an investor could, for example, buy a call, sell a put and the underlying stock and investing the net proceeds at the risk free rate until expiry *t*. If the net proceeds received today are greater than Xe^{-rt}, then an arbitrage opportunity has been utilised.

At maturity, the value of European options is a function of the underlying asset and the option's strike price:

- In the case of a call option, if the value of the underlying share is below the option's exercise price, the option will be worthless and expire unexercised. The more the share price exceeds the exercise price, the greater the value of the call option.

- In the case of a put option, as long as the value of the underlying share is above the strike price, the option will be worthless; the further the share price falls below the exercise price, the more valuable the put option becomes.

Having derived the equations for valuing European call and put options, Black and Scholes (1973) pointed out that under the following condition:

1. Company's capital structure comprises ordinary shares and bonds only;
2. The bonds are pure discount bonds (i.e. pay no coupons);
3. No dividends are payable until after the bonds mature;
4. At maturity, the company sells its assets. If the proceeds are sufficient, bondholders will be paid off and the residual will be distributed to shareholders. If proceeds are insufficient, shareholders will get nothing.

The value of the company's shares at the bonds' maturity date will be either:

- Assets less bonds if the value of the assets is greater than the value of the bonds; or

- Zero, if the value of the assets is less than the value of the bonds.

In other words, the value of an ordinary share can be viewed as the value of a call option on the company's assets, with an exercise price equal to the value of the company's liabilities. The value of the firm's bonds can be viewed as the value of the company's assets less the value of the call option represented by the ordinary shares of the company.

The option pricing approach and bond valuation approach developed by Black and Scholes (1973) led to a range of theories and models for valuation equity derivatives, bonds and credit derivatives. The interplay between volatility, time to maturity and the call-put parity all have direct effect on structural credit models that are based on Black and Scholes' original work. Merton (1974) built on the approach developed by Black and Scholes (1973) and developed the first closed form solution for pricing zero coupon corporate bonds.

2.1.1.2. Pricing Zero-Coupon Risky Bonds: Merton (1974)

Although Black and Scholes (1973) developed the formulae for pricing European call and put options and outlined the conditions under which such formulae could be used to value equity and debt, it was Merton (1974) who first detailed a formula for pricing risky corporate debt. The foundation of Merton's approach is that the value of corporate debt is a function of:

1. The risk-free rate of return;
2. The terms of the debt issue (e.g. maturity, seniority, call options, coupon);
3. The probability of default.

Merton called his theory of pricing defaultable bonds the "risk structure of interest rates". The theory was developed as an extension of the Black and Scholes (1973) option pricing method.

The assumptions used by Merton (1974) are similar to those used by Black and Scholes (1973):

1. Perfect markets with no distortions.
2. Investors believe they can buy or sell as much of any asset as they wish.

3. Borrowing and lending rates are the same.
4. Short selling is allowed and possible.
5. Trading is done continuously.
6. The value of firms is independent of their capital structure, i.e. the Modigliani and Miller (1958) theorem holds.
7. The term structure of interest rates is flat and known with certainty.
8. The firm has a simple capital structure comprising a single zero coupon bond and ordinary equity.
9. No dividends or refinancing is allowed prior to the bond's maturity.
10. The firm's value dynamics can be described using a diffusion-type stochastic process (smooth random process):

$$dV = (\alpha V - \delta)dt + \sigma_V V dz$$

Where:

a = instantaneous expected rate of return on the firm per unit of time

V = value of the firm

δ = total monetary payout by the firm per unit of time to either shareholders or debt holders if positive.

σ_V = the instantaneous standard deviation of the return on the firm's assets.

dz = standard Wiener process.

In Merton's framework, the firm promises its bondholders to repay the bond's face amount D at time T. The repayment is done by selling the firm's assets at the bond's maturity date and using the proceeds to repay bondholders. Any residual amount remaining after bondholders have been repaid is paid to shareholders. It follows, therefore, that the value of the firm is the value of its debt, B, plus the value of its equity, E. Both B and E can only be non-negative due to the limited liability structure of a firm. The value of the company's debt at maturity date ($t = 0$) is given by:

Eq. 2.4 $$B(V,0) = \min(V, D)$$

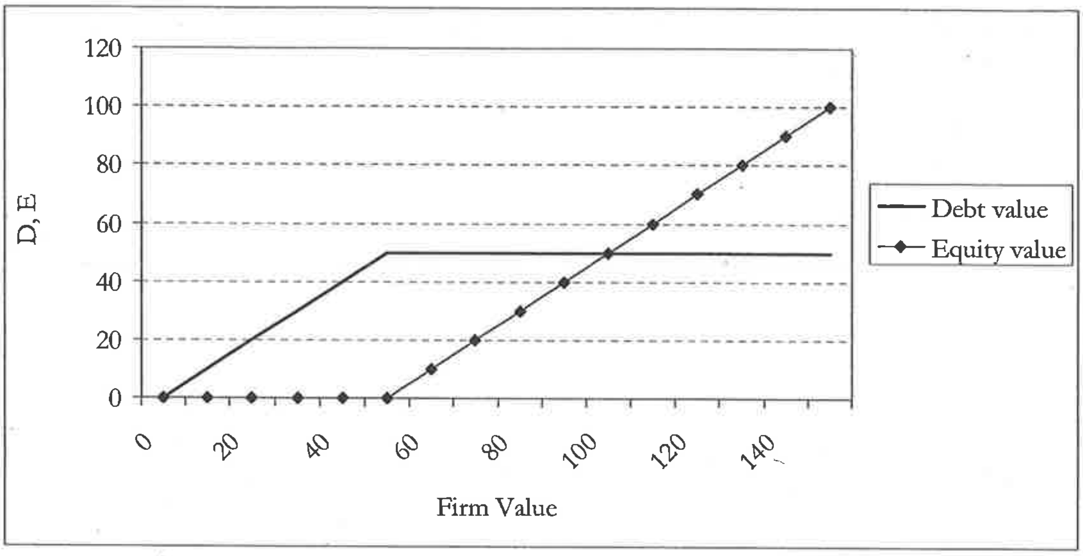
Merton also viewed the problem from a different angle, noting that that the value of the firm's equity is equal to the value of its assets less the value of its liabilities:

$E(V,t) = V - B(V,t)$. At the bonds' maturity date, the value of the firm's equity will be the greater of zero or the residual asset value after bondholders have been paid:

Eq. 2.5 $$E(V,0) = \max(0, V - D)$$

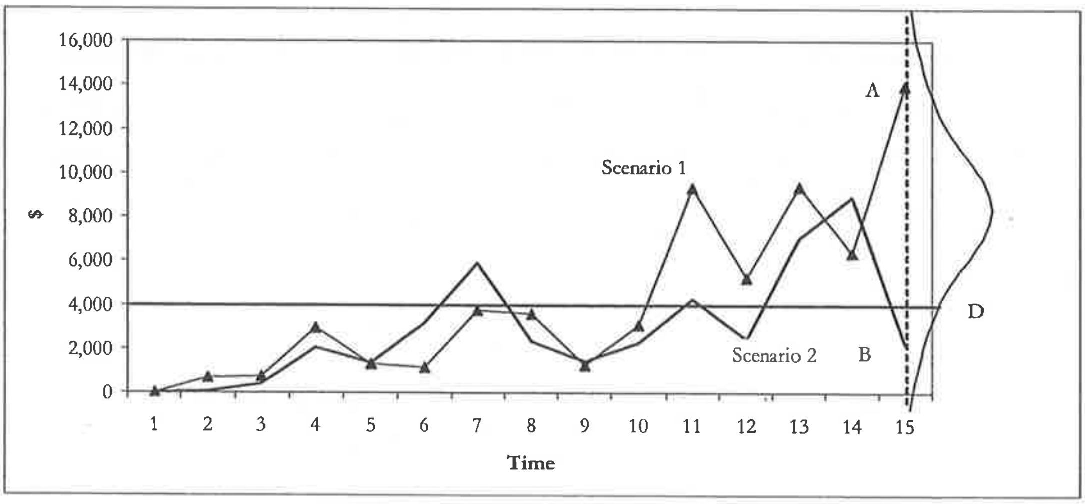
The possible payout functions faced by the firm's debt holders and shareholders (assuming principal debt outstanding equals 50) are shown in Figure 2.1 below:

Figure 2.1: Debt and Equity Value at Maturity



The payout profile for shareholders as described by Equation 2.5 and in Figure 2.1 is the same as the payout profile of a European call option on a non-dividend paying common share. In Merton's approach the option is referenced on the firm's value and the bond's face amount D is the option's strike price. Merton's approach is demonstrated graphically in Figure 2.2 below:

Figure 2.2: A Graphical Representation of Merton's Approach



In Figure 2.2, the firm's value follows a random process with positive drift. It is assumed that the firm's total debt outstanding is $4,000 and that the value of its assets can take a variety of paths, two of which are shown in the diagram:

- In scenario 1 the firm's asset value at maturity is at point A, above the amount of debt outstanding. In this scenario bondholders will receive the payment due to them and equity holders will receive the residual amount, which is positive.
- In scenario 2 the firm's asset value at maturity is at point B, below the amount of debt outstanding. In this scenario there is insufficient value in the firm's assets to repay bondholders in full. Bondholders will receive a payment equal to B which is less than the amount of debt outstanding D; their recovery rate is B/D. Equity holders receive nothing in scenario 2.

The normal distribution curve in Figure 2.2 above represents all the possible values of the firm's assets at maturity, and the probability of default is represented by the area under the curve from line D downwards.

Merton used the framework developed by Black and Scholes (1973) to express the value of a firm's equity as:

Eq. 2.6 $$E(V,t) = V_A N(d_1) - De^{-r\tau} N(d_2)$$

Where:

N(.) denotes the cumulative normal density function,

V_A = value of the firm's assets

σ_V = asset value volatility.

D = the face value of the firm's bonds

r = risk free rate

τ = time to maturity

$$d_1 = \frac{\ln\left(V_A/D\right) + \left(r + \frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma_V\sqrt{\tau}$$

From the relationship between the firm's equity value, asset value and debt value, Merton also derived an expression for the value of corporate debt as:

Eq. 2.7 $$B(V,t) = De^{-rt}\left\{N\left[h_2\left(d,\sigma_V{}^2 t\right)\right] + \frac{1}{d}N\left[h_1\left(d,\sigma_V{}^2 t\right)\right]\right\}$$

Where:

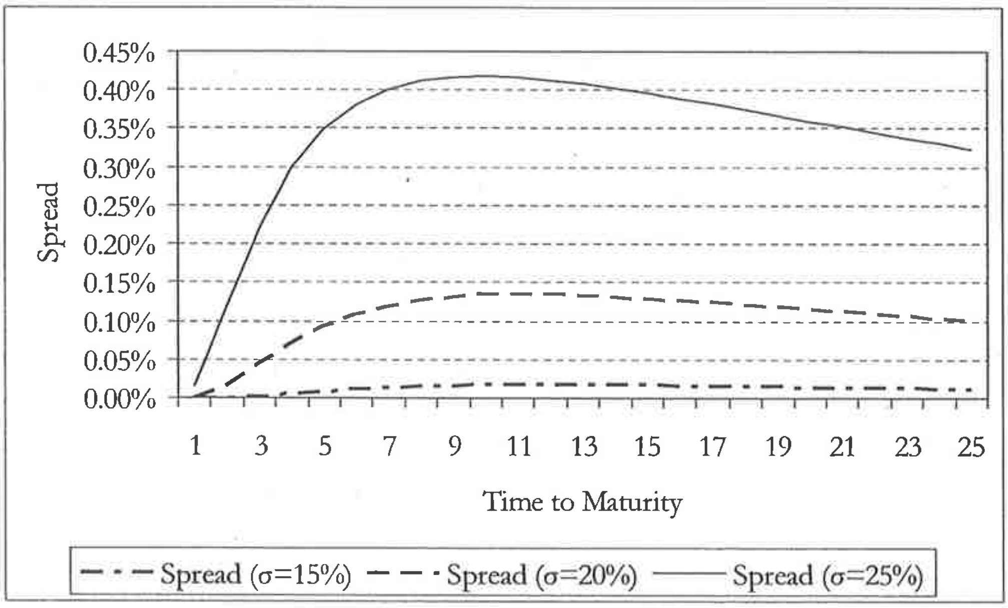
$$d \equiv De^{-rt} \Big/ V$$

$$h_1 = -\left[\tfrac{1}{2}\sigma_V{}^2 t - \ln(d)\right] \Big/ \left(\sigma_V \sqrt{t}\right)$$

$$h_2 = -\left[\tfrac{1}{2}\sigma_V{}^2 t + \ln(d)\right] \Big/ \left(\sigma_V \sqrt{t}\right)$$

In Equation 2.7 the value of corporate debt is expressed as a function of the firm's value V, the time to maturity t, the face value of the debt D, the instantaneous variance of the return on the firm σ_V^2 and the default risk free interest rate r. Using this approach, Merton showed that the value of debt is an increasing function of the current market value of the firm and the promised payment at maturity, and a decreasing function of the time to maturity, the leverage of the firm and the default risk-free rate of return r. In Figure 2.3 below, the spread of a risky zero coupon bond (yield differential between risky and risk-free bonds) is shown as a function of time to maturity for three levels of asset value volatility:

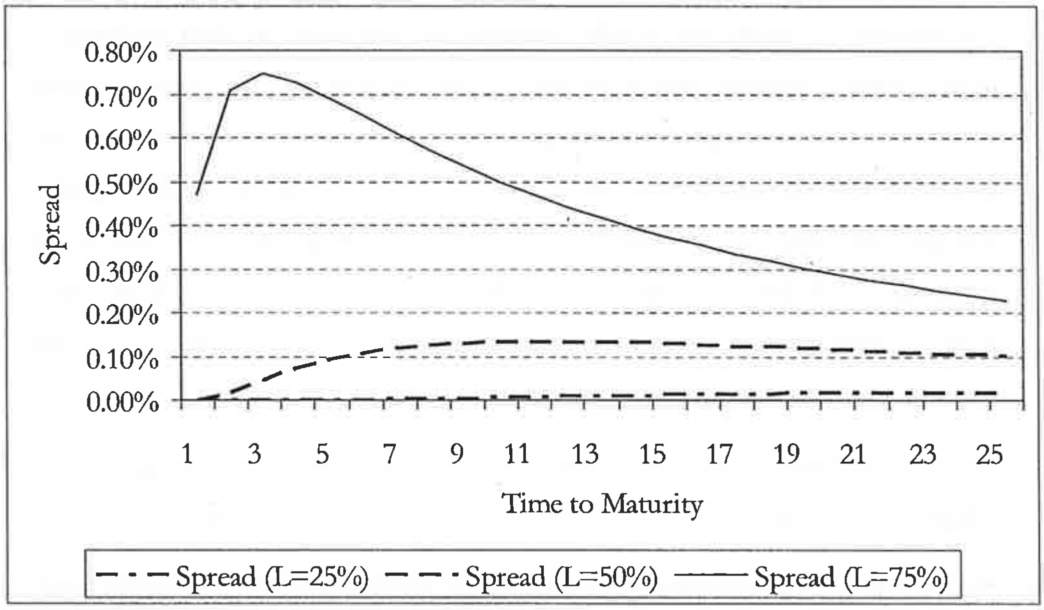
Figure 2.3: Bond spreads over time for different volatility levels



As can be seen from Figure 2.3 above, the greater the asset volatility the greater the spread generated by the Merton model. The shape of the spread curve for every level of volatility is hump-backed. This shape is generated because of the competing effects of volatility and time to maturity. As discussed in the Section 2.1.1.1 the value of a put option rises and then falls the longer the time to maturity because in the short term the effect of volatility on the likelihood of exercising the option is greater than the effect of time, but the longer the time to maturity, the stronger its effect on the likelihood of exercise. In Merton's model, the put option (i.e. default) is more likely to be exercised the shorter the time to maturity. This is also due to the positive drift assumed in the firm value process, which leads to lower likelihood of default the longer the time to maturity is.

The credit spreads generated by Merton's model are also positively linked to leverage – the higher the leverage, the greater the spreads generated by the model, as can be seen in Figure 2.4 below:

Figure 2.4: Bond spreads over time for different leverage levels



As can be seen in Figure 2.4 above, for low-leveraged companies, the credit spreads estimated by the model are upward sloping over time, starting with zero. The higher the leverage, the higher the spreads generated by the model. It should be noted that for highly leveraged firms, the shape of the spread curve is hump-backed. This is because the option embedded in the model is closer to the money (i.e. more likely to have value and thus be exercised) and this effect dominates the effects of time to maturity and positive drift of asset values. As time to maturity increases, the effect of the positive drift strengthens and the option becomes less in the money, thus model generated spreads decline. This type of credit spreads was observed empirically by Sarig and Warga (1989). Intuitively, low-leveraged firms are less likely to default in the short term, but in the long term, something negative can happen and increase their likelihood of default, thus investors would demand greater compensation for taking longer dated risk. Conversely, highly leveraged firms are more likely to default in the short term, for example if they fail to reduce leverage, but in the longer term there is likelihood that management will be able to execute its business plan, reduce leverage and thus the likelihood of default will decline.

Whilst the Merton (1974) model appeared to be able to generate credit spreads that are supported by empirical observations, the model has several key limitations:

- Default can only occur at the bonds' maturity – the assumption that the firm's capital structure comprises one class of zero coupon bonds and one class of equity prevents the possibility of default prior to the bonds' maturity.
- No account of restrictions usually imposed on companies by their creditors – Merton's model assumes that at the bonds' maturity date the firm's assets are sold and the creditors are repaid with the proceeds. However, the model ignores the possibility that shareholders sell the company's assets before the bond's maturity and pay the proceeds to themselves. In order to avoid such situations, bond indentures and loan documents tend to impose restrictions on borrowers, typically specifying restrictions on dividends, minimum leverage ratios and restrictions on asset sales.
- No coupon bonds – Merton's model was developed in order to price zero coupon bonds, whilst most bonds issued by companies are coupon paying bonds.
- Non-stochastic interest rates – the assumption that interest rates are known with certainty does not hold in reality, where risk-free rates fluctuate, adding a further element of volatility to bond pricing.
- No dividends, no new debt – Merton assumed that the firm pays no dividends, nor can it borrow more. As a result, the likelihood of default in the Merton model declines over time, because at the same time that no new debt is allowed, the firm's asset value follows a random process with positive drift; in other words, the firm's leverage as measured by debt to total asset value decreases over time.

In order to address some of these limitations, other academics built on Merton's work and developed more advanced structural credit models. The most notable contributions to structural models are described in the following sub-sections.

2.1.1.3. Bond Indenture Provisions: Black and Cox (1976)

The general valuation framework provided by Black and Scholes (1973) and the structural model proposed by Merton (1974) suffer from certain limitations that restrict their use in practice. One of the limitations of Merton's approach is that it does not take into account restrictions imposed on companies by their creditors, which are found in the indentures governing bonds and loans. Black and Cox (1976) developed a structural model framework in order to take account of certain commonly used bond indenture provisions. The assumptions used in developing Black and Cox's framework are similar to those used by Black and Scholes (1973) and Merton (1974) but the main departure from earlier assumptions is that dividend payments are allowed.

Black and Cox (1976) viewed corporate debt securities as having four sources of values:

1. value at maturity, assuming no bankruptcy prior to that date;
2. value prior to maturity if the firm's value falls below a certain (low) level, thus triggering reorganisation such as bankruptcy;
3. value prior to maturity if the firm's value rises above a certain (upper) level, prompting the firm to call the bonds in order to refinance itself at lower cost;
4. value of payouts that could potentially be received from holding a corporate bond.

The first three values are mutually exclusive. In a risk-neutral world, the value of a risky debt security will be the sum of the discounted expected values of each of the sources of value. Black and Cox (1976) denoted the lower boundary of the firm's value at time t as $C_1(t)$ and the upper boundary as $C_2(t)$. Bankruptcy is triggered if the firm's value reaches $C_1(t)$ and early refinancing of the bonds is triggered if the firm's value reached $C_2(t)$ prior to maturity.

The first aspect of bond covenants analysed by Black and Cox (1976) was the issue of creditor safety. Safety covenants, also known as maintenance covenants, allow bondholders to force a restructuring of the firm if it fails to perform according to a pre-determined set of measures such as failure to pay interest when due, or failing to maintain certain financial ratios. Clearly, if shareholders were allowed to sell the firm's assets in order to pay interest, such covenants would not provide the safety required, as the underlying asset would be gradually diminished. Black and Cox (1976) viewed safety covenants as those that trigger restructuring if the firm's value falls below a certain level. Thus it is the dynamics of the firm's value, rather than interest payments that are important for their analysis. Therefore, the Black and Cox approach is based on zero coupon bonds, but it allows a certain continuous dividend denoted by aV to be paid to shareholders. Given the assumptions of continuous trading, Black and Cox (1976) defined the bankruptcy trigger level (the lower boundary) as:

Eq. 2.8 $$C_1(t) = Ce^{-\gamma(T-t)}$$

Where γ denotes the growth rate of the boundary level over time.

The bankruptcy threshold $C_1(t)$ is an absorbing barrier – once the firm's value reached it, it cannot increase again. The probability that the firm's value V will not reach the bankruptcy threshold (i.e. the probability of non-default) in a given time interval t to τ, τ =< T, is given by:

Eq. 2.9 $$P_{nd} = N\left(\frac{\ln V - \ln K + \left(r - a - \frac{\sigma_V^2}{2}\right)(\tau - t)}{\sqrt{\sigma_V^2(\tau - t)}}\right) - \left(\frac{V}{Ce^{-\gamma(T-t)}}\right)^{1-\frac{2(r-a-\gamma)}{\sigma_V^2}} \times N\left(\frac{2\ln Ce^{-\gamma(T-t)} - \ln V - \ln K + \left(r - a - \frac{\sigma_V^2}{2}\right)(\tau - t)}{\sqrt{\sigma_V^2(\tau - t)}}\right)$$

Where:

N(.) denotes standard cumulative normal distribution function

V denotes the firm's value

K denotes the absorbing bankruptcy barrier

a denotes the continuous dividend payout ratio

r denotes the constant risk free rate of interest

σ_V denotes the firm's asset value volatility

t denotes the starting time of the valuation

T denotes the bonds' maturity date.

By setting $K = Ce^{-\gamma(T-\tau)}$ the probability that the firm will not be reorganised before time τ can be calculated.

Black and Cox (1976) also provide the valuation formula for a zero coupon bond containing a safety covenant:

Eq. 2.10
$$B(V,t) = De^{-r(T-t)}\left[N(z_1) - y^{2\theta-2}N(z_2)\right] + Ve^{-a(T-t)} \times \left[\begin{array}{l} N(z_3) + y^{2\theta}N(z_4) + y^{\theta+\xi e^{a(T-t)}}N(z_5) + y^{\theta-\xi e^{a(T-t)}}N(z_6) - \\ y^{\theta-\eta}N(z_7) - y^{\theta-\eta}N(z_8) \end{array}\right]$$

Where:

$$y = Ce^{-\gamma(T-t)}/V$$

$$\theta = \left(r - a - \gamma + \sigma_V^2/2\right)/\sigma_V^2$$

$$\delta = \left(r - a - \gamma + \sigma_V^2/2\right)^2 + 2\sigma_V^2(r-\gamma)$$

$$\xi = \sqrt{\delta}/\sigma_V^2$$

$$\eta = \sqrt{\delta - 2\sigma_V^2 a}/\sigma_V^2$$

$$z_1 = \left[\ln V - \ln D + \left(r - a - \sigma_V^2/2\right)(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_2 = \left[\ln V - \ln D + 2\ln y + \left(r - a - \sigma_V^2/2\right)(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_3 = \left[\ln V - \ln D - \left(r - a + \sigma_V^2/2\right)(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_4 = \left[\ln V - \ln D + 2\ln y + \left(r - a + \sigma_V^2/2\right)(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_5 = \left[\ln y + \xi\sigma_V^2(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_6 = \left[\ln y - \xi\sigma_V^2(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_7 = \left[\ln y + \eta\sigma_V^2(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

$$z_8 = \left[\ln y - \eta\sigma_V^2(T-t)\right]/\sqrt{\sigma_V^2(T-t)}$$

The formula provided in Equation 2.10 holds for all default threshold levels that are less than or equal to the bond principal outstanding, i.e. $Ce^{-\gamma(T-t)} \leq De^{-r(T-t)}$. In order to model a default threshold that is a constant fraction of the present value of the contractual final payment *D*, the lower boundary threshold can be set to: $Ce^{-\gamma(T-t)} = \rho De^{-r(T-t)}$, where $0 \leq \rho \leq 1$ is the constant fraction of the present value of *D*.

The approach developed by Black and Cox (1976) for valuing bonds with safety covenants shows that the value of the bond is an increasing function of ρ. As the default threshold value increases, the bonds can be considered 'safer' because the permitted decline in the firm's value prior to default decreases. Another aspect of Black and Cox's model is that as asset value volatility and/or the dividend payout ratio increase, the value of the bonds converges on the default threshold, not on zero. Given that the purpose of safety covenants is to prevent shareholders from taking excessive value out of the firm, this feature of the model also reflects real world dynamics.

The framework provided by Black and Cox (1976) focused primarily on zero coupon bonds. In their paper, Black and Cox (1976) extended their framework to value subordinated debt and perpetual coupon paying bonds, demonstrating that a restriction on the level of assets that can be sold in order to pay bond coupon and dividends increases the value of the bonds. Black and Cox (1976) acknowledged two key limitations of their approach:

1. Introducing bankruptcy costs and taxes could change the solutions obtained, although the interplay between shareholders and creditors may not differ dramatically from that described by Black and Cox.

2. If the firm's value followed a jump process rather than a stochastic diffusion process, there may be little value in the safety covenants because the firm value could arrive at a level below the lower boundary without first passing through it.

Other limitations of the Black and Cox approach include:

- Focus on zero coupon and perpetual bonds, whilst the majority of bonds outstanding are coupon paying with finite life
- Non-stochastic interest rates
- No transaction costs
- Perfect and continuous markets in all securities.

2.1.1.4. Pricing Coupon Bonds Using Compound Options: Geske (1977)

Two significant limitations of Merton's model are that it was developed to price zero coupon bonds and it assumes that default can only occur at the bond's maturity date. Geske (1977) sought to address these limitations by developing a model for pricing risky coupon bonds that also incorporates the possibility of default before maturity date. Geske (1977) built on the work done by Black and Scholes (1973) and Merton (1974) and used a methodology of pricing compound options in order to obtain a general valuation equation for a risky coupon bonds, where default could occur at any coupon date.

Geske (1977) argued that when a company's capital structure comprises bonds and shares, the shares of that company can be viewed as a compound option. At every coupon date, shareholders can choose to either buy the next option by paying the coupon or hand-over the firm to its bondholders by defaulting on the coupon payment. The final option for shareholders is to repurchase the firm from bondholders by paying the bonds' principal outstanding.

In developing his valuation approach, Geske (1977) assumed the firm has only one class of shares and one issue of bonds outstanding. The coupon bond has n interest payments of X Dollars each. The number of interest payments due before bond maturity T is $n-1$. The n[th] interest payment is due at maturity, together with principal repayment. Coupon payments are financed by issuing equity and bankruptcy occurs when the firm cannot raise more equity.

Geske (1977) began his analysis by considering the firm's shares at time t_{n-1}, just after the final coupon payment. At that point in time, the boundary condition for valuing the company's shares is that at time T, the share's value will be $S_T = max(V_T - D,0)$, where V_T is the firm's value at time T and D is the face value of the bonds plus interest due. The boundary condition for the coupon bond at time T is given by $B_T = min(V_T,D)$. In this framework, the value of the firm's equity at time t_{n-1} is given by the Black and Scholes (1973) formula for pricing a European call option:

Eq. 2.11 $$E_{t_{-1}} = V_{t_{n-1}} N(d_1) - (c + D)e^{-r(T - t_{n-1})} N(d_2)$$

Where:

c denotes the final coupon payment due on maturity date;

D denotes the bond principal due at maturity date T;

$$d_1 = \frac{\ln\left(V_{t_{n-1}} / c+D\right) + \left(r + \sigma_V^2 / 2\right)(T - t_{n-1})}{\sigma_V \sqrt{T - t_{n-1}}}$$

$$d_2 = d_1 - \sigma_V \sqrt{T - t_{n-1}}$$

The value of the coupon bond, just after the final individual interest payment was made, is found by subtracting the value of the share from the value of the firm:

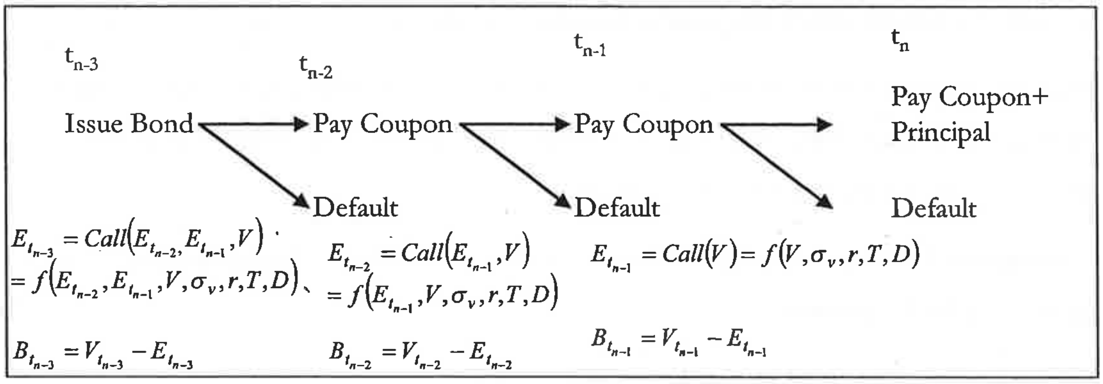
Eq. 2.12 $$B_{t_{n-1}} = V_{t_{n-1}} - E_{t_{n-1}} = V_{t_{n-1}} N(-d_1) + (c+D)e^{-r(T-t_{n-1})} N(d_2)$$

Eq. 2.12 is also a way of valuing a discount bond whose total principal due is equal to $c+D$.

The next stage is to look at periods before t_{n-1}. Geske denoted c_t as the coupon payment at date t. At all dates prior to final coupon payment, i.e. $t < t_{n-1}$, the company's equity is a compound option. By making the one-but-last coupon payment at time t_{n-2} the company's shareholders are buying a call option on the equity up to the date prior to the final coupon payments – in other words, they are buying an option referenced on another option. If shareholders are rational, they would only choose to pay that coupon payment if the call on the equity in the future was worth more than the actual coupon due on that date.

It should be noted that at time t_{n-1} the equity of the firm is a function of time to maturity, asset value volatility, asset value, leverage and principal amount of debt outstanding. At time t_{n-2} the value of the equity is a function of the same variables and the price of the equity itself at time t_{n-1}. The further we go back, the greater the complexity of the calculations required because in each step the firm's equity becomes a compound option referenced on the compound options of the preceding periods. Geske's approach is described diagrammatically in Figure 2.5 below, for a bond paying three coupons of c dollars each, the last one payable at maturity, together with the principal D outstanding:

Figure 2.5: A diagrammatic representation of Geske's model



Geske showed that the value of the coupon bond at time t_{n-2} is given by:

Eq. 2.13
$$B_{t_{n-2}} = V_{t_{n-2}}\left(1 - N_2\left(h_{n-1} + \sigma_V\sqrt{T - t_{n-1}}, h_n + \sigma_V\sqrt{T - t_{n-2}}; \rho\right)\right) + (D + c)e^{-r(T - t_{n-2})}N_2(h_{n-1}, h_n; \rho) + c_{t_{n-1}}e^{-r(T - t_{n-1})}N_1(h_{n-1})$$

Where:

$$h_n = \frac{\ln\left(V_{t_{n-2}} / D + c\right) + \left(r - \frac{\sigma_V^2}{2}\right)(T - t_{n-2})}{\sigma_V\sqrt{T - t_{n-2}}}$$

$$h_{n-1} = \frac{\ln\left(V_{t_{n-2}} / V_{t_{n-1}}\right) + \left(r - \frac{\sigma_V^2}{2}\right)(T - t_{n-1})}{\sigma_V\sqrt{T - t_{n-1}}}$$

$$\rho = \sqrt{\frac{t_{n-1} + t_{n-2}}{T - t_{n-1}}}$$

Also in Equation 2.13, $N_1(.)$ denotes the univariate normal distribution function, and $N_2(.)$ denotes the bivariate normal distribution function. As one moves further back in time, the equations proposed by Geske include trivariate, quadravariate and higher multivariate normal distributions. Given the difficulties in modelling complex compound options, Geske's model for bond valuation has not been tested to determine whether the bond prices and yields it generates match the shape and dynamics of bond prices and yields in the real world, and it has not been tested empirically until relatively recently (see Eom, Helwege and Huang (2003)).

2.1.1.5. Stochastic interest rates: Longstaff and Schwartz (1995)

Longstaff and Schwartz (1995) built on the work of Black and Cox (1976) by developing a model for valuing risky debt that incorporates both default risk and interest rate risk.

Their model also allows for deviation from strict absolute priority of instruments within the capital structure, which was assumed in other models.

Although many of the assumptions used by Longstaff and Schwartz (1995) are similar to those used by Black and Cox (1976), it is worthwhile summarising them here as part of the framework for the model's development:

Assumption 1: V represents the total value of the firm's assets. The dynamics of V are represented by the process:

Eq. 2.14 $$dV = \mu V dt + \sigma V dZ_1$$

Where σ, the variance of V, is constant, Z_1 is a standard Wiener process.

Assumption 2: The short term default-risk free interest rate, represented by r follows a Vasicek (1977) process:

Eq. 2.15 $$dr = (\alpha - \beta r)dt + \eta dZ_2$$

In equation 2.15 above a, β, η are constants and Z_2 is a standard Wiener process. The instantaneous correlation between dZ1 and dZ2 is represented by ρdt.

Assumption 3: The value of the firm is independent of its capital structure. In other words, the Modigliani and Miller (1958) proposition holds. This also implies that coupon payments and principal payments do not affect V.

Assumption 4: As in Black and Cox (1976), there is a threshold value K for the firm so that if V =< K, financial distress occurs. One of the important implications of this assumption is that default occurs for all debt contracts simultaneously, a more realistic assumption given normal cross-default provisions in debt indentures.

Assumption 5: If reorganization occurs, the security holder receives *(1-w)* x face value of the debt security at maturity. In this case, *w* represents the percentage write-down on a security in case of reorganization. For securities of a limited liability company *w* =<1 . Longstaff and Schwartz assume that *w* is constant, but argue that their framework can be extended to include a stochastic *w*, as long as the risk of *w* is unsystematic.

Assumption 6: There are perfect frictionless markets in which securities can be traded continuously. In this environment, the value of a risk-free discount bond is given by the Vasicek (1977) model:

Eq. 2.16 $$D(r,T) = e^{(A(T)-B(T)r)}$$

Where:

$$A(T)=\left(\frac{\eta^2}{2\beta^2}-\frac{\alpha}{\beta}\right)T+\left(\frac{\eta^2}{\beta^3}-\frac{\alpha}{\beta^2}\right)\left(e^{-\beta T}-1\right)-\frac{\eta^2}{4\beta^3}\left(e^{-2\beta T}-1\right)$$

$$B(T)=\frac{1-e^{-\beta T}}{\beta}$$

In the equation 2.16 above, a, β, and η are constants as in assumption 2 above.

Using this framework, the price of a risky zero coupon bond $P(V,r,T)$ can be represented by:

Eq. 2.17 $$P(V,r,T)=1-wI_{y\le T}$$

In equation 2.17 above, I is an indicator function that takes the value of 1 if V reaches K during the life of the bond and zero otherwise.

In addition, Longstaff and Schwartz (1995) denote X as V/K and represent the value of a risky zero coupon bond as:

Eq. 2.18 $$P(X,r,T)=D(r,T)-wD(r,T)Q(X,r,T)$$

Where:

$$Q(X,r,T,n)=\sum_{i=1}^{n}q_i$$

$$q_1=N(a_1)$$

$$q_i=N(a_i)-\sum_{j=1}^{i-1}q_jN(b_{ij}),i=2,3,4...n$$

$$a_i=\frac{-\ln X-M\left(iT/n,T\right)}{\sqrt{S\left(iT/n\right)-S\left(jT/n\right)}}$$

$$M(t,T)=\left(\frac{\alpha-\rho\sigma\eta}{\beta}-\frac{\eta^2}{\beta^2}-\frac{\sigma^2}{2}\right)t+\left(\frac{\rho\sigma\eta}{\beta^2}+\frac{\eta^2}{2\beta^3}\right)e^{-\beta T}\left(e^{\beta t}-1\right)+$$
$$\left(\frac{r}{\beta}-\frac{\alpha}{\beta^2}+\frac{\eta^2}{\beta^3}\right)\left(1-e^{-\beta t}\right)-\frac{\eta^2}{2\beta^3}e^{-\beta T}\left(1-e^{-\beta t}\right)$$

$$S(t)=\left(\frac{\rho\sigma\eta}{\beta}+\frac{\eta^2}{\beta^2}+\sigma^2\right)t-\left(\frac{\rho\sigma\eta}{\beta^2}+\frac{2\eta^2}{\beta^3}\right)\left(1-e^{-\beta t}\right)+\frac{\eta^2}{2\beta^3}\left(1-e^{-2\beta t}\right)$$

The terms for q_i are solved recursively. The first term on the right-hand side of equation 2.18 is the value of a default risk free bond; the second term on the right hand-side represents the discount for the risk of default which is made up of:

$wD(r,T)$ = the write-down in case of default

$Q(X,r,T)$ = the risk-neutral probability of default

The Longstaff and Schwartz (1995) model incorporates stochastic interest rates and allows departure from the absolute priority assumptions of earlier models, but it was designed primarily to price zero coupon bonds, it assumes that the default barrier is exogenously determined and is constant and that the recovery rate in default is also constant. A model that addresses some of these limitations was developed by Leland and Toft (1996) and is described in detail in the following subsection,

2.1.1.6. Endogenous bankruptcy process: Leland and Toft (1996)

Leland and Toft (1996) built on the work done by Merton (1974), Black and Cox (1976), Leland (1994) and Longstaff and Schwartz (1995) to develop a structural model that differs from earlier models in three key aspects:

1. It attempts to estimate the optimal capital structure of a firm, not only to price risky debt;
2. The bankruptcy process is triggered endogenously – the bankruptcy barrier is not fixed exogenously of the firm.
3. Risk free interest rates are assumed to be non-stochastic, as Kim, Ramaswamy and Sundaresan (1993) showed that stochastic interest rates have a limited effect on credit spreads whilst complicating the analysis significantly.

The starting point for Leland and Toft (1996), as in Merton (1974) is that the value of a firm's assets follows a continuous diffusion process:

Eq. 2.19 $$\frac{dV}{V} = (\mu - \delta)dt + dz\sigma_v$$

Where μ is the expected return on the firm's assets, δ is the (constant) payout to security holders, σ_v is the asset value volatility and dz is a standard Brownian motion. The asset value process continues indefinitely unless V falls below a certain level V_B which triggers bankruptcy. Initially, keeping V_B constant, Leland and Toft (1996) express the value of a risky bond B as the sum of the discounted expected values of the coupon flow, the expected discounted value of principal repayment and the expected discounted value of the fraction of the assets that will be allocated to holders of debt with maturity t if bankruptcy occurred before t expires. Leland and Toft (1996) express the value of D as:

Eq. 2.20 $$D = \begin{cases} \int_0^t e^{-rs} c(t)[1 - F(s;V,V_B)] + \\ e^{-rs} p(t)[1 - F(t;V,V_B)] + \\ \int_0^t e^{-rs} p(t)V_B f(s;V,V_B)ds \end{cases}$$

In Equation 2.20:

The first term on the right hand side represents the expected value of the coupon flow *c(t)* which are paid at time *s*, *s* being between *0* and *t*, weighed by the probability of not defaulting, where *F(s)* is the cumulative distribution function of the first passage time to bankruptcy.

The second term on the right hand side represents the expected value of principal repayment, again weighted by the probability of not defaulting.

The third term is the expected discounted value of the fraction of the assets that will be allocated to holders of debt with maturity t if bankruptcy occurs. The function $f(s; V, V_B)$ represents the density of the first passage time s to V_B from V with drift $(r-\delta)$.

Leland and Toft (1996) provided a closed form solution to the value of a risky bond B given by:

Eq. 2.21 $$B = \frac{c(t)}{r} + e^{-rt}\left[p(t) - \frac{c(t)}{r}\right][1 - F(t)] + \left[p(t)V_B - \frac{c(t)}{r}\right]G(t)$$

Leland and Toft took the expression for *F(t)* from Harrison (1990):

Eq. 2.22 $$F(t) = N[h_1(t)] + \left(\frac{V}{V_B}\right)^{-2a} N[h_2(t)]$$

And the expression for *G(t)* is taken from Rubinstein and Reiner (1991):

Eq. 2.23 $$G(t)=\int_0^t e^{-rs}f(s;V,V_B)ds=\left(\frac{V}{V_B}\right)^{-\alpha+z}N[q_1(t)]+\left(\frac{V}{V_B}\right)^{-\alpha+z}N[q_2(t)]$$

Where:

$$q_1(t)=\frac{\left(-b-z\sigma_v^2t\right)}{\sigma_v\sqrt{t}} \quad ; \quad q_2(t)=\frac{\left(-b+z\sigma_v^2t\right)}{\sigma_v\sqrt{t}}$$

$$h_1(t)=\frac{\left(-b-\alpha\sigma_v^2t\right)}{\sigma_v\sqrt{t}} \quad ; \quad h_2(t)=\frac{\left(-b+\alpha\sigma_v^2t\right)}{\sigma_v\sqrt{t}}$$

$$\alpha=\frac{\left(r-\delta-\sigma_v^2/2\right)}{\sigma_v^2} \quad ; \quad b=\ln\left(\frac{V}{V_B}\right) \quad ; \quad z=\frac{\left(\left(\alpha\sigma_v^2\right)^2+2r\sigma_v^2\right)^{0.5}}{\sigma_v^2}$$

One of the assumptions associated with exogenously determined default barriers is that default is assumed to occur when V falls below a fixed value V_B. Leland and Toft (1996) argued that prior to maturity V might fall below V_B but the amount of funds required to avoid default and the potential appreciation in equity if default is avoided are sufficient in order to persuade shareholders to contribute sufficient funds to the company in order to pay the coupon and avoid default. At maturity, when repayment to debt holders is significantly higher than simply the coupon, the firm's asset value V needs to be higher than V_B in order to avoid bankruptcy.

In order to introduce a constant, endogenously determined level of V_B, Leland and Toft assume the firm issues debt continuously. The principal amount of new debt issued is constant, the maturity is T and the rate of issuance is given by $p = P/T$, where P is the total principal of all outstanding bonds. The same amount of principal is repaid when earlier issues mature. As a result, at any given moment the total amount of debt outstanding is P and it has a uniform distribution over time. The bonds with principal p pay a coupon $c = C/T$, so that total coupon payments for the firm equals C per year. The total debt service requirement every year is equal to $C + P/T$ (i.e. the annual coupon and the principal repayments of bonds maturing). The total value of debt at the time the new debt with maturity T is issued is denoted by $D(V;V_B,T)$. The cost of bankruptcy is represented by a and therefore the remaining value allocated to bondholders is given by $(1-a)V_B$. The value of a company's total risky debt can thus be expressed as:

Eq. 2.24 $D(V;V_B,T)=\frac{C}{r}+\left(P-\frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT}-I(T)\right)+\left((1-\alpha)V_B-\frac{C}{r}\right)J(T)$

Where:

$$I(T)=\frac{1}{rT}\left(G(T)-e^{-rT}F(T)\right)$$

$$J(T)=\frac{1}{z\sigma_v\sqrt{T}}\left[-\left(\frac{V}{V_B}\right)^{-\alpha+z}N[q_1(T)]q_1(T)+\left(\frac{V}{V_B}\right)^{-\alpha+z}N[q_2(T)]q_2(T)\right]$$

$G(T)$, $F(T)$, z, q_1 and q_2 are as defined in equations 2.22 and 2.23 above.

Leland and Toft (1996) also expressed the value of the firm as the total value of its assets and the value of its tax benefits less the value of bankruptcy costs in perpetuity. They assumed that tax benefits accrue at a rate of τC per year as long as $V > V_B$, where τ is the tax rate. The value of the firm can thus be expressed as:

Eq. 2.25 $$v(V;V_B)=V+\frac{\tau C}{r}\left(1-\left(\frac{V}{V_B}\right)^{-x}\right)-\alpha V_B\left(\frac{V}{V_B}\right)^{-x}$$

Where $x=\alpha+z$

The value of the equity of a firm can also be calculated within the Leland and Toft (1996) framework as the value of the firm less the value of its debt:

Eq. 2.26 $$E(V;V_B,T)=v(V;V_B)-D(V;V_B,T)$$

In order to determine the equilibrium bankruptcy barrier V_B, Leland and Toft equate the partial derivative of the firm's equity with respect to V to zero:

Eq. 2.27 $$\left.\frac{\partial E(V;V_B,T)}{\partial V}\right|_{V=V_B}=0$$

They show that the solution is independent of time, supporting their assumption of a constant V_B:

Eq. 2.28 $$V_B=\frac{C/r\left(A/rT-B\right)-AP/rT-\tau Cx/r}{1+\alpha x-(1-\alpha)B}$$

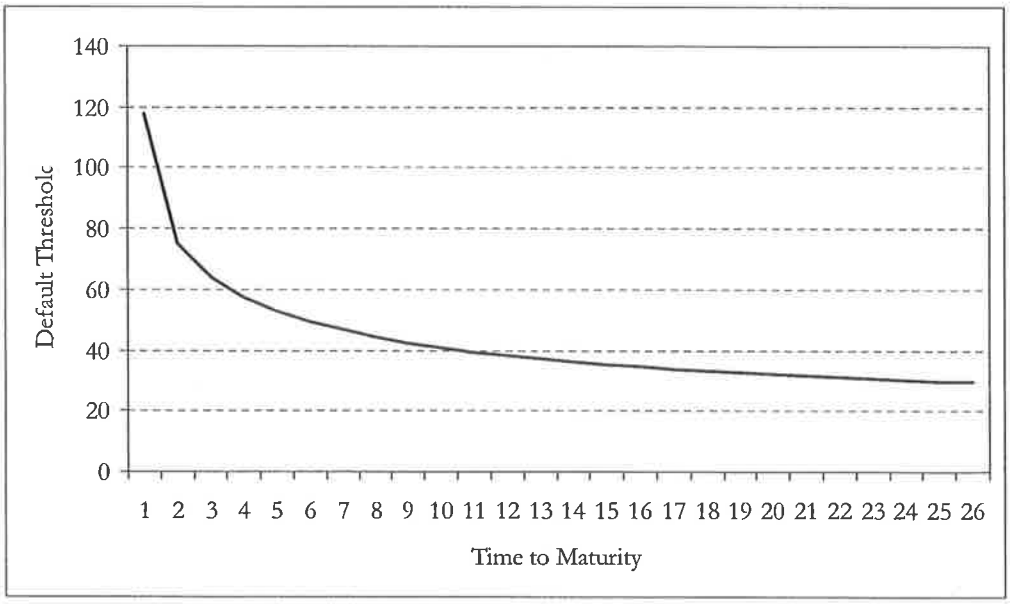
Where $A=2ae^{-rT}N\left[a\sigma_v\sqrt{T}\right]-2zN\left[z\sigma_v\sqrt{T}\right]-\frac{2}{\sigma_v\sqrt{T}}n\left[z\sigma_v\sqrt{T}\right]+\frac{2e^{-rT}}{\sigma_v\sqrt{T}}n\left[a\sigma_v\sqrt{T}\right]+(z-a)$

$$B = -\left(2z + \frac{2}{z\sigma_v^2 T}\right) N\left[z\sigma_v\sqrt{T}\right] - \frac{2}{\sigma_v\sqrt{T}} n\left[z\sigma_v\sqrt{T}\right] + (z-a) + \frac{1}{z\sigma_v^2 T}$$

The terms used in equation 2.28 have been defined in the preceding equations, with the exception of the notation *n(.)* which represents the standard normal density function.

V_B is a function of the debt maturity chosen, unlike models that base the bankruptcy trigger event on cash flow or net worth, which tend to assume that V_B is independent of debt maturity (e.g. Kim, *et al.* (1993) and Longstaff and Schwartz (1995)). Figure 2.6 below demonstrates the dynamics of V_B and time to maturity T using the Leland and Toft (1996) model for a hypothetical firm (the parameters used in this hypothetical example were $P = 60, p = 30$, $C = 3.0, c = 1.5, r = 0.075, \delta = 0.07, \tau = 0.35, a = 0.50, t = 5, V = 100, \sigma_V = 0.25$):

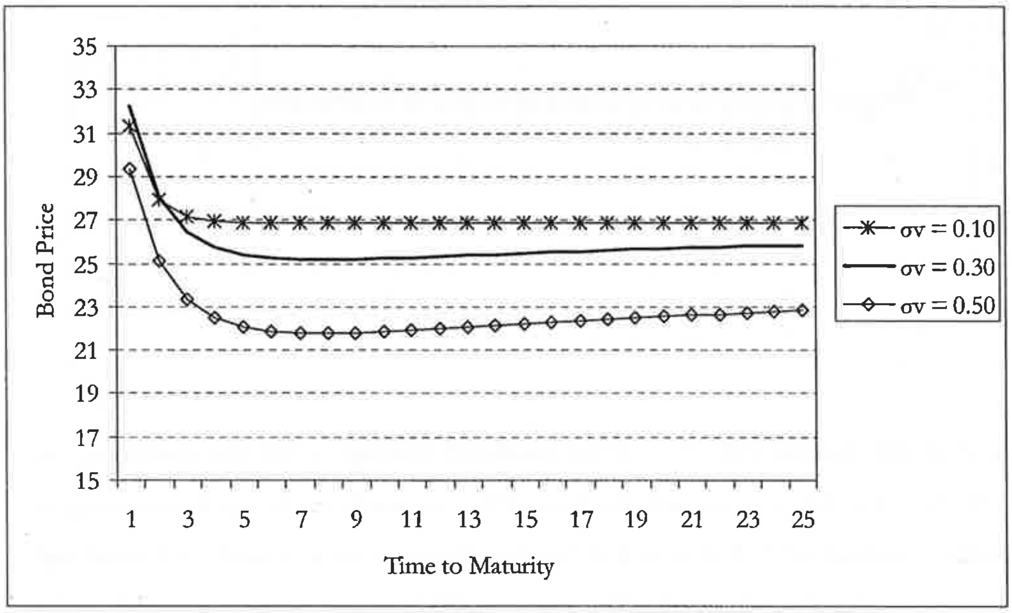
Figure 2.6: V_B as a function of time to maturity



Leland and Toft's model allows firms with long term debt maturity profiles to have negative net worth during the life of the debt, as there is sufficient time for their asset value to recover, thus avoiding bankruptcy. However, as T approaches zero, V_B approaches P/(1-α); in the example shown in figure 2.6 above, P/(1-α) = 120. As long as α > 0 (i.e. in all cases), bankruptcy could occur even if $V = V_B > P$. Leland and Toft (1996) explain this by arguing that bankruptcy is triggered because the anticipated appreciation in equity as a result of making a contribution to avoid default is lower than the actual contribution required.

The model developed by Leland and Toft (1996) provides similar bond pricing patterns to those generated by earlier models from Merton (1974) onwards. In Figure 2.7 below, the price of a bond as a function of time to maturity T is shown using three levels of asset value volatility: $\sigma_V = 0.10$, $\sigma_V = 0.30$, and $\sigma_V = 0.50$:

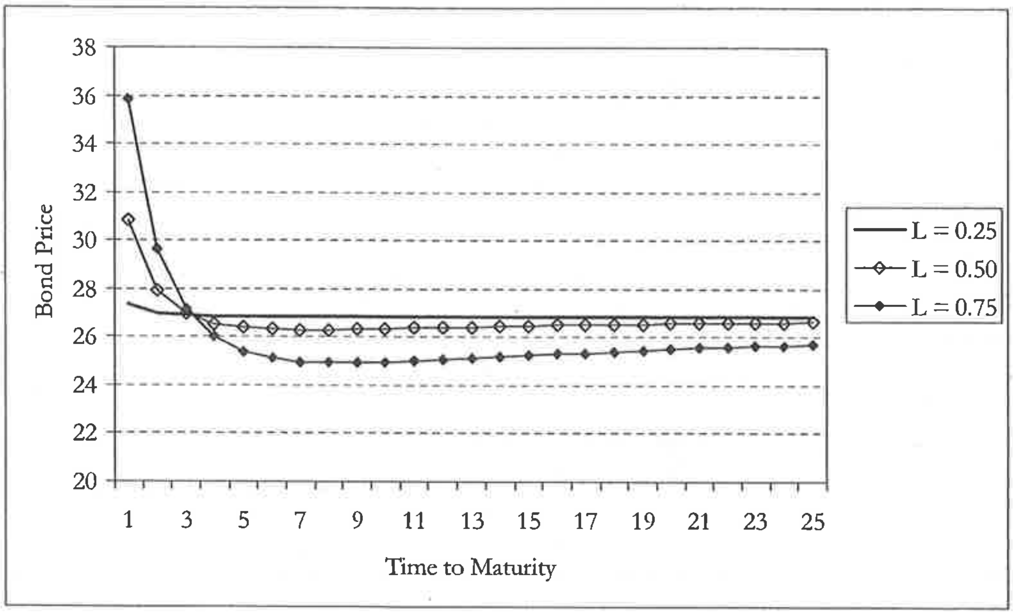
Figure 2.7: Bond prices as a function of time to maturity and σ_V



As can be seen from Figure 2.7, for low volatility levels, bond prices decrease over time, but as asset value volatility increases, bond prices first decline and then increase, the longer the time to maturity is. This pattern of prices is akin to the upward sloping and hump-shaped spread curves shown for the Merton model.

Applying different levels of leverage ($L = P/V$) to the Leland and Toft model results in similarly shaped curves, as shown in Figure 2.8 below. For low-leveraged firms (L = 0.25) the bond price curve is downward sloping, implying an upward sloping spread curve. The higher the leverage, the more U-shaped the bond price curve becomes, implying a hump-backed spread curve.

Figure 2.8: Bond prices as a function of time to maturity and leverage



Using their model, Leland and Toft (1996) examined leverage ratios that maximize the value of the firm for different maturity lengths. They showed that the optimal leverage is an increasing function of T, but also a decreasing function of a, r, and σ_v. Leland and Toft also examined debt value and debt capacity. Debt capacity was measured as the point at which the value of a company's debt is maximized as a function of leverage. For short maturities, Leland and Toft (1996) found that debt capacity is lower than for longer maturities, because there is less time for the firm's value to develop. Optimal debt capacity is also a declining function of a, and σ_v. For low-leveraged firms, Leland and Toft found that the value of debt declines as σ_v and/or r increase, but for highly leveraged firms, their model shows that debt value increases as σ_a and/or r increased. This anomaly occurs because as σ_a and/or r rise, the level of V_B (the default threshold) declines, reducing the likelihood of default.

The model developed by Leland and Toft (1996) is designed to price individual coupon bonds, value the firm's total debt and its equity and incorporate an endogenously determined default barrier. In contrast with earlier models, the Leland and Toft model also assumes that the absolute amount of the firm's debt remains unchanged over time, as the firm continues to issue debt to replace debt that needs to be repaid. However, as debt remains constant and the asset value V has a positive drift $(\mu - \delta)$, the ratio V/V_B increases over time and the likelihood that V will fall below V_B decreases. A structural credit model that allows firms to deviate from their long term leverage ratio has been developed by Collin-Dufresne and Goldstein (2001) and is described in the subsection below.

2.1.1.7. Mean-reverting leverage and stochastic interest rates: Collin-Dufresne and Goldstein (2001)

The assumptions of a simple capital structure, no new debt, strict priority in bankruptcy and non-stochastic interest rates, all of which underpin many of the structural models described above, do not hold in practice. Malitz (2000) found that most debt covenants allow firms to add equal priority debt and that many bond indentures allow firms to add even senior secured debt in the future. Therefore, although major changes in the capital structure of a company are not normally possible without balance sheet restructuring, as long as firms keep to certain credit quality ratios, they can and do add more debt occasionally. This empirical finding conflicts with one of the underlying assumptions of the original Merton model, namely no new debt. Collin-Dufresne and Goldstein (2001) demonstrated that the limitation on issuing new debt results in a downward sloping term structure of credit spreads for highly leveraged firms, which also contradicts empirical findings (e.g. Helwege and Turner (1999)).

Collin-Dufresne and Goldstein (2001) developed a structural model that aimed to address some of the limitations of the original Merton model and several of its later versions. Their version of a structural model allows firms to deviate from their long term leverage ratio for a limited period of time, i.e. issue more debt during the period under consideration, and incorporates stochastic interest rates.

The starting point of Collin-Dufresne and Goldstein (2001) is the same as for the Longstaff and Schwartz (1995) model – the dynamics of the initial firm value V over time can be expressed as the process:

Eq. 2.29 $$\frac{dV_t}{V_t} = (r - \delta)dt + \sigma_v dz(t)$$

Where r is the default risk free interest rate, δ is the pay-out ratio of the firm (e.g. dividend rate), σ_V is the asset value volatility, all of which are assumed to be constant, and $z(t)$ is a geometric Brownian motion.

Default occurs if $V_T < D$ where D is the face value of the debt outstanding at time T. The cost of bankruptcy is represented by a, a constant proportion of V_T in default.

Given these assumptions, Collin-Dufresne and Goldstein (2001) expressed the value of a risky zero coupon bond:

Eq. 2.30
$$P_{disc} = De^{-rT} N\left(d^1_{(\Gamma,T)}\right) + (1-\alpha)\frac{V_0}{D} e^{(r-\delta)} N\left(-d^2_{(\Gamma,T)}\right)$$

Where:

$N(.)$ represents the standard normal cumulative distribution function;

Γ denotes the inverse leverage ratio V_0/D;

$$d^1_{(x,y)} = \frac{\ln(x) + \left(r - \delta - \frac{1}{2}\sigma_v^2\right)y}{\sigma_v \sqrt{y}}$$

$$d^2_{(x,y)} = \frac{\ln(x) + \left(r - \delta + \frac{1}{2}\sigma_v^2\right)y}{\sigma_v \sqrt{y}}$$

The first term on the right-hand side of equation 2.30 represented the present value of the bond's principal due at maturity, adjusted for the probability of not defaulting until maturity. The second term on the right hand side of equation 2.30 represents the present value of the amount bondholders might recover in bankruptcy, adjusted for the cost of bankruptcy and for the probability of default prior to maturity.

In order to allow a firm to increase its leverage at some point τ between $t = 0$ and T, Collin-Dufresne and Goldstein assume the firm can issue a zero coupon bond D_N with the same maturity T as the previously issued debt, and that its face value is determined so as to bring the firm back to its original target leverage ratio. The amount of new debt is therefore determined by:

Eq. 2.31
$$V_\tau / D + D_N = V_0 / D = \Gamma$$

Collin-Dufresne and Goldstein (2001) also assumed that proceeds from the new debt are used to repurchase existing shares, leaving the firm's value unchanged. Under these assumptions, Collin-Dufresne and Goldstein (2001) expressed the price of a risky zero coupon bond as:

Eq. 2.32
$$\begin{aligned} P_{disc} = {} & De^{-rT} N_2\left(d^1_{(\Gamma,T)}, -d^1_{(1,\tau)}, -\sqrt{\tau/T}\right) + (1-\alpha)V_0 e^{-\delta T} N_2\left(-d^2_{(\Gamma,T}, -d^2_{(1,\tau)}, \sqrt{\tau/T}\right) \\ & + DN\left(d^1_{(\Gamma,T-\tau)}\right)N\left(d^1_{(1,\tau}\right) + (1-\alpha)V_0/D \, e^{-\delta(T-\tau)} N\left(d^2_{(\Gamma,T-\tau)}\right)N\left(d^1_{(1,\tau)}\right) \end{aligned}$$

In equation 2.32, the term N(.) denotes the univariate standard normal cumulative distribution function, $N_2(.)$ represents the bivariate standard normal cumulative distribution function, and $d^1_{(x,y)}$ and $d^2_{(x,y)}$ are the same general functions as in equation 2.30. Collin-Dufresne and Goldstein show that by allowing a firm to issue debt just one time during the life of the original bond issue, significantly higher credit spreads can be generated.

Collin-Dufresne and Goldstein (2001) developed their model further to incorporate stationary leverage ratio but unlike earlier models, they allowed the default barrier to change over time in order to revert back to the firm's long term leverage ratio. This flexibility within the model incorporates the assumption that if the firm's asset value rises (and its leverage falls) it will issue more debt to revert back to its original leverage ratio. A further enhancement of the model developed by Collin-Dufresne and Goldstein (2001) was to incorporate stochastic interest rates using the Vasicek (1977) model and linking the dynamics of the default threshold to the firm's value and to the risk free interest rate.

The incorporation of mean-reverting leverage and linked default threshold, as well as the accommodation of stochastic interest rates resulted in a complex model of limited use because it is designed to price zero coupon bonds only, and it allows firms in increase leverage only once during the period analysed.

All of the structural models described thus far assume that the firm's value follows a smooth diffusion process. From this assumption it follows that default is always predictable. However, empirical evidence by Jones, Mason and Rosenfeld (1983) and others showed that Merton-type models tend to underestimate the credit spreads observed in the market for shorter dated bonds, suggesting that market participants demand a higher risk premium (credit spread) than that estimated by a smooth diffusion model. In order to incorporate short term uncertainty of default, hybrid models have been developed, incorporating smooth diffusion processes and jumps in asset value. One such model, proposed by Zhou (1997) is described in the subsection below.

2.1.1.8. Departure form smooth diffusion processes: Zhou (1997)

One of the underlying assumptions of most structural credit models is that the firm's value follows a smooth diffusion process. As a result, it follows that credit spreads generated by such models always start at zero and increase as a function of time to maturity because default is never unexpected. Empirical tests by Jones, *et al.* (1983), Jones, *et al.* (1984) and Helwege and Turner (1999) found that corporate bond spreads predicted by the Merton (1974) model tend to be lower than those observed in the market, and that structural models tend to generate negligible credit spreads for shorter debt maturities. Empirical studies by Fons (1994) and Sarig and Warga (1989) found that credit spreads not are always upward sloping – some firms display flat or even downward sloping spread curves. Zhou (1997) argued that the reason most structural credit models tend to underestimate credit spreads and predict upwards sloping spread curves is the assumption of a smooth diffusion process to describe the dynamics of the firm's asset value. Zhou (1997) argued that the reason observed credit spreads in the market are positive even for very short debt maturities is that investors take account of negative surprises, which by definition, are not incorporated into a continuous smooth diffusion process. Zhou (1997) proposed a hybrid model, where the firm's value follows a jump-diffusion process as a way of generating non-negligible credit spreads for short maturity debt.

The assumptions used in developing the hybrid model are:

The firm's value V follows a jump-diffusion process:

Eq. 2.33 $$dV/V = (\mu - \lambda\varsigma)dt + \sigma_V dZ_1 + (\Pi - 1)dY$$

Where:

μ denotes the drift factor of the firm's asset value;

$\Pi > 0$ denotes the jump amplitude, with expected value of $\varsigma + 1$

ς denotes the of the value of the jump component

σ_v denotes the constant volatility of the firm's asset value

Z_1 is a standard Brownian motion

dY denotes a Poisson process with intensity parameter λ

The jump amplitude Π is assumed to be a log-normal random variable that can be described using its average and variance:

Eq. 2.34 $$\ln(\Pi) \sim N(\mu_\pi, \sigma_\pi^2)$$

Given the link between v and Π, Zhou (1997) showed that the log-normality of Π means that:

Eq. 2.35 $$\varsigma = e^{\left(\mu_\pi + \sigma_\pi^2/2\right)} - 1$$

In addition, Zhou assumed that:

- The jump risk is firm-specific and can be diversified away, thus commanding no risk premium.
- The Modigliani and Miller (1958) theorem holds and the firm's value is independent of its capital structure.
- The firm defaults when its asset value falls below a positive value *K*. In structural models using smooth diffusion processes, when the firm hits the default threshold for the first time, its asset value is equal to *K*, thus bondholders may expect to recover an amount *K*, adjusted for any bankruptcy costs. In a jump-diffusion model such as that of Zhou (1997) the asset value could be anywhere between 0 and *K* because of the jump process, which may result in asset value "skipping" K and falling significantly below it in a single jump.
- The firm has a simple capital structure comprising equity and a zero coupon bond.
- If default occurs prior to the bond's maturity, bondholders receive [1-$w(X_t)$]xD, where D denotes the face value of the bond, w denotes loss given default, and X is the ratio of the firm's value V to the default threshold K.

The closed-form solution proposed by Zhou (1997) for the pricing of a risky zero coupon bond is given by:

Eq. 2.36 $$\begin{aligned} B &= e^{-rT} E^Q\left[I_{X_T>1} + \left(1 - w(X_T)\right) I_{X_T \le 1}\right] \\ &= e^{-rT} - e^{-rT} E^Q\left[w(X_T) \middle| X_T \le 1\right] F_T^Q(1|X) \end{aligned}$$

Where:

I denotes an indicator function that takes the value 1 if its governing condition is true and 0 otherwise.

E^Q denotes the expectations set based on information currently available

$F_T^Q(\xi|X)$ denotes the probability of event XT=<ξ conditional on current X. In Equation 2.36 this is the probability of default.

Equation 2.36 above shows that the value of a risky zero coupon bond is the value of a risk-free zero coupon bond less the expected loss given default weighted by the probability of default under current available information.

Zhou (1997) proposed to solve for the probability of default as:

Eq. 2.37 $$F_T^{/Q}(\xi|X)=\sum_{i=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^i}{i!}\times N\left(\frac{\ln(\xi)-\ln(X)-\left(r-\sigma_V^2/2-\lambda v\right)T-i\mu_\pi}{\sqrt{\sigma_V^2 T+i\sigma_\pi^2}}\right)$$

The variables required for Equation 2.37 have been defined for Equation 2.33 above.

Zhou (1997) expanded his model to provide a general approach for pricing bonds when default can occur at any time. However, there was no closed-form solution to the general case and a proposed method for solving it is by using a Monte-Carlo simulation. Zhou (1997) argued that structural pricing models that incorporate both a jump component and a diffusion process in the firm's asset value process generate more varied shapes of the term structure of credit spreads than those generated by pure-diffusion based models.

A jump-diffusion model may be capable of explaining some of the features from empirical observations that are not explained by pure-diffusion type models such as short term probability of default and short term credit spreads. However, as there are no closed-form solutions to it, such models have not gained popularity. Furthermore, the data required in order to estimate parameter inputs for a hybrid model is significant and not easily available. In addition to the usual difficulties of estimating V and σ_v, practitioners also need to determine the Poisson distribution of asset values and the intensity parameter λ from market observations. The likelihood of a surprise default in the very near future is usually very small and observations to estimate this likelihood are very few and far between, thus parameter estimation for jump-diffusion models appears to be more difficult and less accurate than for pure diffusion structural models, and their performance is yet to be tested empirically.

2.1.2. Practical Applications of Structural Models

Structural credit models were developed in order to provide practitioners with a method of estimating the fair value of a credit risky asset. There have been several attempts to implement structural model as commercial applications in order to make them more widely available for practitioners. Saunders and Allen (2002) identify KMV's and

Moody's Models as those built on the structural credit model framework, whilst Meissner and Nielsen (2002) identify KMV' Portfolio Manager™ and RiskMetrics™' CreditMetrics™ as commercial applications for estimating portfolio risk using building blocks based on structural models. This section provides an overview of the principles underlying the implementation of KMV's Credit Monitor® and RiskMetrics™' CreditGrades™, which are commercially available tools used in order to estimate individual assets' credit risk, and are then used to feed into the portfolio risk management tools.

2.1.2.1. KMV's Credit Monitor®

KMV, a consultancy based on San Francisco and now part of Moody's Investors Service was one of the pioneers of quantifying credit risk using the structural model and providing commercial applications for practical implementation. KMV's approach to modelling individual asset's credit risk is described in Crosbie and Bohn (2003), Kealhofer (2003) and Kealhofer (2003). This section provides an overview of KMV's approach to estimating individual asset's credit risk.

KMV identified three standalone risks that combine to form credit risk:

1. Default probability ('DP') – the likelihood of default occurring during a given time period.
2. Loss given default ('LGD') – how much will be lost if a borrower defaulted.
3. Migration risk – the effect on value of a change in default probability.

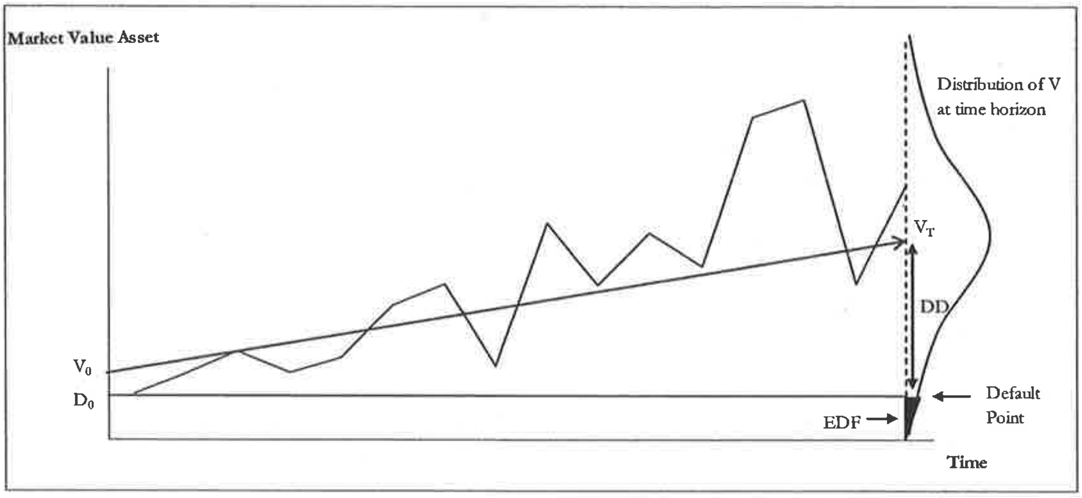
Of these risks, KMV consider the probability of default to be the most important, but also the most difficult to estimate.

The underlying approach used by KMV is based on the Merton (1974) model, but has been expanded by Oldrich Vasicek and Stephen Kealhofer, and is referred to as the Vasicek-Kealhofer model. The model itself is proprietary to KMV, but is described in Crosbie and Bohn (2003). The Vasicek-Kealhofer model assumes that the firm is a perpetual option, and that the default point is an absorbing barrier. Default is triggered when the firm's asset value is equal the default point. The Vasicek-Kealhofer model incorporates multiple liability classes, including short term liabilities, long-term liabilities, convertible debt, preferred equity and common equity. Furthermore, it is assumed that when the firm's asset value reaches a certain size, the convertible debt is converted to equity, thus diluting existing equity. Dividends are also explicitly included in the model.

When default occurs, it is assumed that all debt securities default at the same time, regardless of seniority in the capital structure. Unlike many structural models, the objective of the Vasicek-Kealhofer model is to calculate an Expected Default Frequency (EDF™).

The KMV model combines the distribution of the asset value and the time horizon specified by the user in order to determine the firm's distance to default ('DD') and its EDF™. Figure 2.9 below provides a graphical overview of the model's components:

Figure 2.9: The KMV approach – a graphical representation



The key elements of KMV's approach, which are shown in Figure 2.9 above are:

- V_0: the firm's asset value at the outset of the analysis. V follows a random process over time as indicated by the broken, thin curve. The positive drift of the asset value dynamics is indicated by the line V_0V_T. At the time horizon of the analysis, the values that V can take are assumed to be normally distributed, as shown by the normal distribution curve.
- D_0: the firm's default point, which is equal to the book value of the firm's liabilities.
- *DD*: the firm's distance to default, which can be measured as the number of standard deviations the expected asset value at time T will be from the default point.
- *EDF*: the expected default frequency, which is the area under the curve which falls below the default point.

In order to calculate the expected default frequency, the KMV model required three main inputs:

1. The market value of the firm's assets, denoted as V – in theory, this would be the present value of all future free cash flows discounted at an appropriate rate.
2. Asset risk – the uncertainty associated with asset value, which can be denoted as σ_V – the standard deviation of the asset values. Given that asset value is an estimate, σ_V is also an estimate.
3. Leverage – the amount of a company's contractual liabilities. Unlike the company's assets, for which the current market value is needed, the firm's liabilities are included at book value, given their contractual nature.

If, over time, the value of a company's assets converges on the book value of its liabilities, the likelihood of default increases. Crosbie and Bohn (2003) state that based on their study of defaults, firms do not generally default when their asset value is equal to the book value of their liabilities. Instead, they argue that many firms stay in business and service their debts, taking advantage of the breathing space afforded by their long term liabilities. Therefore, a firm's default point, according to Crosbie and Bohn (2003), lies somewhere between the amount of total liabilities and current liabilities.

When estimating a firm's default probability, KMV consider a relevant net worth ('RNW') which is:

Eq. 2.38 $$RNW = V - DP$$

Where V denotes the market value of the firm's assets and DP is the default point. Default occurs when RNW equals zero. Asset risk, σ_V, is related to the size of the company and the nature of the industry it operates in. Some industry sectors tend to have stable earnings and will tend to have lower asset value volatility than those sectors that display fluctuating earnings over time. KMV utilise the relationship between asset volatility and equity volatility through the firm's leverage. If a firm carries no debt, its equity volatility would equal its asset value volatility. However, leverage increases equity volatility.

KMV combine asset value, asset value volatility and leverage to estimate a firm's distance to default ('DD'):

Eq. 2.39 $$DD = \frac{V - DP}{V \times \sigma_v}$$

In Equation 2.39, *V* and *DP* are defined as in Equation 2.38 and σ_v represents asset value volatility. If the probability distribution of asset values was known, the default probability could be computed directly from the distance to default. Alternatively, if the default rate for a given distance to default was known, then the probability of default could be derived form it.

For publicly quoted companies, KMV utilize the approach that equity can be viewed as a call option on the firm's assets. Using the observed equity price *E*, equity price volatility σ_e and book value of liabilities *X*, KMV derive the underlying asset value and asset value volatility – an approach similar to that used to determine implied equity volatility from share prices and option prices observed in the market. As the Vasicek-Kealhofer model is not publicly available, the implementation of the KMV approach is explained using the Black and Scholes framework. In the Black and Scholes (1973) framework, the value of a call option is expressed as:

Eq. 2.40 $$E = VN(d_1) - e^{-rT} XN(d_2)$$

Where:

$$d_1 = \frac{\ln\left(V/X\right) + \left(r + \sigma_v^2/2\right)T}{\sigma_V \sqrt{T}}$$

$$d_2 = d_1 - \sigma_v \sqrt{T}$$

V, *X* and σ_v are as before, asset value, book value of liabilities and asset value volatility respectively, and *N(.)* is the cumulative standard normal distribution function.

Utilising the relationship between equity price volatility σ_e (observed) and asset value volatility (unobserved), as shown below, it is possible to solve for asset value through substitution of Equation 2.41 below into Equation 2.40 above:

Eq. 2.41 $$\sigma_e = \frac{V_a}{E} \Delta \sigma_a$$

In practice, the relationship expressed in Equation 2.41 is instantaneous and in order to obtain a long term measure of asset value and asset value volatility, KMV solve for asset value volatility through a process of iteration. Having calculated *V* and σ_v, KMV then proceed to determine the distance to default *DD* as in Equation 2.39 above.

The probability of default is the probability that *NRW* will reach zero during a specified time horizon. Using the Black and Scholes (1973) framework, this is expressed as:

Eq. 2.42 $$P_{d,T} = N\left(-\frac{\ln\left(V/X\right)+\left(\mu-\sigma_v^2/2\right)}{\sigma_v\sqrt{T}}\right)$$

Where μ represents the expected return on the firm's assets and all other inputs are as defined before. The distance to default is the number of standard deviations that the firm is from default. In the Black and Scholes framework, this is expressed as:

Eq. 2.43 $$DD = \frac{\ln\left(V_a/X\right)+\left(\mu-\sigma_a^2/2\right)}{\sigma_v\sqrt{T}}$$

KMV argue that using normal standard distribution to calculate the probability of default is inappropriate, because the default point itself is a random variable – as firms approach bankruptcy, their liabilities tend to change dramatically. In order to overcome this, KMV estimate the relationship between *DD* and the probability of default by fitting the distance to default obtained for a particular firm on to a database of historical default and bankruptcy cases. KMV's proprietary database has over 250,000 company years of data and around 4,700 default cases. A lookup function searches for the proportion of defaulted firms that had a *DD* measure equal to the one calculated for the firm being analysed, *T* number of years before default. The number of firms fitting that description, divided by the total number of firms in KMV's database equals KMV's default probability, or *EDF*.

Empirical testing of the KMV model has largely been done by Kealhofer (2003), Kealhofer (2003), and Crosbie and Bohn (2003). All three studies show that the KMV model is effective in predicting credit quality changes when measured against changes in credit ratings assigned by agencies such as Moody's and Standard & Poor's, and that the EDF™ measure is more powerful in predicting default than rating agencies' categories. It should be noted, however, that all three authors work at KMV, and Mr. Kealhofer is one of KMV's founders, and are thus likely to have a vested interest in proving the superiority of KMV's model to other methods of estimating default. Independent studies of the robustness of KMV's model performance are not widely available. However, Oderda, Dacorogna and Jung (2003) compared the performance of KMV's CreditMonitor and Moody's RiskCalc to credit ratings and found that the models contained relevant information that was not available by ratings alone, and that on average, the models predicted default some 10 months before it occurred.

It is worth noting, however, that most empirical studies of structural models focus on the ability of such models to predict credit spreads and bond prices, but the empirical tests of KMV focused on the model's ability to predict credit quality changes, and the relative out-performance vis-à-vis the rating agencies may be testament to the overall superiority of structural models in predicting changes in credit quality, rather than the superiority of the KMV model only. Perhaps the strongest evidence for the success of KMV in implementing a structural model commercially and the strengths of the structural model's approach is the fact that Moody's Corporation, a leading credit rating agency, bought KMV in 2002.

2.1.2.2. RiskMetrics™ CreditGrades™

CreditGrades™ was developed by the RiskMetrics Group, Inc. in 2002. An earlier development by the RiskMetrics Group was CreditMetrics® which is a model for calculating portfolio credit risk that was launched in 1997. Whilst individual credit risk within the CreditMetrics model is represented by its credit rating and the rating transition matrix associated with each rating, the focus of CreditGrades is on individual asset's credit risk assessment. The RiskMetrics Group, which is sponsored by JP Morgan, Goldman Sachs and Deutsche Bank, launched CreditGrades in 2002, in response to a wave of credit defaults in 2001 and the increased sophistication of banks' capital adequacy and risk management regulation.

CreditGrades is a publicly available application of the structural credit model for quantifying credit risk. The focus of CreditGrades is on tracking credit spreads and providing an indication of credit quality deterioration in a timely manner. CreditGrades was designed to price credit default swaps – a credit derivative instrument that aims to facilitate the trading of pure default risk, without the added complications of liquidity, call and put options and convertibility which are associated with many bond issues. In order to achieve this aim, the data used to calibrate CreditGrades is from market observed variables such as credit spreads and equity volatility. The explanation of CreditGrades' methodology is based on information contained in Finger (2002).

As in all structural models, CreditGrades assumes that V, a company's asset value, follows a stochastic process as shown below:

Eq. 2.44 $$\frac{dV_t}{V_t} = \sigma_V dW_t + \mu_D dt$$

Where W is a standard Brownian motion, σ_V is the volatility of V and μ_D is the asset drift, assumed to equal zero.

Default occurs when V crosses a default barrier for the first time. The default barrier is the amount of assets remaining once default occurred (the recovery value) expressed as L x D, where L is the recovery rate and D is the debt outstanding. In CreditGrades, the debt is calculated per share, but in order to keep the explanation in this section as concise as possible, this aspect of the model is omitted here.

In order to overcome the issue of unrealistic short term credit spreads estimated by pure diffusion processes assumed by theoretical structural models, CreditGrades apply randomness to the recovery value L, based on the findings of significant variance of recovery rates (e.g. Altman and Kishore (1996), Altman and Arman (2002), and Varma, Cantor, Hamilton, Ou, Bodard de, Theodore, Keegan, Menuet and West (2005)). It is assumed that L is log-normally distributed with mean $\bar{L}$ and standard deviation λ. Thus:

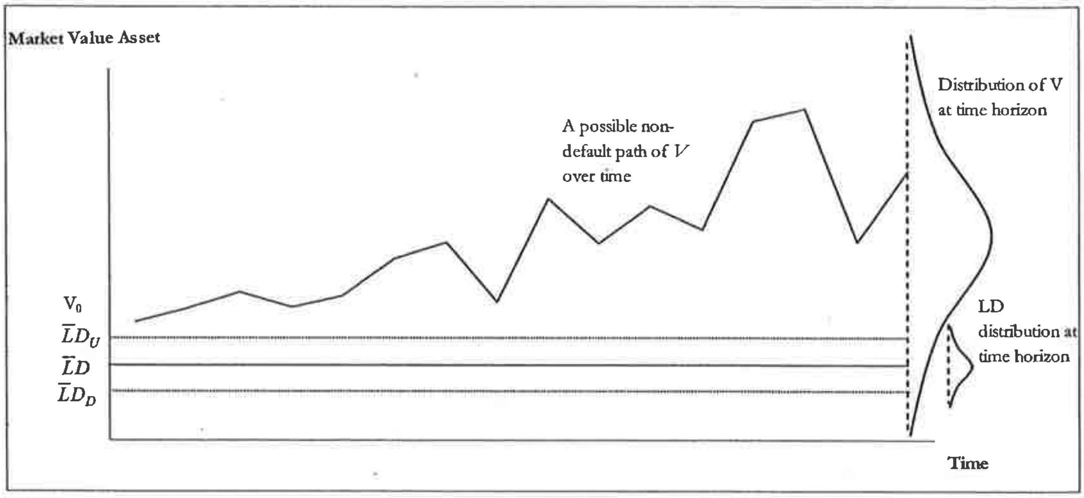
Eq. 2.44 $$\bar{L} = EL$$

Eq. 2.45 $$\lambda^2 = Var \ln(L)$$

Eq. 2.46 $$LD = \bar{L}De^{\lambda Z - \lambda^2/2}$$

In Equation 2.46 Z is a standard normal random variable that captures the uncertainty of recovery levels and is independent of W. At time t = 0, Z is unknown and it only becomes apparent during default. Thus even if there is some level of L that does not change through time, LD, the default barrier, can be hit unexpectedly because of the presence of Z. As Figure 2.10 below demonstrates, there are two causes of uncertainty in the CreditGrades approach: the distribution of asset value over time, and the distribution of the recovery value, represented by the distribution of LD.

Figure 2.10: The CreditGrades approach – a graphical representation



In Figure 2.10, the variance of the recovery rate is demonstrated by the variance attributed to the default threshold $\bar{L}D$, which is shown as a range between an upper limit of $\bar{L}D_U$ and a lower limit of $\bar{L}D_D$. The default threshold is assumed to be normally distributed around a mean $\bar{L}D$. Therefore, even if asset value V does not reach $\bar{L}D$ the firm can still default if, at some point, the default threshold reaches $\bar{L}D_U$ when V falls to that level. Alternatively, even if V hits a level equal to $\bar{L}D$, default may be not occur, if, at that time, the default threshold is actually at $\bar{L}D_D$.

Within the framework of CreditGrades, default will not occur as long as the asset value did not cross the default barrier:

Eq. 2.47 $$V_0 e^{\sigma_V W_t - \sigma_V^2 t/2} > \bar{L}D e^{\lambda Z - \lambda^2/2}$$

The process by which the firm's assets develop over time can also be presented as:

Eq. 2.48 $$X_t = \sigma_V W_t - \lambda Z - \frac{\sigma_V^2 t}{2} - \frac{\lambda^2}{2}$$

The initial condition set out in Equation 2.47 can then be written as:

Eq. 2.49 $$X_t > \ln\left(\bar{L}D / V_0\right) - \lambda^2$$

When $t >= 0$, X_t is normally distributed with mean and variance as shown below:

Eq. 2.50 $$E(X_t) = -\frac{\sigma_V^2}{2}\left(t + \lambda^2 / \sigma_V^2\right)$$

Eq. 2.51 $$Var(X_t) = \sigma_V^2\left(t + \lambda^2 / \sigma_V^2\right)$$

The survival probability that $\overline{X}$ will be greater than the default barrier up to time t is then given by the closed form formula:

Eq. 2.52 $$P(t) = N\left(-\frac{A_t}{2} + \frac{\ln(d)}{A_t}\right) - dN\left(-\frac{A_t}{2} - \frac{\ln(d)}{A_t}\right)$$

Where:

$$d = V_0 e^{\lambda^2} / LD$$

$$A_t = \sqrt{\sigma_V^2 t + \lambda^2}$$

Once the survival probability has been estimated, it is used in order to determine the price of credit. In order to calculate the price of credit, two further inputs are needed: the risk-free interest rate r and the recovery rate R on the underlying credit. It is important to note that R is not the same as L; L denotes the general recovery rate for the firm, whereas R is the recovery rate for a specific instrument in the capital structure (e.g. a senior secured loan, or a senior unsecured bond). In order to estimate the fair price of credit risk, CreditGrades solves for a continuously compounded spread c^* such that the expected premium payments associated with the credit risk are equal to the expected loss payouts. The equation used by CreditGrades is given by:

Eq. 2.53 $$c^* = r(1-R)\frac{1 - P(0) + e^{r\xi}(G(t+\xi) - G(\xi))}{P(0) - P(t)e^{-rt} - e^{r\xi}(G(t+\xi) - G(\xi))}$$

Where r and R are the risk free rate and the instrument specific recovery rate respectively, *P(t)* is the survival probability to time t and $\xi = \lambda^2 / \sigma_V^2$. The value of G in equation 2.53 is given by:

Eq. 2.54 $$G(x) = d^{z+\frac{1}{2}} N\left(-\frac{\ln(d)}{\sigma_V\sqrt{x}} - z\sigma_V\sqrt{x}\right) + d^{-z+\frac{1}{2}} N\left(-\frac{\ln(d)}{\sigma_V\sqrt{x}} + z\sigma_V\sqrt{x}\right)$$

In the formula above (developed by Rubinstein and Reiner (1991)), $z = \sqrt{\frac{1}{4} + 2r / \sigma_V^2}$.

The inputs required in order to implement the model include an initial value V_0 and asset volatility σ_V, both of which are unobservable, but are linked to observed equity price E and the equity price volatility σ_E through the process:

Eq. 2.55 $$\sigma_E = \sigma_V \frac{V}{E}\frac{\partial E}{\partial V}$$

The distance to default can be defined as the number of standard deviations between the firm's current value and the default barrier, defined as η:

Eq. 2.56 $$\eta = \frac{1}{\sigma_V}\ln\left(\frac{V}{LD}\right) = \frac{V}{\sigma_E E}\frac{\partial E}{\partial V}\ln\left(\frac{V}{LD}\right)$$

As can be seen, η can be expressed as a function of asset volatility (σ_V, unobservable) and equity volatility (σ_E, observable). By focusing on the behaviour of η near the boundary conditions, it is possible to derive the values of the unobservable variables. The first boundary condition describes the value of V near the default threshold LxD. As default approaches, the value of equity E approaches zero, thus at the extreme:

Eq. 2.57 $$V\big|_{E=0} = LD$$

Near that point, i.e. at the boundary, E still has value, but it is affected by the change in the value of the firm with respect to a small change in the share price:

Eq. 2.58 $$V \approx L \times D + \frac{\partial V}{\partial E}E$$

Substituting the approximation of V into equation 2.56 we obtain

Eq. 2.59 $$\eta \approx 1/\sigma_E$$

The second boundary condition is at the other extreme, when the value of the firm is far from the default barrier. At that point E/V approaches 1. Substituting into equation 2.56 results in another approximation for η:

Eq. 2.60 $$\eta \cong 1/\sigma_E \ln\left(\frac{E}{LD}\right)$$

The expression for V and η that satisfies both the near default boundary conditions and the far from default boundary conditions simultaneously are

Eq. 2.61a $$V = E + LD \text{ and}$$

Eq. 2.61b $$\eta = \frac{E + LD}{\sigma_E E} \ln\left(\frac{E + LD}{LD}\right)$$

Therefore, at time $t = 0$ the value of the firm is:

Eq. 2.62 $$V_0 = E_0 + \bar{L}D$$

Where E_0 is the stock price at time $t = 0$. This also provides a solution for σ_V, the unobservable volatility of the firm's assets:

Eq. 2.63 $$\sigma_V = \sigma_E \frac{E}{E + \bar{L}D}$$

Assuming asset volatility is stable, as stock price decreases, the equity volatility increases, as the company approaches default. Equity volatility is dependent on stock price and this is especially evident in the volatility skew observed in the equity options markets for highly leveraged companies. CreditGrades use a reference share price and reference equity volatility to determine an asset volatility and keep it stable over a given time period.

The assumption of asset value process with a drift of zero means that over time the firm issues more debt to maintain a constant level of leverage (alternatively, the firm pays out excess cash in dividends), thus the debt has the same drift as the stock price.

Following the derivation of estimation functions for the unobservable inputs, CreditGrades utilizes the following formula for calculating survival probability based on market observable parameters:

Eq. 2.64 $$P(t) = N\left(-\frac{A_t}{2} + \frac{\ln(d)}{A_t}\right) - dN\left(-\frac{A_t}{2} - \frac{\ln(d)}{A_t}\right)$$

Where:

$$d = \frac{E_0 + \bar{L}D}{\bar{L}D} e^{\lambda^2}$$

$$A_t = \sqrt{\left(\sigma_E^* \frac{E^*}{E^* + \bar{L}D}\right)^2 t + \lambda^2}$$

E_0 is the initial stock price;

E^* is the reference stock price;

σ_E^* is the reference stock price volatility;

D is the level of debt (per share in CreditGrade);

$\overline{L}$ is the overall debt recovery rate;

λ is the standard deviation of the default barrier, expressed in percents.

In calculating the debt of the company, CreditGrades includes all long and short term borrowings, convertible bonds, capital leases, under-funded pension obligations and preferred shares. Operating liabilities such as accounts payables and deferred taxes are not included.

The equation for the survival probability is a function of both the distance to default η and the uncertainty attached to the default barrier itself, λ. The longer the time horizon, the greater the influence of the distance to default, whereas in the short term, the main driver of default probability is the volatility of the default barrier λ.

At the time of writing, there appears to be no publicly available independent empirical study of the accuracy of CreditGrades. The only comparative study that is publicly available is by Finger (2002), which compared data for 107 firms on two dates (30th January and 15th November 2001) using the following models:

1. CreditGrades with 1,000 day historical volatility estimator (i.e. based on 1,000 daily equity returns);
2. CreditGrades with equity volatility input from the longest-maturity at-the-money options;
3. Moody's RiskCalc;
4. Risk-neutral default probabilities implied by actual CDS spreads.

Finger (2002) used Cumulative Accuracy Profiles to compare the models and found that CreditGrades' ranking indicates significant information on how the market actually ranks credit risk, and that the model using implied volatility performed better when ranking the worst credits, whilst the model using historical volatility ranked the good quality credits better. As in the empirical tests of the KMV model, the model's ability to classify good and bad credits successfully is an encouraging start, but a true test of the structural model's success is whether it is able to predict bond prices (and/or credit spreads) that are reflected in the market. The next section describes some of the most notable empirical tests conducted on structural models.

2.1.3. Empirical Testing of Structural Models

The purpose of the structural models described in section 2.1.1 is to provide a practical approach for pricing risky bonds using an approach that is grounded in financial and economic theory. In the early days of structural models development, more emphasis was placed on theoretical development than on empirical testing due to the difficulties of obtaining reliable bond pricing data. As more accurate and extensive market data became available and computing power increased, both the implementation and the testing of such models became more robust. This section provides a chronological overview of the most notable empirical tests of structural credit models that has been conducted so far.

One of the earliest empirical tests of structural models was conducted by Jones, *et al.* (1984), who applied the Merton model to a sample of firms and secondary bond prices. The purpose of the study was to test the predictive power of a structural credit model for pricing debt securities and it built on an earlier study by Jones, *et al.* (1983). Jones, *et al.* (1984) used monthly data for 27 publicly traded firms from January 1977 to January 1981. The set of firms used comprised 18 investment grade companies and nine non-investment grade firms. The data selection process aimed to achieve a sample of firms that were as close as possible to the simple capital structure assumed by Merton (1974), thus Jones, *et al.* (1984) selected firms with the following characteristics:

1. Simple capital structure (no preference shares, no convertible bonds, and as few bond issues as possible).
2. Small proportion of private debt to total capital
3. Small proportion of short term debt to total capital
4. Traded debt (i.e. bonds) rated by the major rating agencies.

Jones, *et al.* (1984) used bond prices from Standard & Poor's Bond Guide, bond covenant and indenture data were taken from Moody's Bond Guide and the risk-free rate used was the one-year forward rate implied from treasury yield curves. In order to estimate V and σ_v Jones, *et al.* (1984) used two methods:

- Method 1: using a monthly time series of the firm's value based on 24 trailing months of data, where V was estimated as the sum of the company's market capitalisation and the market value of its debt. Using these time series the logarithmic total return on V and the standard deviation of returns were calculated.

- Method 2: An approach based on several steps, where initially the estimate for σ_v from Method 1 were used to generate values for the firm's value V and the firm's equity E. Then a three-months standard deviation of return on equity was calculated, denoted by σ_E. The final step was to use the relationship between σ_V, σ_E and leverage ($\sigma_E = \sigma_V \frac{\partial E}{\partial V}\frac{V}{E}$) in order to solve for σ_V.

Jones, *et al.* (1984) drew several conclusions from their study:

1. The Merton model outperformed a "naïve" model (which assumed no default risk) using 99% significance level. In other words, the pricing errors generated by the structural model were smaller than pricing errors generated by the "naïve" model. However, for the investment grade sub-sample, there was virtually no statistical difference between the pricing errors generated by the structural model and by those generated by the "naïve" model.
2. The Merton model used in the study overpriced bonds by an average of 4.5%. The largest absolute errors were found when pricing sub-investment grade bonds.
3. The Merton model appeared to be more suited for valuing the debt of high risk firms because probability of default is a significant component of their pricing.
4. A significant relationship was found between pricing errors and maturity, equity variance, leverage and time period.
5. Using non-stochastic interest rates in the Merton model is a contributor to bond pricing errors.

Ogden (1987) conducted an empirical test of Merton's model that sought to overcome the problem of reliable secondary bond prices encountered by Jones, *et al.* (1983) and Jones, *et al.* (1984). Instead of using secondary market prices, Ogden (1987) used prices from new bond offerings. The sample used by Ogden was selected using the following criteria:

1. All companies in the sample were publicly quoted, with share price history going back at least 30 months prior to the bond issue date.

2. Only new issue corporate bond data was used, thus providing timely information regarding market clearing bond pricing and rating.
3. Each bond in the sample had to have at least 10 years remaining until maturity, contain call provisions and sinking fund provisions.
4. The bond's market value must be not less than 10% below its par value.
5. The firms in the sample had to have simple capital structure, as specified in Merton (1974), thus effectively excluding all public utilities, financial institutions and transportation companies, which tend to have complex capital structures.

Using these criteria, Ogden (1987) obtained a data sample comprising 57 bonds issued between 1973 to 1985.

Ogden (1987) used Moody's Industrial Manuals from 1973 to 1985 in order to obtain issuer- and bond information. The estimates for V and σ_V that were needed as inputs to the structural model were obtained using a methodology similar to that used by Jones, *et al.* (1984).

Ogden (1987) used a probit analysis to test the power of structural model variables in explaining rating variation. He found that the inputs V and σ_V explain around 78.6% of variations in ratings, and that the natural logarithm of the firm's size, as represented by $\ln(V)$ increases the predictive power of the probit model as represented by R^2 from 0.786 to 0.855. The argument for the explanatory power of firm size in predicting credit rating and default probability is that smaller firms are more likely to default than larger firms. Ogden used regression analysis to determine whether factors used by structural models explain market yields. The regression suggested that structural model factors can explain around 60% of market yield variations, but it also resulted in a large positive intercept, suggesting that 40% of yield variations are explained by factors that are not included in structural models, and that the Merton model underestimates credit spreads. Ogden (1987) also identified non-stochastic interest rates as a weakness of the Merton model, arguing that interest rate environment will affect the propensity to exercise call options.

Sarig and Warga (1989) conducted an empirical test of the risk structure of interest rates paid by corporate borrowers. Sarig and Warga (1989) took advantage of an increase in the issuance of discount bonds by corporations in the United States in the mid 1980s in order to separate the effect of default risk on corporate bond yields whilst avoiding the added complications of sinking fund provisions, call options and multiple payments during the life of the bond. The data set used comprised monthly price data (where available) for 137 zero coupon bonds issued by forty two US firms in the period from February 1985 to September 1987. Bond prices were taken from a database of Shearson Lehman Brothers (an investment bank). Sarig and Warga (1989) noted that actual trade prices are not available for every trading day because most corporate bonds are traded over-the-counter between traders rather than through a central exchange. It is therefore possible for a bond not to trade for days, weeks and sometimes months. During the time a bond is not traded, its price is based on a trader's estimate of what the fair price would be. In order to ensure that prices used in their study reflect market clearing prices (i.e. traded prices) Sarig and Warga (1989) filtered out the 'trader' price and only used those prices that reflected the closing price on a day a bond actually traded. Other exclusions from the study included bonds whose rating changed during the sample period, bonds whose reported price represented an anomaly (e.g. where the reported price was lower (higher) than the price of a zero coupon bond with shorter (longer) maturity from the same issuer), and callable bonds.

Sarig and Warga (1989) subtracted the yield of zero coupon US Government bonds with similar maturities from the yields of corporate bonds in their sample in order to obtain a terms structure of risk premia. They found that the term structure of risk premia, also known as the "spreads" are downward sloping for highly leveraged firms, humped for medium leveraged firms and upward sloping for low leveraged firms. Sarig and Warga (1989) compared the shape of the risk premia term structure obtained from their study with the theoretical term structure generated by Pitts and Selby (1983) and concluded that the term structure of risk premia generated by the Merton (1974) model are similar to those they observed from empirical data.

Helwege and Turner (1999) argued that the study by Sarig and Warga (1989) suffers from sample selection bias due to the selection of maturity – within the same rating category, safer firms will tend to issue longer dated securities. Helwege and Turner (1999) sought to address this bias by choosing sets of bonds issued by the same firm but with different maturities, focusing on sub-investment grade bonds. The final data set comprised 64 groups totalling 163 bonds between 1977 and 1994. Helwege and Turner (1999) concluded that the term structure of credit spreads generated by sub-investment grade companies tends to have a positive slope, as most bond sets in the sample tended to show a strict increase in credit spreads for an increase in maturity.

Wei and Guo (1997) conducted the first empirical comparison between two structural models: Merton (1974) and Longstaff and Schwartz (1995). The data used by Wei and Guo comprised prices for six Eurodollar bond yields taken from Reuters every Thursday during 1992 and thirty three US Treasury Bill prices taken from the *Wall Street Journal* during 1992. Thus the study was based on 53 days of observations. Empirical credit spreads were calculated by subtracting the yield on Treasury Bills from the corresponding yield on a Eurodollar bond. Theoretical spreads for the same yield maturities were calculated using the Merton (1974) model and the Longstaff and Schwartz (1995) model. The theoretical spreads were compared with the empirical spreads and Wei and Guo (1997) demonstrated that:

1. Despite its relative simplicity, the Merton model provides a better explanation of credit risk in four out of five tests carried out, as long as the asset value volatility parameter is allowed to fluctuate. When keeping all variables constants, both the Merton and the Longstaff and Schwartz models performed equally unsatisfactorily compared with empirical data.
2. Both Merton's and Longstaff and Schwartz' models can generate humped credit risk term structure starting from zero because the predictability of default is built into them through the firm value dynamics, and thus they may not be ideal for estimating credit spreads on money-market securities.
3. For long-dated maturities the credit term structure generated by the Merton model converges on a constant whereas that generated by the Longstaff and Schwartz model converges on zero. The Longstaff and Schwartz model thus appears less suitable for Eurodollar term structure estimation.

Wei and Guo (1997) noted that the Longstaff and Schwartz model requires a larger number of parameter estimation and its implementation is more complicated. Therefore, although their results show that Merton's model, when asset volatility is allowed to fluctuate, appears to outperform the Longstaff and Schwartz model, Wei and Guo (1997) suggested that this result might be different if the required parameters for both models were recalculated for every period tested.

Ericsson and Reneby (2004) argued that the difference in performance between structural models and reduced form models as noted by Jarrow, Lando and Turnbull (1995), Lando (1998) and Duffie and Singleton (1999) was due to parameter estimation issues, and that if the parameters needed for a structural model were estimated accurately, the perceived performance advantage of reduced form models would be eliminated. Ericsson and Reneby (2004) tested a first passage to default version of the structural model approach that was developed by Leland (1994) and enhanced it to allow departure from absolute priority rule and future debt issuance. The data sample used in the study comprised 5,594 dealer quotes for 141 US corporate bonds between January 1994 and February 1998. In order to estimate asset value volatility and asset value, Ericsson and Reneby (2004) used a maximum likelihood method suggested by Duan (1994). The principle underlying this approach is to use the prices of several related derivatives in order to deduce the characteristics of the unobserved underlying process. Ericsson and Reneby (2002) evaluated the maximum likelihood approach and compared it with traditional methods such as Merton (1974) and Briys and De Varenne (1997) and found that the maximum likelihood method is less biased and more efficient. In their empirical study, Ericsson and Reneby (2004) used a time series of equity prices and the formula for valuing equity as a put option in order to arrive at a maximum likelihood estimate of asset value and asset value volatility. Ericsson and Reneby (2004) found that their model was able to price credit spreads one-month in advance with an average spread error of 1.7 basis points (around 0.1% overestimation of bond prices), and the standard deviation of the spread error was 19 basis points. For sub-investment grade bonds the spread error was 3.7 basis points, but the relative spread error was 1.3% compared with 2.2% for the whole sample and 3.1% for the highest rated companies in the sample. The pricing errors obtained by Ericsson and Reneby (2004) are similar or better than those obtained by Duffee (1999) and Bakshi, Madan and Zhang (2001) for reduced form models. For longer time horizons (up to four years ahead), the spread error in the study by Ericsson and Reneby (2004) increased to -13 basis points for the sample as a whole (compared with -1.7 basis points for the one-month time horizon), and the relative spread error increased to -18% from -2.2%, suggesting that the structural model tended to over-price bonds the longer the time horizon.

The most extensive comparison of structural models to date is that by Eom, *et al.* (2004), who tested five structural models: Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). Eom, *et al.* (2004) used a sample of 182 US corporate bond prices as at the last trading day in December of each year from 1986 to 1997. Asset value volatility was estimated using historical equity volatility and leverage (150 trading days, although other periods were also tested) and using bond-implied volatility (using the previous month's bond price to derive it). Eom, *et al.* (2004) concluded that:

1. All structural models tested exhibit some prediction accuracy problems.
2. The Merton (1974) and Geske (1977) models tend to underestimate spreads when compared with empirical data.
3. The Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models generate credit spreads that are generally too high compared with empirical data.
4. Incorporating stochastic interest rates and financial distress costs tend to increase the models' inaccuracy, raising a question over the added value generated by using more complex models that incorporate stochastic interest rates.
5. The type of leverage measure incorporated in the models effects the under- or over-estimation of credit spreads. Eom, *et al.* (2004) used total liabilities divided by total liabilities plus equity value as their measure of leverage. However, when using a measure of leverage suggested by KMV (see Crosbie. and Bohn (2003)), which places greater emphasis on short term debt, structural models, not surprisingly, generated lower credit spreads. However, Eom, *et al.* (2004) suggested that by adding a constant to the spreads generated it may be possible to fit the models' predictions on to market data and obtain similar levels of spread predictions but with lower variance.
6. Eom, *et al.* (2004) tested the models using bond-implied asset value volatility. The resulting pricing errors were smaller than those generated using historical equity volatility and leverage. Despite the improved accuracy, Eom, *et al.* (2004) question the wisdom of using bond-implied volatility as it may incorporate not only asset value volatility but also other errors in structural models, especially if bond prices do not change much between estimation periods.
7. The Leland and Toft (1996) model appeared to be an exception when compared with the other models in the study in that it consistently overestimated spreads on most bonds in the study. Eom, *et al.* (2004) argue that this overestimation is not sensitive to parameter estimation, and it is thus an issue within the model specification itself. They argued that the assumption of a continuous coupon is

at the root of the problem – because the Leland and Toft (1996) model uses leverage and coupon to define risk, rather than asset value volatility.

Qi, Liu and Wu (2004) sought to address some of the criticisms levelled at the Leland and Toft (1996) model by Eom, *et al.* (2004) by implementing a calibrated version of the Leland and Toft (1996) model and testing it against average spreads for AAA through to B rated bonds based on empirical data from Huang and Huang (2003). Qi, *et al.* (2004) sought to calibrate the unobserved variables of the model so that as many of the model-generated variables match historical data. The model-generated variables on which Qi, *et al.* (2004) focused were default probability, equity risk premium, and recovery ratio. The calibrated model used by Qi, *et al.* (2004) generated credit spreads in the range of 15 – 22 basis points for AAA rated, bonds and 52 – 68 basis points for BBB rated bonds. Unlike the results obtained by Eom, *et al.* (2004), the calibrated Leland and Toft model did not generate excessive spreads for any bond category. Despite the calibration, Qi, *et al.* (2004) note that the Leland and Toft (1996) model underestimated credit spreads vis-à-vis historically observed spreads for all bond rating categories.

From the empirical studies described above, it appears that structural models' credit spread estimates contain a range of prediction errors, and that in general, structural models tend to underestimate credit spreads, especially for shorter maturities when compared with observed credit spreads. Sarig and Warga (1989) found that the term structure of credit spreads generated by a structural model is similar to that observed in the market, but Helwege and Turner (1999) found evidence that contradicts those findings for sub-investment grade bonds. Most studies described in this section found that the explanatory power of structural models is significant. The empirical studies of structural models described above also highlight the importance of parameter estimation – as more sophisticated methods for estimating model inputs are utilised, so it is possible to assess the modes' accuracy better and to try to improve it. These findings may support the view that the theory that underlies structural models is responsible for explaining most of the elements that affect default probability, hence the significant explanatory power of structural models. The findings also seem to support the view that market observed credit spreads are influenced by additional factors which are not incorporated in structural models. Elton, Gruber, Agrawal and Mann (2001) sought to examine the factors that influence credit spreads and found that spreads can be almost entirely explained by the loss due to expected default, state and local taxes which are payable (in certain jurisdictions) on corporate bonds but not on Government bonds, and a premium required to compensate for bearing the systematic risk of holding a corporate bond. However, Elton, *et al.* (2001) found that for a 10 year single A-rated corporate bond, expected loss on default accounted for only 17.8% of the spread, while taxes accounted for 36.1%, and a further 46.17% of the spread is explained by other factors, mostly the compensation for bearing systematic risk of holding a corporate rather than a Government bond. Collin-Dufresne, Goldstein and Martin (2001) examined the changes in credit spreads of individual bond yields and found that the factors used by structural models explain only around a quarter of the spread variation, and that the residuals in the regression analysis of these factors are highly correlated. Collin-Dufresne, *et al.* (2001) were unable to explain satisfactorily the systematic factor these results suggested by any other financial or economic variables. It also appeared from the results that macro-factors and external shocks of supply and demand for bonds are more important in determining credit spread changes than firm-specific variables.

All the empirical studies described above encountered two common problems:

1. data collection for bond pricing is difficult due to the nature of bond trading in the secondary market; and
2. parameter estimation, especially the unobservable inputs, which are seen as key to the accuracy of the models' output.

The challenge of accurate bond pricing data has improved over time to the efforts of regulators to improve transparency and oversight of the bond markets (e.g. TRACE in the United States) and the desire of data providers such as Bloomberg and Reuters to improve their services to end-users by providing bond price histories.

The challenge of parameter estimation remains a focus for practical implementation of structural models and for reducing the price prediction errors. The earliest method used by Jones, *et al.* (1983) proved to be the most popular but also led to significant pricing estimation errors. More recent studies such as those by Ericsson and Reneby (2004) and Qi, *et al.* (2004) achieved smaller pricing errors by using alternative parameter estimation methods and calibrating model inputs to observed data.

There is clearly scope for further empirical research as well as theoretical development of structural models, including improved estimation of unobservable inputs. There may also be scope for altering the focus of structural models from trying to predict credit spreads with accuracy to being part of a larger model incorporating additional factors.

2.2. Reduced Form Models

Reduced form or intensity-based models were developed as an alternative to structural models. Reduced form models do not attempt to model the behaviour of the firm's assets or predict when it is more economically beneficial for shareholders to put their company into liquidation rather than repay its debts. Instead, such models assume that default occurs in an unpredictable manner, and is determined exogenously. The most notable contributions to the development of these models were made by Jarrow and Turnbull (1995), Jarrow, *et al.* (1995), Das and Tufano (1996) and Duffie and Singleton (1999). The key inputs required for reduced form models are interest rates, hazard rates (the intensity function of default) and recovery rates. Reduced form models deal only with the firm's debt and their intensity functions are inferred from market prices or from historical observations; they are not, however, grounded in economic theory, nor do they attempt to explain the interplay between shareholders and creditors. This section provides a brief overview of the key elements used in reduced form models, beginning with the approach developed by Jarrow and Turnbull (1995), followed by enhancements to the reduced form model proposed by Jarrow, *et al.* (1995) and Das and Tufano (1996). It ends with an attempt to reconcile reduced form models and structural models proposed by Jarrow and Protter (2004).

Jarrow and Turnbull (1995) argued that there are practical obstacles to implementation of structural models because the assets underlying financial securities are often not tradable and thus their value and volatility cannot be observed. Furthermore, when applying a Merton (1974) type model to firms with complex capital structures, all the financial liabilities of different seniority levels and different maturities need to be valued simultaneously, introducing significant computational difficulties.

Jarrow and Turnbull (1995) built on the foreign currency analogy developed by Jarrow and Turnbull (1991) in which the stochastic term structure or risk-free interest rates and the stochastic credit risk spreads are taken as given. The framework for developing the Jarrow and Turnbull (1995) credit pricing model comprises the following:

1. A frictionless economy with trading horizon $[0, \tau]$. The time 0 to τ can be discrete or continuous.
2. A class of default-free zero coupon bonds. $P_0(t,T)$ denotes the t-time dollar value of a zero coupon bond maturing at time T. It is assumed that these zero coupon bonds have strictly positive values and that they are default free.

3. Given the existence of risk-free zero coupon bonds, it is possible to construct a money-market account by investing a dollar in the shortest maturity zero coupon bond and rolling the investment over at maturity. The value of such a money-market account at time *t* is denoted by *B(t)*.

4. A class of risky zero coupon bonds that are subject to default risk. These bonds, denoted *XYZ*, have a time *t* dollar value of $v_1(t,T)$.

Using the foreign exchange analogy, Jarrow and Turnbull (1995) decomposed the XYZ bonds into two elements:

- A zero coupon bond denominated in a hypothetical, non-dollar currency called XYZ;
- A price in dollars of XYZ.

Therefore it is possible to define:

Eq. 2.65 $$e_1(t) \equiv v_1(t,t)$$

where $e_1(t)$ represents the time *t* dollar value of one promised XYZ currency delivered immediately – the spot exchange rate of dollar/XYZ. If XYZ bond is not in default, the exchange rate equals 1 as the XYZ bonds promise to pay one dollar at maturity. If XYZ is in default, the exchange rate might be less than 1, as the payout to bondholders could be reduced. In this approach, the risk of default (i.e. the risk of being paid less than promised at maturity) is represented by the exchange rate $e_1(t)$.

It is then possible to represent the risky-zero coupon bonds in XYZ currency as:

Eq. 2.66 $$p_1(t,T) \equiv {v_1(t,T)}/{e_1(t)}$$

In other words, the promised dollar value of XYZ bonds divided by the exchange rate of dollar/XYZ currency. In XYZ currency, this class of bonds cannot default, as it will always payout in XYZ currency units. The risk to bondholders is that the conversion of XYZ currency units into dollars will result in a loss in dollar terms, if the conversion ratio is less than 1 due to default. Therefore, the dollar value of XYZ bonds can be represented by:

Eq. 2.67 $$v_1(t,T) = p_1(t,T) \times e_1(t)$$

In order to explain the reduced form model, it is worthwhile to begin with a two period discrete model, which can later be generalised to continuous time.

It is assumed that the risk-free bond price process is governed by spot interest rate only. The one period spot rate of interest at time $t=0$ is denoted as

Eq. 2.68 $$r(0) = \frac{1}{p_0(0,1)}$$

It is further assumed that in period 1, interest rates can move either up or down:

Eq. 2.69a $$r(1)_u = \frac{1}{p_0(1,2)_u}$$

Eq. 2.69b $$r(1)_d = \frac{1}{p_0(1,2)_d}$$

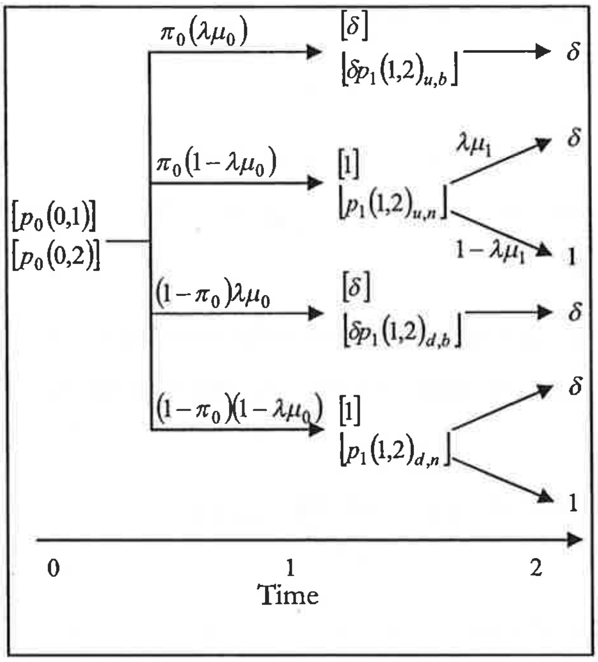
The moves up or down are determined by the risk-neutral probability μ_0 and $(1-\mu_0)$.

The value of $p_0(0,1)$ at maturity equals 1 in both up and down scenarios, as the interest rate during period 0 is known. The value of $p_0(0,2)$ is a function of $r(0)$ and $r(1)$ and the probabilities of an up or down move in interest rates. At the end of period 2, $p_0(0,2)$ must also equal 1.

In the case of risky bonds, two states are assumed possible: default and non-default. continuing with the foreign exchange analogy, the spot exchange rate at time 0 is $e(0) = 1$, but at future time periods 1 and 2 $e(t)$ can be either 1 if no default occurred, or δ, a fixed payoff in default. It should be noted that the state of default is absorbing, i.e. if default occurred in time period 1, the bonds remain in default in time 2. The probability of default in time 1 is denoted as $\lambda\mu_0$ and in time 2 as $\lambda\mu_1$.

The XYZ bond can be affected by a move up or down in interest rates and by the probability of default. However, as mentioned above, in XYZ currency, the bonds always pay the promised amount; the risk of a loss given default is represented by the exchange rate or pay-out ratio. Combining the possible states of the XYZ bonds and the pay-out ratio, the value of XYZ bond in dollar units is represented in Figure 2.11 below:

Figure 2.11: Risky bond payout scenarios over two time periods



As can be seen from the diagram above, the price of a risky bond is a function of the payout ratio, interest rates and default probability. The assumptions of no arbitrage opportunities, complete markets and discrete time economy mean that the probabilities governing the changes in interest rates and default are unique and that the current value of traded securities equals their expected value. Jarrow and Turnbull (1995) show that the value of a risky zero coupon bond can thus be expressed as:

Eq. 2.70 $$\tilde{E}_1(e_1(2)) = \begin{cases} \delta \\ \lambda\mu_1\delta + (1-\lambda\mu_1) \end{cases}$$

Where $\tilde{E}_t(.)$ denotes the time t conditional expected value under risk-neutral probabilities, δ denotes the payout ratio if default occurred, and $\lambda\mu_1\delta + (1-\lambda\mu_1)$ denotes the weighted average payouts in default and in non-default. In other words, at time 1, the expected payout in time 2 is either the payout at default or the weighted average payout of default and non-default. The expected payout in time 2, as viewed from time 0 is given by:

Eq. 2.71 $$\tilde{E}_0(e_1(2)) = \lambda\mu_0\delta + (1-\lambda\mu_0)\times[\lambda\mu_1\delta + (1-\lambda\mu_1)]$$

The expression above shows that the expected payout in time 2, viewed from time 0 is equal to the probability adjusted payment δ at time 1 if default occurs, and the probability of not going bankrupt at time1 multiplied by the expected payout at time 2. The value of a risky zero coupon bond can be more generally expressed as:

Eq. 2.72 $$v_1(t,T) = p_0(t,T) \times \widetilde{E}_1(e_1(T))$$

The value of a risky zero coupon bond is its expected payout at time T, denoted as $\widetilde{E}_1(e_1(T))$, discounted to the present by the risky-free zero coupon bond price denoted as $p_0(t,T)$.

Having developed their model in discrete time, Jarrow and Turnbull (1995) showed it can be generalised in continuous time environment. The value of risky zero coupon bonds in continuous time can be expressed as:

Eq. 2.73 $$v_1(t,T) = p_0(t,T) \times \widetilde{E}_1(e_1(T)) = \begin{cases} \left(e^{\lambda_1\mu_1(T-t)} + \delta_1\left(1 - e^{\lambda_1\mu_1(T-t)}\right)\right) \times p_0(t,T) \\ \delta_1 p_0(t,T) \end{cases}$$

The continuous time expression for the value of a risky zero coupon bond is very similar to the one used in discrete time. If default occurs, the bonds are worth δ_1 discounted by the risk-free rate $p_0(t,T)$; if default does not occur, the value of a risky bond equals the weighted average of the payouts in default and in non-default, both discounted to the present by the risk-free rate $p_0(t,T)$.

In the continuous time expression, the parameters needed in order to compute the value of a risky zero coupon bond are δ_1 and $\lambda_1\mu_1$. The payout ratio process δ_1 is assumed to equal 1 until bankruptcy, and then to equal some other value which is less than 1. In Jarrow and Turnbull (1995) the payout ratio is assumed to be constant, although they argue that this assumption can be relaxed at the cost of added computational complexity which may not be offset by the additional accuracy. The excess return expected in order to account for the risk of default is denoted by $\lambda_1\mu_1$. This denotes the Poisson bankruptcy process where μ_1 is assumed to be a positive constant, independent of the spot interest rate process.

Jarrow, *et al.* (1995) built on the model developed by Jarrow and Turnbull (1995) by specifying the bankruptcy process as a finite-state Markov process in the firm's credit ratings. The assumptions used by Jarrow, *et al.* (1995) are the same as in Jarrow and Turnbull (1995). The building blocks of the Jarrow, *et al.* (1995) model are specified in the discrete time case. $B(t)$ denotes a money market account with accumulative returns at the spot interest rate $r(i)$:

Eq. 2.74 $$B(t) = e\left(\sum_{i=0}^{t-1} r(i)\right)$$

A risk-free zero coupon bond is denoted as *p(t,T)* and is expressed as the expected discounted value of a dollar received with certainty at time *T*:

Eq. 2.75 $$p(t,T) = \widetilde{E}_t\left(\frac{B(t)}{B(T)}\right)$$

A risky zero coupon bond is denoted as *v(t,T),* and is expressed as the expected, discounted value of a risky dollar received at time *T*:

Eq. 2.76 $$v(t,T) = \widetilde{E}_t\left(\frac{B(t)}{B(T)} \times \left[\delta I_{\{\tau \le T\}} + I_{\{\tau > T\}}\right]\right)$$

Where:

δ denotes the recovery value in default;

τ denotes the time of default;

$I_{\{\tau \le T\}}$ denotes an indicator function taking the value of 1 if default occurs prior to maturity, or zero at all other times.

$I_{\{\tau > T\}}$ denotes an indicator function taking the value of 1 if default occurs after the bonds mature, or zero at all other times.

Given the assumption of a set of unique probabilities $\widetilde{Q}$ under which the expected values of all securities are equal to their current values, the value of a risky zero coupon bond can be written as:

Eq. 2.77 $$v(t,T) = p(t,T) \times \left(\delta + (1-\delta)\widetilde{Q}_t(\tau > T)\right)$$

Where $\widetilde{Q}_t(\tau > T)$ is the survival probability, i.e. the probability that default occurs after maturity.

In the discrete time case, Jarrow, *et al.* (1995) modeled the distribution of time to default using a finite state space S = (1…K), representing the possible credit classes, with 1 denoting the highest credit quality class and K representing default. Using this approach, the set of actual probabilities Q can be denoted as a KxK transition matrix:

$$\text{Eq. 2.78} \qquad Q = \begin{pmatrix} q_{1,1} & q_{1,2} & q_{1,3} & \cdots & q_{1,K} \\ q_{2,1} & q_{2,2} & q_{2,3} & \cdots & q_{2,K} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{K-1,1} & q_{K-1,2} & q_{K-1,3} & \cdots & q_{K-1,K} \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

Where $q_{i,j}$ denotes the actual probability of moving from state i to state j in one time interval. The state of bankruptcy is assumed to be absorbing, hence the last row in matrix Q is zero for all states representing a move from bankruptcy to a better class of credit and 1 for the probability of moving from the state of bankruptcy in one period to a state of bankruptcy in the next period.

In complete markets, no-arbitrage discrete time environment, Jarrow, *et al.* (1995) argued that the transition matrix from time t to time $t+1$ is represented by:

$$\text{Eq. 2.79} \qquad \widetilde{Q}_{t,t+1} = \begin{pmatrix} \widetilde{q}_{1,1}(t,t+1) & \widetilde{q}_{1,2}(t,t+1) & \widetilde{q}_{1,3}(t,t+1) & \cdots & \widetilde{q}_{1,K}(t,t+1) \\ \widetilde{q}_{2,1}(t,t+1) & \widetilde{q}_{2,2}(t,t+1) & \widetilde{q}_{2,3}(t,t+1) & \cdots & \widetilde{q}_{2,K}(t,t+1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{q}_{K-1,1}(t,t+1) & \widetilde{q}_{K-1,2}(t,t+1) & \widetilde{q}_{K-1,3}(t,t+1) & \cdots & \widetilde{q}_{K-1,K}(t,t+!) \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

Where $\widetilde{q}_{i,j}(t,t+1) = \pi_i(t) q_{i,j}$ and $\pi_i(t)$ is a function of time representing a risk premium that transforms the actual probability of default into a risk-neutral default probability. The discrete time model developed by Jarrow, *et al.* (1995) has also been expanded to the continuous time case. However expanding on this approach is beyond the scope of this thesis, and the interested reader is referred to the original paper by Jarrow, *et al.* (1995).

As in structural models, reduced form models also require a set of inputs. Jarrow, *et al.* (1995) suggested the following methods for estimating the parameters required for their model:

- Stochastic risk-free forward rates can be generated using models such as those developed by Heath, Jarrow and Morton (1992);
- Recovery rate in default can be taken from empirical studies such as those published by Moody's Investor Service and other rating agencies;
- The credit rating transition matrices can also be taken from publications by Moody's Investor Service or other rating agencies. However, transition matrices based on historical data provide actual rather than risk-neutral transition probabilities. In order to adjust historical transition probabilities, Jarrow, *et al.*

(1995) suggest estimating the risk premiums implied by bonds traded in the market.

Das and Tufano (1996) built on the model proposed by Jarrow, *et al.* (1995) by allowing the recovery rate in default to vary. One of the limitations of the model developed by Jarrow, *et al.* (1995) is that by keeping the recovery rate constant, the only variability in credit spreads is driven by changes to the bond's credit ratings. Das and Tufano (1996) argued that in the debt markets, credit spreads change even if ratings have not.

Das and Tufano (1996) used assumptions similar to those used by Jarrow, *et al.* (1995) regarding the discrete-time, discrete space credit quality transition matrix $\tilde{Q}_{t,T}$. However, in order to incorporate recovery rate uncertainty, they specified the stochastic recovery rate as β as the payout bondholders receive in default. The stochastic recovery rate is a function of time, interest rates and an additional state variable X_2 which, in turn, is correlated with X_1, a state variable governing the probability of default. Das and Tufano (1996) expressed the stochastic recover rate process as:

Eq. 2.80 $$\beta_{t,T} = \left(1 + \frac{1-\beta_t}{\beta_t} e^{\sigma_\beta X_2 \sqrt{T-t}}\right)^{-1}$$

Combining the transition matrix $\tilde{Q}_{t,T}$, the risk-free zero coupon bond $B(T)$ and the recovery rate $\beta_{t,T}$, Das and Tufano (1996) expressed the value of a risky zero coupon bond as:

Eq. 2.81 $$P(r,X,T) = B(r,T) - [1 - \beta(r,X,T)]Q(r,X,T) \times B(r,T)$$

The price of a credit-sensitive bond is thus expressed as the value of a risk-free zero coupon bond less an adjustment for credit risk. The adjustment for credit risk comprises stochastic recovery rates and default probability, both of which are functions of interest rates, time to maturity and a state variable X which expresses the riskiness of the debt.

The model developed by Das and Tufano (1996) incorporates three types of risk: interest rate risk, default risk and recovery risk. In the Das and Tufano model recovery payment after default is assumed to happen as near default as possible, whereas in Jarrow, *et al.* (1995) the recovery payout is assumed to be received at the bond's maturity date. However, both models were developed in order to price zero-coupon bonds, and are thus not directly applicable to coupon paying bonds. Additionally, Das and Tufano (1996) developed their model in discrete time only, and implementing it in practice requires the development of binomial trees to represent the path of interest rates and recovery rates.

Das and Tufano (1996) provided examples of how their model can be used to price credit sensitive instruments. The parameters used were taken from a range of historical sources including studies by Altman (1992) and rating transition matrices and risk-premiums published by rating agencies. As an alternative to using historical data, Das and Tufano (1996) suggested that the parameters required for their model can be inferred from market prices. The proposed methods for estimating the inputs required for reduced form models highlight one of their key weaknesses: using historical data for ratings transitions and recovery rates does not necessarily result in risk-neutral default probabilities and reduces the various states that a bond can move into to those covered by the rating scale used; in order to derive parameters such as implied rating transition probability and implied payout in default, a significant amount of market data is needed (e.g. a whole series of bonds with different maturities for every step in the rating scale) – such data is not widely available and where it can be found, it may contain "noise" such as liquidity premiums.

The development of reduced form models generated debate among academics regarding which approach is better – reduced form or structural models (e.g. Jarrow, van Deventer and Wang (2003). However, Jarrow and Protter (2004) proposed a different view, arguing instead that structural models and reduced form models are not as dissimilar as usually thought. Jarrow and Protter (2004) argue that the key difference between the two types of models is the assumption of available information:

- Structural models assume that the information available to the firm's managers is known to all, thus the firm's default time is predictable.
- Reduced form models assume that information available to the market is not as detailed as that available to the firm's managers, thus default cannot be predicted and occurs as a surprise.

It is worthwhile noting, however, that the main criticism of structural models by Jarrow and Protter (2004) is that the firm's asset value is unobservable, and that such models require continuous observation of the firm's asset value – both of these are not available to the market. This weakness of structural models has been flagged by the early developers of reduced form models. Jarrow and Protter (2004) claim that reduced form models are better when the purpose of using the model is risk management, pricing and hedging, because the parameters of reduced form models are inferred from the markets, which they assume are at equilibrium. The weaknesses of reduced form models are overlooked by Jarrow and Protter (2004):

- No foundation in economic and financial theory;
- Assumption that market prices are available for a range of instruments and maturities in order to infer market implied risk premiums, intensity functions and recovery rates;
- The reduction of an infinite number of possible outcomes into a limited number of possible credit states;
- No expressed view regarding the firm's equity and total debt value, only focusing on a single bond issue;
- Most reduced form models deal with zero-coupon bonds only, which in the real world account for a small portion of total bonds outstanding, making the inference of market observed risk parameters even more difficult.

Despite these weaknesses, some commercial applications have been developed using techniques based on reduced form methodology. The most notable of these are CreditMetrics™ which was developed by Gupton, Finger and Bhatia (1997) at J. P. Mogan and CreditRisk+ which was originally developed by Credit Suisse Financial Products (see Gundlach and Lehrbass (2004)). Both applications are concerned with measuring the risks of a portfolio of risky credit instruments and use credit quality transition matrices to help determine the fair value of an individual risky instrument within the portfolio..

This section provided a brief overview of reduced form models. Reduced form models are relatively easy to implement and do not seek to explain how or why a firm defaults – these advantages made the approach popular with practitioners. However, a key weakness of the reduced form approach is that it is not grounded in economic or financial theory, and it does not seek to explain why default happens.

3. Hypotheses Development

The literature review in the previous section highlighted the strengths and weaknesses of structural credit models when tested empirically and when compared with the alternative approach presented by reduced form models. One of the key strengths of structural models is their grounding in the theory of finance. Such models seek to derive an arbitrage-free price of risky debt securities from the capital structure of the firm. A key weakness of such models is the number of variables that need to be estimated in order to obtain a result. Whilst many of the variables are observable, two key variables are unobservable: the asset value V and the volatility of asset returns σ_V.

The concept of volatility is at the heart of many financial models, as they seek to describe the distribution of a certain variable (e.g. stock price, interest rate, bond price, etc). Neftci (2000) highlighted that one of the ways in which distribution models can be classified is using the notion of "moments". Normally distributed random variables can be expressed in terms of their first two moments:

1. The first moment is the expected value of the random variable, also known s the mean; and
2. The second moment is the variance of that variable around its mean.

The first moment of a random variable can be thought of as its average – the value it tends towards, whilst the second moment describes the way the variable's distribution is spread out. The square root of the second moment is known as the standard deviation of the variable and it measures the average deviation of observations from the mean. This standard deviation is referred to as volatility.

The role of volatility in capital structure theory is to introduce an element of risk. As highlighted by Leland (1994) and Leland and Toft (1996), asset value volatility can lead to an optimal capital structure for a company, as it affects the values of the firm's equity and debt. The greater the volatility, the more widely distributed are the asset values. Increased volatility of asset values increases the probability that asset value will cross the default threshold and trigger bankruptcy, thus debt values will tend to decrease when volatility increases. The value of equity, on the other hand, tends to rise, all other things being equal, when volatility increases, as equity is viewed as a call option on the firm's assets, and that call's probability of being "in-the-money" increases with volatility. The trade-off between higher bankruptcy probability and increased likelihood of the equity being "in-the-money" is the driver of capital structure optimisation in the Leland (1994) approach.

Despite the importance of asset value V and the volatility of asset returns σ_V, the literature of structural models deals with the complexity of estimating such variables by assuming that they are constant and known. Listed below are a few examples from the seminal articles in the field of structural models regarding the assumptions about σ_V and V:

"The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is log normal. The variance rate of the return on the stock is constant." (Black and Scholes (1973), p.640).

Merton (1974) derived his version of the structural model by assuming that the instantaneous standard deviation of return on the firm σ_V is related to σ_y the instantaneous standard deviation of a security F whose value is a function of the firm's value V and time T through the equation: $(\alpha - r)/\sigma_V = (\alpha_y - r)/\sigma_y$, where a and a_y denote the expected rate of return on the firm and the security respectively and r denotes the risk-free rate. If σ_y is constant, as in Black and Scholes (1973), then σ_V must be constant too. Furthermore, V was assumed to follow a continuous diffusion process with constant expected payouts, interest rates and volatility of returns.

Geske (1977) assumed that investors agree about the variance of changes in the value of the firm σ_V and it is expressed as a constant, not as a time dependent variable.

Longstaff and Schwartz (1995) assume that σ_V is constant and that V follows a continuous diffusion process with constant drift and constant volatility.

Leland and Toft (1996) assumed that "As in Merton (1974), Black and Cox (1976) and Brennan and Schwartz (1978), the firm has productive assets whose unleveraged value V follows a continuous diffusion process with constant proportional volatility σ" (Leland and Toft (1996) p. 989)

Collin-Dufresne and Goldstein (2001) also assumed that the risk-free rate, the firm's payout rate and asset value volatility are all constant and that V follows a geometric Brownian motion.

Empirical tests of structural models also assumed that σ_V is constant and employed various methods for estimating V:

Jones, *et al.* (1984) assumed the σ_V is constant and estimated V as the sum of a firm's market capitalization, the market value of traded debt and the estimated market value of non-traded debt. σ_V was estimated using a monthly time series for the value of the firm over a 24 months period and by using the relationship $\sigma_E = \sigma_V \frac{\partial E}{\partial V} \Big/ E$ where σ_E denotes the standard deviation of the firm's equity E.

Ericsson and Reneby (2002) used a maximum-likelihood approach developed by Duan (1994) to estimate the unobservable inputs required for solving structural models, using as inputs equity prices and bond prices. Although they noted some cross-sectional variation when estimating σ_V, Ericsson and Reneby (2002) did not attempt to characterize asset value and asset value volatility. In a later study, Ericsson and Reneby (2004), using the same method for estimating the unobservable parameters of structural models, showed that such models pricing errors compare well with those achieved by reduced form models. This result could indicate that some of the pricing errors observed by earlier empirical studies of structural models are due to parameter estimation difficulties.

Based on the assumptions used when theoretical versions of the structural model were developed and the pricing errors and findings of empirical tests of such structural models, it appears that the assumptions and estimates of unobservable inputs is of great importance to the accuracy of such models, which in turn leads to accurate debt and equity valuations and to capital structure optimization. A two-part research question arises from the observation that current estimation of unobservable inputs results in inaccurate debt pricing:

1. Is there a difference between the values of V and σ_V calculated using equity price and volatility? and
2. Is the assumption that σ_V is constant over time correct?

From this research question, four hypotheses were developed in order to identify the characteristics of the unobservable inputs V and σ_V:

Hypothesis 1: Bond-implied asset values are the same as equity-implied asset values.

H_0: Asset values calculated using equity prices and accounting values are the same as asset values implied by structural models using bond prices.

H_A: Asset values implied by bond prices using structural models are different to those calculated using equity prices and accounting values.

Hypothesis 2: Bond-implied asset value volatility is the same as equity-derived asset value volatility.

H_0: Asset value volatility calculated using equity prices is the same as asset value volatility implied by structural models using bond prices.

H_A: Asset value volatility calculated using equity price inputs is different to that implied by bond prices.

Hypothesis 3: Bond-implied asset value volatility is stable over time.

H_0: Asset value volatility is stable over time.

H_A: Asset value volatility is unstable over time.

Hypothesis 4: The bond-implied asset value volatility is not correlated with the bond's time to maturity.

H_0: The correlation between bond-implied asset value volatility and bond time to maturity equals zero.

H_A: The correlation between bond-implied asset value volatility and bond time to maturity does not equal zero.

The first three hypotheses were developed to answer the research question directly, whilst the fourth hypothesis aims to test whether there is a relationship between bonds' maturity and bond implied asset volatility, as this could affect the conclusions drawn from the third hypothesis.

4. Research Methodology

4.1. Introduction

This section describes the research methodology undertaken in conducting this research. The study deals with financial data observed and derived from financial markets, and is grounded in the positivistic school of research philosophy. The characteristics of positivistic research, as identified by Easterby-Smith, Thorpe and Lowe (2003), include: independent observers, progress through hypotheses and deductions, measurable concepts and simple units of analysis. A positivistic research, according to Easterby-Smith, *et al.* (2003), is generalised through statistical probabilities and the samples used are large and random. This study sets out four hypotheses that are tested using statistical methods; no interference has been made when data was collected, and the sample used is considered large by the standards set out in McClave, Benson and Sincich (2001). The units of analysis are asset value and asset value volatility – both are quantifiable. The research subject is on the dynamics of financial instruments over time. Based on the characteristics of research identified by Easterby-Smith, *et al.* (2003) a longitudinal empirical quantitative methodology appears to be most suited for the study undertaken.

The rest of this section provides details on the following aspects of the research: methodology, research model, population and sample, data collection, model inputs, model implementation and data derivation.

4.2. Methodology

The research's aims were to observe the dynamics of asset value V and asset value volatility σ_V in the context of structural credit models. Theoretical versions of structural models have been developed since Merton (1974) and although some empirical studies have been undertaken to test their usefulness in terms of predicting bond prices and yields, none of the research conducted to date sought to address the issue of asset value and asset volatility dynamics over time as implied by bond and equity prices.

In order to achieve the research aim, a study of asset value and asset volatility dynamics over time was undertaken by comparing two methods of calculating them:

1) Utilising the theoretical relationship between equity volatility and asset volatility using time series of equity prices as suggested by Jones, *et al.* (1984) and used by Eom, *et al.* (2003)

2) Calculating the implied asset value and volatility from a time series of bond and equity prices by using the Leland and Toft (1996) model.

The structural approach to credit risk attempts to model the behaviour of a company's assets and its leverage over time, seeking to identify the likelihood and timing of hitting a default threshold. Given the element of time, a longitudinal study is called for. Therefore, a time-series data of observed parameters was used in order to estimate the unobserved input parameters needed for such models and observe their dynamics over time. The following section provides further details of the research model.

4.3. Research Model

In order to derive the implied values of V and σ_V from observed bond prices, a system of two equations is needed, so that a set of two equations and two unknowns can be solved. It is therefore necessary to find a version of the structural model that provides closed form solutions to the value of equity and the value of bonds. Whilst Merton (1974) provides closed form solutions to equity and debt and the model is relatively simple to implement, it is only applicable to zero coupon bonds, is most suitable for firms with a simple capital structure and is based on the assumption of strict adherence to the capital structure seniority rules it thus appeared unsuitable for the purpose of this research. Based on the types of structural models that were described in the literature review, the model developed by Leland and Toft (1996) is the most suitable for the purpose of the research presented here.

In the research undertaken, all the parameters except for V and σ_v where either observed or derived from previous studies and the values of V and σ_v were derived from solving a system of equations. The steps taken in order to obtain the system of two equations and two unknowns is shown in Figure 4.1 below:

Figure 4.1: Observable inputs, observed outputs and derived unobservable inputs

Step 1:

Collect observable inputs

Observable inputs:
$r, c, p, C, P, t, T, \alpha, \delta, \tau, \rho$

Step 2:

Calculate the firm's total value, bankruptcy barrier and total value of risky debt as functions of V and σ_v

Firm's total value:
$$v = f_1(r, \tau, C, V, K, \alpha, \sigma_V, \delta)$$

Bankruptcy barrier:
$$K = f_2(r, T, C, P, \alpha, V, \sigma_V, \delta)$$

Value of total debt:
$$D = f_3(r, C, P, V, K, \alpha, \sigma_V, \delta, t)$$

Step 3:

Using the functions for the firm's total value, total debt value and bankruptcy barrier, calculate the value of the firm's risky bond and equity value as functions of V and σ_v

Value of a risky bond:
$$B = f_4(r, c, p, V, K, \rho, \sigma_V, \delta, T)$$

Firm's equity value:
$$E = f_5(v, D)$$

Step 4:

Using observed values for B and E, solve simultaneously the equations for B and E to derive the unobserved values of V and σ_v

The parameters required as inputs in order to solve the equations above are:

r = the risk free interest rate

δ = the cash payout ratio to security holders (both dividends and interest as a proportion of market value).

α = loss given default.

T = total debt time to maturity.

t = individual bond's time to maturity.

P = total principal of all outstanding debt instruments issued by the company.

p = principal outstanding of an individual bond.

C = total coupon payments made by the firm in a year.

c = coupon paid by an individual bond.

$N(.)$ denotes the cumulative standard normal distribution function.

$n(.)$ denotes standard normal density function.

V = the firm's asset value, unobserved.

σ_v = the firm's asset value volatility, unobserved.

Leland and Toft's model is described in technical detail in the literature review. It is worthwhile reviewing the concepts underlying the model in this section before describing the implementation method used. Leland and Toft (1996) assume that the firm's asset value follows a continuous diffusion process that continues undisturbed until asset value falls to a default-trigger value. The default triggering level is determined endogenously and is a function of debt maturity, total debt outstanding, total coupon payable after tax, the risk-free interest rate, asset value volatility and loss given default. It is also assumed that the firm issues a constant amount of debt with constant maturity that replaces maturing debt, so that the overall amount of the firm's debt remains constant over time. If the default-triggering value is reached, bondholders takeover the firm and recover a residual value of its assets. Using this approach, Leland and Toft (1996) model a risky coupon paying bond by adding three components:

1. Discounted expected value of coupon cash flow – the contractual coupon cash flow is adjusted for the probability of non-default prior to maturity.
2. The expected discounted value of the principal repayment, again, adjusted for the probability of non-default prior to maturity.
3. The discounted recovery value in bankruptcy which bondholders expect, adjusted for the probability of default at any time prior to maturity.

The model provides an analytical framework for modelling equity and debt. The inclusion of an endogenously determined bankruptcy barrier and the smooth pasting condition which applies to the option pricing methodology used by Leland and Toft (1996) lead to the following three possible scenarios:

1. When asset value V reaches the default-triggering barrier K i.e. when $V = K$, the additional cash flow needed in order to keep the firm solvent equals the marginal increase in the firm's equity value that will result from making that incremental cash flow available. Thus existing shareholders will be prepared to provide this cash themselves or be diluted through a cash injection by new shareholders. The incremental cash flow needed comprises after-tax coupon payments and principal redemption of maturing debt, net of funds raised by issuing new debt in order to keep the firm's total principal outstanding unchanged.
2. As long as $V > K$, the marginal increase in the firm's equity value will be greater than the incremental cash flow needed in order to keep the firm solvent, so default will be avoided.
3. When $V < K$, the marginal increase in equity value will be smaller than the cash flow needed in order to sustain the firm as a going concern, and it will default.

The smooth pasting condition ensures that the firm's equity is greater than or equal to zero; it cannot be negative.

The endogenously determined bankruptcy barrier is a function of, among other things, debt maturity. In the Leland and Toft model, firms that have long term debt structures will tend to have bankruptcy triggers that are loess than their debt principal, and thus may continue to operate even when their net worth is negative. When debt structure is short term and bankruptcy costs are greater than zero, default can occur even if net worth is positive, because the anticipated equity appreciation does not justify additional contribution for debt servicing. Thus in the Leland and Toft (1996) model default occurs when cash flow is insufficient to service or repay debt.

The Leland and Toft (1996) model provides closed form solutions to the valuation of the firm's total value, a single risky bond, the firm's total debt and the firm's equity value. The firm's equity value is a function of the firm's total value less the value of its debt. Common to all the functions in the Leland and Toft model is the requirement to specify values for the firm's asset value and asset value volatility, two unobservable inputs. However, by observing the market price of a single bond and the equity value of the firm, it is possible to find values for asset value V and asset value volatility σ_v that would satisfy the equations for the firm's bond price and its equity value simultaneously. The system of equations summarized in Figure 4.1 above is effectively a set of two equations and two unknowns. The two equations are for value of a company's bond B and the value of the company's equity E. The two unknowns are V and σ_V. It is worthwhile noting that solving the system of equations in Figure 4.1 above also yields the firm's total value v and the endogenously determined default barrier K. These are obtained as 'by-products' of the solution to the system of equations used to find V and σ_V and although they are not the focus of this research, can be used as the basis for further research into the characteristics of the Leland and Toft (1996) model and structural models in general.

It is worth noting that not all systems of two equations and two unknown yield a unique solution – some can have multiple solutions. When the two equations are linear, the point at which they intersect is unique, thus there is only one set of variables that will solve simultaneously both equations. When the equations are not linear, the problem of multiple solutions becomes more apparent:

- If the two functions are monotone with respect to both variables, then each function will have only one unique solution.
- If one or both functions are non-monotone but simple non-linear equations, a common approach, described, for example, by Courant and Robbins (1996), involves subtracting one equation from the other, obtaining an expression for one variable in terms of the other, substituting into one of the original equations and solving one equation with one unknown, then deciding based on the characteristics of the variables whether to accept only one solution or more than one.
- For more complex formulae Courant and Robbins (1996) suggest that graphical approaches may be used, to identify which solution is the one sought – this could be based on certain boundary conditions or limits, such as no negative solutions.

Although both equity and debt valuations in the Leland and Toft (1996) model are monotone increasing functions of asset value V, all other things being equal, the effect of volatility is not always the same:

- Debt value falls when σ_V increases for low – and medium leverage firms. For very high leverage firms, debt value increases when σ_V increases because as σ_V increases, K decreases and the firm's survival probability increases.
- Unlike in the Merton (1974) framework, where equity is viewed as a European call whose value increases with volatility (all other things being equal), the Leland and Toft (1996) equity valuation formula assumes that default can happen at any

time and that the option's strike value (the bankruptcy barrier) varies with the firm's volatility. Leland and Toft (1996) show that for short and intermediate term debt, increased volatility benefits neither debt holders nor equity holders, and that increased volatility will lead to higher equity valuation only for very long term debt or when bankruptcy is near. Equity holders benefit before bondholders from increasing risk as bankruptcy nears

From these observations it is reasonable to conclude that for firms that are not near bankruptcy and where the debt maturity is short or intermediate, debt value is a monotone decreasing function of σ_v and equity value is a monotone increasing function of σ_v. It follows that using a system of two equations and two unknowns is likely to result in a unique solution under these conditions.

Leland and Toft (1996) provide closed form solutions to the values of the firm's equity, total debt and individual bonds. The formulae for each of these values are provided below.

The value of a risky bond is given by:

Eq. 4.1 $$B = \frac{c}{r} + e^{-rt}\left[p - \frac{c}{r}\right] \times [1 - F(t)] + \left[\rho K - \frac{c}{r}\right] \times G(t)$$

The value of the company's total debt is given by:

Eq. 4.2 $$D = \frac{C}{r} + \left[P - \frac{C}{r}\right] \times \left[\frac{1 - e^{-rT}}{rT} - I(T)\right] + \left[(1 - \alpha)K - \frac{C}{r}\right] \times J(T)$$

The total value of the firm is given by:

Eq. 4.3 $$v = V + A_1 V + A_2 V^{-(a+z)} - \alpha K\left(\frac{V}{K}\right)^{-(a+z)}$$

The value of the firm's equity is given by:

Eq. 4.4 $$E = v - D$$

The formulae needed in order to solve equations 4.1 – 4.4 are:

Eq. 4.5 $$F(t) = N[h_1(t)] + \left(\frac{V}{K}\right)^{-2a} \times N[h_2(t)]$$

Eq. 4.6 $$G(t) = \left(\frac{V}{K}\right)^{-a+z} \times N[q_1(t)] + \left(\frac{V}{K}\right)^{-a-z} \times N[q_2(t)]$$

where:

$$a = \frac{r - \delta - \sigma_V^2/2}{\sigma_V^2} \qquad b = \ln\left(\frac{V}{K}\right) \qquad z = \frac{\left[\left(a\sigma_V^2\right)^2 + 2r\sigma_V^2\right]^{0.5}}{\sigma_V^2}$$

$$h_1(t) = \frac{\left(-b - a\sigma_V^2 t\right)}{\sigma_V \sqrt{t}} \qquad h_2(t) = \frac{\left(-b + a\sigma_V^2 t\right)}{\sigma_V \sqrt{t}}$$

$$q_1(t) = \frac{\left(-b - az_V^2 t\right)}{\sigma_V \sqrt{t}} \qquad q_2(t) = \frac{\left(-b + z\sigma_V^2 t\right)}{\sigma_V \sqrt{t}}$$

Eq. 4.7 $$I(T) = \frac{G(T) - e^{-rT}F(T)}{rT}$$

Eq. 4.8 $$J(T) = \frac{1}{z\sigma_V \sqrt{T}} \times \left[-\left(\frac{V}{K}\right)^{-a+z} \times N[q_1(T)]q_1(T) + \left(\frac{V}{K}\right)^{-a-z} \times N[q_2(T)]q_2(T) \right]$$

Eq. 4.9 $$A_1 = \frac{\tau C}{r} \times \frac{a+z}{a+z+1} \times \frac{1}{V_T}$$

Eq. 4.10 $$A_2 = -\frac{\tau C}{r} \times \frac{a+z}{a+z+1} \times \frac{K^{a+z+1}}{V_T}$$

Eq. 4.11 $$K = \frac{C/r \times \left(\frac{A}{rT} - B_1\right) - AP/rT}{1 + (a+z)\left(\tau C/rV_T + \alpha\right) - (1-\alpha)B_1}$$

Eq. 4.12

$$A = 2ae^{-rT}N\left(a\sigma_V\sqrt{T}\right) - 2zN\left(z\sigma_V\sqrt{T}\right) - \frac{2}{\sigma_V\sqrt{T}}n\left(z\sigma_V\sqrt{T}\right) + \frac{2e^{-rT}}{\sigma_V\sqrt{T}}n\left(a\sigma_V\sqrt{T}\right) + z - a$$

Eq. 4.13 $$B_1 = -\left(2z + \frac{2}{z\sigma_V^2 T}\right) \times N\left(z\sigma_V\sqrt{T}\right) - \frac{2}{\sigma_V\sqrt{T}}n\left(z\sigma_V\sqrt{T}\right) + z - a + \frac{1}{z\sigma_V^2 T}$$

The parameters required as inputs in order to solve the equations above have already been described earlier in this section, as well as in the literature review.

The approach used to estimate V and σ_v from equity prices is that suggested by Jones, *et al.* (1984) and used by Eom, *et al.* (2003):

V_t was calculated by adding the firm's market capitalization to its total debt.

σ_{ve} was derived from the equation:

Eq. 4.14 $$\sigma_e = \sigma_v \frac{V_t}{E_t} \frac{\partial E_t}{\partial V_t}$$

Where $$\frac{\partial E_t}{\partial V_t} = N\left(\frac{\ln\left(V_t/D_t\right) + \left(r + \frac{\sigma_e}{2}\right)t}{\sigma_e\sqrt{t}}\right)$$

In order to distinguish between the bond-implied parameters and the equity-derived parameters, asset value and asset value volatility calculated using equity prices have been assigned the subscript *e*.

Eom, *et al.* (2003) estimated σ_{ve} using observed equity volatility 30, 60, 90 and 150 trading days before the observed bond price, and noted that the differences in the average errors among the different measurements were very small. In this study, therefore, equity price volatility is calculated using equity prices during the 150 trading days preceding the bond price observation.

In order to solve the system of equations described in this section, a data set and a range of inputs is required. The population, data set and model inputs are described in the following sections.

4.4. Population and Sample

The research concentrated on bonds issued by Western European companies. By concentrating on bonds issued by companies in one geographical region, it was possible to address a relatively uniform population sample. Although there are differences in accounting standards and reporting practices among European countries, the effects of such differences on the results is likely to be limited as bond prices and equity of a particular company will be traded and valued by the market on the basis of information available for the issuing company. A further reason for the limited effect of accounting differences is that the only data taken from the companies' accounts are total debt outstanding, interest expense, effective tax rate and cash dividend/share buy-back. Focusing on Western Europe as opposed to the whole of Europe was intended to include only developed and mature financial markets.

Within the overall population of European corporate bonds, it is needed to differentiate among different bond types. Some bonds have a conversion option, allowing investors to convert them to shares at a pre-determined share price level, others accrue interest until maturity and are referred to as zero coupon or discount bonds, some bonds contain call options, allowing the issuer to call them at a predetermined premium prior to their contractual maturity, some have put options, allowing investors to put the bonds to the company if certain events happen, some bonds pay a fixed cash coupon, and others pay a fixed margin over a variable money market rate such as LIBOR. In order to create as uniform but as large as possible population sample, this study focuses on straight (i.e. non-callable) cash-paying, non-convertible bonds issued by companies based in Western Europe. By selecting such bonds, bond price distortions that are likely to be caused by conversion and call options were eliminated. To avoid further distortions, only bonds issued by non-Governmental, non-financial, non-utility, corporations were included. Excluding Government-owned entities removes the distortion of State-guarantees; financial firms have very highly leveraged balance sheets and are not normally included in empirical tests of structural models; utilities tend to enjoy natural monopoly status and are highly regulated, thus their bond prices may reflect these externalities.

A further requirement stemming from the research methodology adopted in this study is that the firms issuing the bonds must have shares that are publicly traded; otherwise it will not be possible to compare values of V and σ_v based on equity prices and those derived from bond prices.

Thus the population from which the research sample was taken are publicly quoted companies based in Western Europe that issue cash paying straight bonds and for which a sufficient number of bond prices and equity prices is available.

Given that not all corporate bonds are traded on an exchange, reliable prices for corporate bonds are not always available. This obstacle to empirical research of bond prices was highlighted by Jones, *et al.* (1984), Helwege and Turner (1999, Ogden (1987), Sarig and Warga (1989) and others.

The basic unit of analysis is the values of V and σ_v for companies that are included in the research sample. For the sample to be considered large, it must be, according to McClave, *et al.* (2001), larger than 30. Given that it is proposed to test V and σ_v for each firm and the characteristics of V and σ_v over time and a function of time to maturity, the study required at least thirty bond price points for each individual bond and at least thirty bonds from different firms. In order to observe the characteristics of asset value and asset value volatility over time, this study is based on bonds where daily pricing data was available for three years (approximately 730 data points for each bond), thus more than satisfying the number of data points required by McClave, *et al.* (2001). The following section describes the process by which bond data was collected and filtered in order to match the criteria needed for this study.

4.5. Data Collection

In order to collect the inputs required for the research model from the population identified in the preceding section, the following steps were taken:

Step 1: Bloomberg search for bonds matching the following criteria:

- Coupon Type: Fixed
- Country of issuance: Western Europe (Austria, Belgium, Channel Islands, Cyprus, Denmark, Finland, Greece, Iceland, Ireland, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland)
- Currencies: French Franc, Deutschemark, Italian Lira, Pound Sterling, US Dollar, Austrian Schilling, Belgian Franc, Danish Krone, Euro, Finnish Markka, Greek Drachma, Icelandic Krona, Irish Punt, Luxembourg Franc, Maltese Lira, Dutch Guilder, Norwegian Krone, Portuguese Escudo, Spanish Peseta, Swedish Krona, Swiss Franc.
- Issuer type: Industrial, Telephone, Special Purpose
- Rating: Moody's Investor Service: Aa1 – C; Standard & Poors: AA+ - C

The search was performed on 4th March 2005, and returned 753 bonds

Step 2: retrieving data from Bloomberg to Excel

Every bond on Bloomberg has a unique ISIN number. All the ISIN numbers provided by the search result were keyed into a Bloomberg-linked Excel spreadsheet. This enabled the following data fields to be obtained for each bond:

- Short name: of the entity that issued the bonds.
- Currency: the currency in which the bonds are denominated
- Amount Outstanding, in the currency issued
- Issue Date
- Maturity Date
- Bond Price, as provided by Bloomberg or "#N/A N.A.", the standard Bloomberg database return for bonds that are not priced on Bloomberg.
- Coupon

- Coupon Type: whether fixed, variable or floating. The purpose of this field was to provide a check that all the bonds selected by the filter are indeed fixed coupon bonds.
- Step Coupon: all the bonds returned "#N/A N Ap", indicating that none of them contained step coupons.
- Corporate-related Equity Ticker: every publicly listed company on Bloomberg has a unique ticker. This field returned either the equity ticker of the entity related to the bond issuer or "#N/A N Ap", meaning that no publicly traded entity is related to the bond issuer.
- Notes: a conditional field that returns the string "Private Company" if the corporate-related equity ticker was #N/A N Ap and the string "Public Company" in all other cases.

Step 3: Filtering the data

The data output obtained in the Excel spreadsheet was then filtered and cleaned up as follows:

Excluding all private companies; all companies that had a "Private Company" label were removed from the data set, as one of the requirements of the model used was to have an equity price. This left 574 bonds out of the initial 753 obtained from the Bloomberg data download.

Excluding bonds for which there is no price on Bloomberg; In order to identify the non-priced bonds, a conditional field was added to the filtered data indicating whether the bonds contained a price or not. The field returned "Not Priced" if the Bloomberg pricing data returned "#N/A N.A." or "Priced" for all other cases. This filter reduced the sample to 444 bonds.

Excluding small-sized issues; in order to include only liquid bonds in the data sample, bonds that have principal outstanding smaller than 100,000,000 of issue currency were excluded from the sample. This action reduced the sample to 391 bonds.

Excluding bonds with less than three years' pricing data potential and those with maturity of less than one year; As bonds near their maturity date, trading activity in them tends to decline as holders await redemption, and prices of such bonds may not always reflect market perception of credit risk. In order to focus the data sample on bonds with maturity of more than one year, bonds whose maturity date was prior to 3rd March 2006 were excluded. The need for three years' pricing data meant that bonds issued after 4 March 2002 needed to be excluded from the data set. These steps reduced the sample further to 179 bonds. One exception was allowed – Portugal Telecom, whose bond matured at the end of February 2006. This exception was made because on the one hand the bond's maturity was only days away from the cut-off date and on the other hand, once the sample was filtered, every additional bond that could have been included would have been useful.

Exclude European subsidiaries of non-European companies; As the study is focused on Western European corporate bonds, those bonds whose associated equity ticker had a US suffix or whose shares are traded on an non-Western European exchange were excluded from the sample, thus reducing the sample size to 150 bonds.

Where bonds were issued in two series (144A and Reg S) then the Reg S prices were used and the duplicate series removed from the sample. This is because European investors tend to trade Reg. S bonds. This reduced the sample size to 148.

For every remaining bond, the data contained on Bloomberg was checked to see that the extent of historical pricing data that is available. Only bonds where three years' pricing data was available were included, and where several such bond issues were available for a single company, the largest bond issue was used in order to ensure that the most liquid issue of the company's bonds is included in the study. As a result of this final filtration, the number of bonds that could be used for the study was reduced to 42.

The list of companies and bonds used in the study is shown Figure 4.2 below:

Figure 4.2: Company and Bond List

	Company Name	Industry Sector	Bond credit rating	Principal	Coupon	Maturity
1	Royal Ahold NV	Food retail	Ba2/ BB	€1500m	5.875%	May-08
2	Akzo Nobel NV	Chemicals	A3/A-	DM1000m	5.375%	Nov-08
3	Assa Abloy AB	Machinery/Engineering	NR/A-	€600m	5.125%	Dec-06
4	Atlas Copco AB	Machinery/Engineering	A3/A-	US$250m	7.750%	Sep-09
5	British American Tobacco Plc	Tobacco	Baa1/BBB+	US$330m	6.875%	May-08
6	BMW AG	Automotives	A1/NR	€750m	5.250%	Sep-06
7	Brisa-Auto Estradas de Portugal SA	Transport infrastructure	A3/A+	€600m	4.875%	Dec-06
8	**BT Group Plc**	Telecommunications	Baa1/A-	US$200m	8.765%	Aug-09
9	Ciba Specialty Chemicals AG	Chemicals	A2/A	CHF300m	3.250%	Apr-09
10	CIR - Compagnie Industriali Riunite SpA	Holding company	NR/BBB-	€400m	5.250%	Mar-09
11	DSM NA	Chemicals	A2/A-	US$250m	6.750%	May-09
12	Deutsche Telekom AG	Telecommunications	Baa1/A-	€2000m	5.250%	May-08
13	Electrolux AB	Household appliance	Baa1/BBB+	€300m	6.000%	Mar-08
14	Fiat SpA	Automotives	Ba3/BB-	€500m	5.500%	Dec-06
15	Imperial Tobacco Group Plc	Tobacco	Baa3/BBB	US$600m	7.125%	Apr-09
16	Royal KPN NV	Telecommunications	Baa1/A-	€1500m	4.750%	Nov-08
17	Metro AG	General retail	Baa1/BBB	€100m	5.900%	May-07
18	Metso Oyj	Machinery/Engineering	Ba1/BB	€156m	6.250%	Dec-06
19	Modern Times Group AB	Media	NR/BB-	€120m	5.500%	Jun-06
20	Norsk Hydro ASA	Oil and Gas	A2/A	€300m	6.250%	Dec-06
21	Royal Philips Electronics NV	Electronics	Baa1/BBB+	€750m	6.125%	May-11
22	Portugal Telecom SGPS SA	Telecommunications	A3/A-	€1,000m	5.750%	Feb-06
23	Repsol YPF SA	Oil and Gas	Baa2/BBB+	€750m	5.750%	Dec-06
24	Scania AB	Automotives	NR/A-	€550m	6.000%	Dec-08
25	Securitas AB	Services	Baa2/BBB+	€500m	6.125%	Mar-08
26	Siemens AG	Machinery/Engineering	Aa3/AA-	€2000m	5.750%	Jul-11
27	**SKF AB**	Machinery/Engineering	A3/A-	US$200m	7.125%	Jul-07
28	Solvay SA	Chemicals	A2/A	€700m	5.250%	Jul-06
29	**Statoil ASA**	Oil and Gas	A1/A	€500m	5.125%	Jun-11
30	Stora Enso Oyj	Paper and Packaging	Baa1/BBB+	€375m	6.375%	Jun-07
31	Suedzucker	Food producers	A3/NR	€500m	5.750%	Feb-12
32	Swedish Match AB	Tobacco	A3/A-	€180m	6.125%	Oct-06
33	**Syngenta AG**	Chemicals	A3/A-	€219m	5.500%	Jul-06
34	TDC A/S	Telecommunications	A3/BBB+	€685m	5.875%	May-06
35	Telecom Italia SpA	Telecommunications	Baa2/BBB+	€3000m	6.125%	Apr-06
36	Telefonica SA	Telecommunications	A3/A	€500m	4.500%	Apr-09
37	**Telenor ASA**	Telecommunications	A2/A-	€400m	5.250%	Jan-07
38	TeliaSonera AB	Telecommunications	Baa1/A-	€203m	4.625%	Apr-09
39	Unilever NV	Food producers	A1/A+	€1000m	5.125%	Jun-06
40	UPM-Kymmene Oyj	Paper and Packaging	Baa1/BBB	€600m	6.125%	Jan-12
41	**VNU NV**	Media	Baa1/BBB	€333m	1.750%	May-06
42	Wolters Kluwer NV	Media	Baa1/BBB+	NLG500m	5.250%	Apr-08

The companies in Figure 4.2 above that are shown in bold were subsequently dropped from the study because not enough solutions were obtained for them, or because the bonds that have been filtered were incorrectly labelled as straight bonds, thus the final number of corporate bonds used in the study was reduced to 36. Once the companies and bonds meeting the study's requirements were identified, the inputs required in order to solve the model were collected. The following section describes the inputs used in the model.

4.6. Model Inputs

The input variables required to implement the research model can be grouped under: firm specific, bond specific and general market inputs, as shown in the table below:

Firm Specific inputs	**Source / Calculation Method**
AD	Accounts publication date
E_i	Market capitalisation as at date i
i	Observation date
P_{AD}	Total debt principal outstanding as at the accounts publication date, including capital leases where disclosed.
C_{AD}	Total debt coupon paid as at the accounts publication date.
T	The term of the company's debt outstanding. T was calculated as the average life of the debt outstanding.
τ_{AD}	Tax rate as at the accounts publication date, calculated by dividing the tax charge/credit by pre-tax profit/loss.
Δ	The Firm's total payout ratio (to shareholders and debt holders). Total payout ratio was calculated as the average of the firm's three-year average cost of debt and the net dividend payout (adjusted for share buy-backs and share issuance) as a percentage of average market capitalisation.
a	The expected loss to debt holders if the company defaulted (Loss Given Default). This was taken from Varma, *et al.* (2005)
Bond specific inputs	
P	Bond's principal outstanding, taken from Bloomberg
c	Bond's annual coupon payable, taken from Bloomberg
ϱ	The percentage of expected recovery rate that is expected to be allocated to holders of the specific bond analysed. Calculated as bond principal divided by total debt outstanding.

t_i	Bond's time to maturity as at observation date *i*.
B_i	The bond's price as at observation date *i*. Bond prices were downloaded from Bloomberg.
General market inputs	
r_i	The short-term (1 year) risk-free rate as at observation date *i*. The risk-free rates were downloaded from Bloomberg for every observation date, corresponding to the currency in which each company reported its financial information.

The inputs were used to derive asset value and asset value volatility from bond prices and from equity prices, as described in the following section.

4.7. Model Implementation and Data Derivation

The Leland and Toft (1996) model used in this study was programmed in Microsoft Excel® so that the inputs for each observation date can be included in one row and the outputs are obtained in the same row. This approach enabled the time-series of daily observations to be listed vertically from the top of the spreadsheet towards the bottom.

The model was also programmed using Maple® 9.5 software package, which is a mathematics software package. One of Maple 9.5's advantages is that it allows the user to enter formulae in mathematical notation and then obtain solutions to them by specifying a range of inputs. Thus the formulae entered into Maple® appear identical to those contained in the original Leland and Toft (1996) paper, with the exception of certain variable notations.

In order to test that the Excel model contains the correct formulae, several hypothetical sets of inputs were used for both the Excel and the Maple version of the Leland and Toft model and their outputs were compared. The outputs were the same in all cases, indicating that the Excel formulae used replicate those used by Leland and Toft.

The Leland and Toft model enables calculation of the default barrier K, the value of the firm's total debt D, the value of a risky coupon bond B, the firm's total value v and the firm's equity value E. As already described in section 4.3 above, all the inputs except for asset value V and σ_v have been collected, as well as the firm's bond value B and its equity value E. Data was derived in the following steps:

1. For every observation date, the Excel Solver function was used to determine which values of V and σ_v would result in a calculated value of B equal to the observed value of B, subject to the constraint that the calculated value of E must equal the observed value of E.
2. Once the Solver sequence determined the values of V and σ_v that satisfy the specified conditions, the next set of inputs was used, again repeating step 1. This was repeated for every observation date until a time series of bond derived values for V and σ_v was obtained.

On occasion the Solver function, which solves multiple equations using an iteration process, could not find a solution. Two causes appeared to trigger this outcome:

1. An insufficient number of iterations were run, in which case the Solver procedure was run again.
2. An invalid value of σ_v was used, usually negative, resulting in an error message. When this happened, the values for V and σ_v were manually reset and the Solver procedure run again, which usually led to an optimal solution.

Whilst the process of running the Solver procedure for each observation data separately was very time consuming, it facilitated quality control to ensure that every data point derived from the inputs represented an optimal solution to the constraint problem.

In order to ensure that the solutions obtained using the Excel Solver procedure represent the optimal outcome, a selection of inputs and the derived values for V and σ_v was used to solve the Leland and Toft equations in Maple® 9.5. The results obtained using Maple were the same as those obtained by Excel once the differences in decimal points used have been taken into account.

The equity-implied values for V and σ_v were derived from the same set of inputs used for obtaining the bond implied values, using the method described in section 4.3 above.

The end result for each bond analysed was a data set comprising:

1. Bond price B;
2. Time to maturity t;
3. Time series of bond-implied asset values V_B;

4. Time series of bond-implied asset value volatility σ_{vB};

5. Time series of equity-derived asset values V_E;

6. Time series of equity-derived asset value volatility σ_{vE};

The table below provides an example of the data obtained:

Date	B	t	V_B	σ_{vB}	V_E	σ_{vE}
03/03/2005	107.15	3.19	36,192	0.338	30,200	0.355
02/03/2005	107.07	3.19	36,277	0.339	30,309	0.358
01/03/2005	107.13	3.19	36,317	0.338	30,332	0.359
28/02/2005	107.19	3.19	36,395	0.338	30,387	0.360
25/02/2005	107.08	3.20	36,483	0.340	30,511	0.362
24/02/2005	107.08	3.21	36,353	0.339	30,387	0.360
23/02/2005	107.17	3.21	36,291	0.339	30,293	0.358
22/02/2005	107.13	3.21	36,301	0.340	30,317	0.359
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
15/03/2002	100.55	6.16	52,170	0.550	50,065	0.758
14/03/2002	100.66	6.16	51,849	0.551	49,715	0.757
13/03/2002	100.82	6.16	51,786	0.547	49,650	0.756
12/03/2002	100.78	6.16	51,810	0.549	49,664	0.756
11/03/2002	100.82	6.17	53,089	0.552	50,945	0.763
08/03/2002	100.43	6.18	52,047	0.554	49,933	0.758
07/03/2002	100.29	6.18	52,102	0.558	49,986	0.758
06/03/2002	100.56	6.18	50,488	0.552	48,329	0.750
05/03/2002	100.82	6.18	50,892	0.550	48,710	0.752
04/03/2002	101.25	6.19	50,396	0.543	48,178	0.749

Source: data analysis, Ahold solution spreadsheet

The time series of inputs and derived outputs cover the same dates, thus allowing the data to be compared and analysed, as described in the following section.

5. Data Analysis

The data obtained in section 4.7 was used to test the hypotheses set out in section 3 above. Given the quantity of data obtained, a range of data analysis methods could be used. In order to identify which statistical tests are most appropriate to the data and whether the sample obtained is large enough, it was useful to review the data analysis methods used in earlier studies of structural credit models. The data analysis tests used in the leading empirical research studies published in the field of structural credit models are summarised below, in chronological order:

Jones, *et al.* (1984) used a sample of bonds from 27 firms over 5 years to estimate the predictive power of a Merton-type model compared with a ratio based model. In total they had 135 independent observations and used a sign test to determine which model has larger absolute pricing errors as well as regression analysis to determine which model and which model has stronger explanatory power (R^2).

Ogden (1987) used a sample of 57 bond prices and regression analysis of model-based spreads and actual bond spreads.

Wei and Guo (1997) used 53 weekly prices of Eurodollar and Treasury bills to compare the Longstaff and Schwartz (1995) and Merton (1974) models. The explanatory power of each model was tested using regression analysis (R^2).

Duffee (1998) used regression analysis to investigate the relationship between Treasury yields and corporate bond yield spreads. The hypotheses in Duffee's paper were tested using *t*-test at the 1% and 10% significance levels.

Helwege and Turner (1999) compared the credit spreads of sets of bonds from the same issuer with different maturities to determine the slope of the yield curve for speculative grade issuers. Their sample consisted of 64 issuers and a total of 163 bonds. They used *t*-test (parametric) and Wilcoxon signed rank test (non-parametric), and both tests arrived at the same results.

Collin-Dufresne, *et al.* (2001) used 688 bonds with at least 25 monthly price quotes for each to test the determinants of credit spread changes. The significance of the various factors was tested using regression analysis and *t*-statistics.

Ericsson and Reneby (2004) conducted an empirical study of structural credit risk models using a sample of 141 US corporate bonds totalling 5,594 quotes. In comparing the predictive power of the models, they used the correlation between estimated and actual bond prices and spreads.

Eom, *et al.* (2003) conducted an empirical study of five structural models using 182 bond prices. The hypotheses regarding the differences of outcomes between the different groups in the study were analysed using *t*-tests.

From the examples above, it appears that most studies in the field of corporate bond spreads use parametric tests, and that sample sizes differ due to data availability. It also appears most studies assume normal distribution of data as no adjustments were made to account for skewness, Kurtosis or heteroskedasticity (where appropriate). Opinions about the effects of skewness and Kurtosis of data on the test results differ, but it appears that as long as the sample is large enough, parametric tests can be used even if the data departs to a degree from the normal distribution assumptions. McDougal and Rayner (2004) tested the effect of normal distribution on the strength of results using the *t*-test, Wilcoxon's signed rank test and the sign-test for different sample sizes. They concluded that for small sample size with skewed distribution, the sign test is stronger than Wilcoxon's signed rank test or the *t*-test. However, all tests become more powerful as the sample size increases. All three tests produce more robust results for flatter (less Kurtosis) data, both Wilcoxon's and *t*-test are more sensitive to skewness than to Kurtosis. Moore and McCabe (2002) and Weiss (1999) suggest that the *t*-test is still appropriate when data are non-normal, as long as *n* (the sample size) is large enough. Both Moore and McCabe (2002) and Weiss (1999) suggest sample sizes of 15, 20, 30 or 40 depending on the degree of non-normality.

This study used 36 corporate bonds issued by 36 different companies. Each bond had daily prices covering three years, thus the number of data points in each time series averaged 746. The sample size compares favourably with previous studies. Jones, *et al.* (1984) used 27 firms and a total of 135 independent price observations, Ogden (1987) using 57 bond prices, and even Ericsson and Reneby (2004) used 141 corporate bonds with 5,594 observations, i.e. an average of 40 observations per bond.

In light of the accepted practices and the views of practitioners mentioned above, the hypotheses in this study were tested using parametric tests, based on methods provided in McClave, *et al.* (2001). However, as a further enhancement to data analysis, some of the hypotheses were also tested using non-parametric tests in order to determine whether both parametric and non-parametric tests yield the same results. The approach taken for testing each hypothesis is described in the paragraphs below.

Hypothesis 1: Bond implied asset values are the same as equity implied asset vales.

H_0: Asset values calculated using equity prices and accounting values are the same as asset values implied by structural models using bond prices.

H_A: Asset values implied by bond prices using structural models are different to those calculated using equity prices and accounting values.

Testing hypothesis 1 entails comparing two samples that are not independent, because once the days for which bond prices are available have been selected, the values for V_E and σ_{vE} are calculated for the same days. McClave, *et al.* (2001) state that when two samples are not independent, the independent samples *t*-test should not be used. Instead, a paired difference *t*-test should be used: for each pair of calculated values for V_B and V_E, the difference between the two values is calculated, thus resulting in a time series of differences. If the equity price-based asset values are the same as the bond price derived asset values, than the difference between them should be zero and the mean of the differences, denoted as μd_V, should also equal zero. Hence Hypothesis 1 can be stated as:

H_0: $\mu d_V = 0$

H_A: $\mu d_V \neq 0$

The test statistic is given by:

Eq. 5.1 $$t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n_d}}$$

Where $\bar{x}_d$ is the sample mean difference, s_d is the sample standard deviation of differences, and n_d is the number of observed differences (also equals the number of paired observations). The acceptance/rejection of the null hypothesis depends on the number of observations and the significance level chosen. The data was tested using two significance levels: $\alpha = 0.05$ and $\alpha = 0.01$. The test results in acceptance or rejection of the Null Hypothesis at two significance levels for each bond tested.

The *t*-test for paired differences assumes that the sampled populations are normally distributed and have equal variances. In order to determine whether the results of the sample are normally distributed, the Kurtosis and Skewness of the difference between V_B and V_E were calculated:

- Kurtosis measures the relative peakedness or flatness of a distribution compared with normal distribution. A positive number indicates relatively peaked distribution; a negative number indicates a relatively flat distribution. In a perfectly normal distribution the Kurtosis measure would be zero.
- Skewness measures the degree of asymmetry of a distortion around the mean. Positive Skewness indicates distribution skewed towards the tail extending to more positive values; negative skewness indicates distribution with an asymmetric tail extending towards the more negative or smaller values tail. In a perfectly normal distribution the skewness measure would be zero.

An alternative to a parametric significance test such as the *t*-test is a non-parametric test such as Wilcoxon's signed rank test, which requires no assumptions regarding the underlying data distribution. Hypothesis 1 was also tested using the Wilcoxon's signed rank test for large samples, where *z* is calculated and the rejection region is where the absolute value of *z* is greater than the critical *z* which is the same as the critical *t* for large samples (samples where n >= 25).

The results of the parametric and non-parametric tests where then cross tabulated to determine whether the null hypothesis can be rejected with the same levels of significance.

Hypothesis 2: Bond-implied asset value volatility is the same as equity-derived asset value volatility.

H_0: Asset value volatility calculated using equity prices is the same as asset value volatility implied by structural models using bond prices.

H_A: Asset value volatility calculated using equity price inputs is different to that implied by bond prices.

Testing Hypothesis 2 also requires a paired difference *t*-test for the same reasons applied to Hypothesis 1. Therefore, defining $\mu d_{\sigma v}$ as the mean of the differences between σ_{vB} and σ_{vE}, Hypothesis 2 can be expressed as:

H_0: $\mu d_{\sigma v} = 0$

H_A: $\mu d_{\sigma v} \neq 0$

The same *t*-test as proposed in Hypothesis 1 was used for testing Hypothesis 2, using the same significance levels ($\alpha = 0.05$ and $\alpha = 0.01$) and degrees of freedom. Because *t*-tests assume that data is normally distributed with equal variance, the Kurtosis and skewness of the differences between σ_{vB} and σ_{vE} were tested, to highlight any potential weaknesses in using parametric tests for non-normally distributed data.

Hypothesis 2 was also tested using Wilcoxon's signed rank test for large samples. The results of the parametric and non-parametric tests were cross tabulated to determine whether the null hypothesis can be rejected with the same levels of significance using both types of tests.

Hypothesis 3: Bond-implied asset value volatility is stable over time.

H_0: Asset value volatility is stable over time.

H_A: Asset value volatility is unstable over time.

The method used for testing Hypothesis 3 at an individual bond level was analysis of variance (ANOVA). Hypothesis 3 could therefore be expressed as:

H_0: $\mu\sigma_{v1} = \mu\sigma_{v2} = \ldots = \mu\sigma_{vp}$

H_A: at least two of the *p* volatility averages are different.

Where $\mu\sigma_{vp}$ denotes the mean variance of a given data sample.

Three samples of bond implied asset value volatility were used to perform the ANOVA:

1. From the first trading day in January 2005 to 3rd March 2005;
2. From the first trading day in January 2004 to 3rd March 2004;
3. From the first trading day in January 2003 to 3rd March 2003.

The ANOVA was carried out using two significance levels: : $\alpha = 0.05$ and $\alpha = 0.01$. If the *F*-ratio obtained from the calculation was greater than the critical value of *F* then the

Null Hypothesis could be rejected. An alternative interpretation of the ANOVA results is to reject the Null Hypothesis whenever the observed significance level P is less than the specified confidence level.

The ANOVA test is based on the assumption that the sampled populations are normally distributed and that the samples are randomly and independently selected.

Once an ANOVA test was carried out for all bonds, the next stage was to test whether the results obtained for the whole bond sample are significant. In order to estimate the significance of the findings, a *z*-value was calculated for the individual P values obtained at the ANOVA tests with significance levels of : $\alpha = 0.05$ and $\alpha = 0.01$ and a one-tail, lower-tailed *t*-test was carried out at the : $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. The null hypotheses for the overall sample tests were that $P_{avg} >= 0.05$ and that $P_{avg} >= 0.01$. Rejecting the null hypotheses would support the conclusion that the variances of σ_{vB} across the whole sample are not equal at the : $\alpha = 0.05$ and $\alpha = 0.01$ significance levels.

Hypothesis 4: Bond-implied asset value volatility is not correlated with the bond's time to maturity.

H_0: The correlation between time to maturity and bond-implied asset value volatility equals zero.

H_A: The correlation between time to maturity and bond-implied asset value volatility does not equal zero.

In order to test Hypothesis 4 the Pearson product moment coefficient of correlation, *r*, was calculated, measuring the strength of a linear relationship between two variables, in this case t and σ_{vB}. The closer r is to zero, the weaker the relationship between the two variables; an r value close to +1 (-1) indicates a strong positive (negative) correlation between the two variables. The hypothesis to be tested was therefore stated as:

$$H_0 : r = 0$$

$$H_A : r \neq 0$$

The result of this test is a correlation coefficient between the bond-implied asset value volatility and time to maturity for every bond.

The next stage of the hypothesis testing was to determine whether the whole population demonstrates correlation between time to maturity and bond implied asset value volatility. In order to test this, the average correlation coefficient for all the bonds in the sample is analysed.

The hypothesis to be tested was therefore stated as:

$$H_0 : \bar{r} = 0$$

$$H_A : \bar{r} \neq 0$$

Where $\bar{r}$ is the average correlation coefficient for all the bonds in the sample. In order to test Hypothesis 4 for the whole sample a value z was calculated for accepting or rejecting the null hypothesis at a given significance level, e.g.:

Eq. 5.2 $$z = \frac{\bar{r} - 0}{\sigma_r}$$

To reject a two-tailed hypothesis with a significance level $\alpha = 0.05$, z must be less than -1.96 or greater than +1.96. To reject the two-tail hypothesis with a significance level $\alpha = 0.01$, z must be less than -2.575 or greater than +2.575.

6. Results

From the data obtained it was possible to test the four hypotheses described in section 3. The results for each of the four hypotheses are described in this section. Appendix 1 contains summary information about each bond in the sample and the results of the statistical tests performed in order to test the hypotheses.

6.1. Hypothesis 1

Hypothesis 1 sought to determine whether bond-implied asset values are the same as equity-derived asset values. In order to test the hypothesis both the paired difference *t*-test and the Wilcoxon's Signed Rank Test were used. If the equity price based asset values are the same as the bond price derived asset values, than the difference between them should be zero and the mean of the differences, denoted as μd_V, should also equal zero. Hence Hypothesis 1 was stated as:

H_0: $\mu d_V = 0$

H_A: $\mu d_V \neq 0$

The hypothesis was tested using two significance levels, $\alpha = 0.01$ and $\alpha = 0.05$. The results of the *t*-test, Wilcoxon's Signed Rank Test, as well as the measure of Kurtosis and skewness for each bond in the sample are shown in the table below:

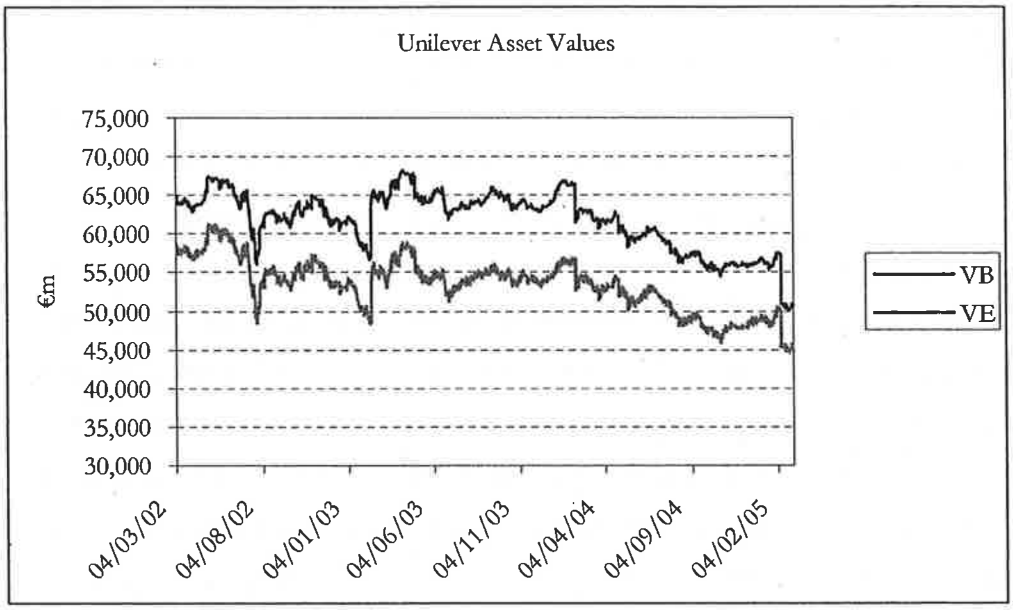
Company	Sector	Credit Rating Moodys/S&P	t-Test Calculated t		Wilcoxon's Signed Rank Test Calculated z	Kurtosis	Skewness
Royal Ahold NV	Food retail	Ba2/ BB	43.53	**	-23.34 **	0.15	0.03
Akzo Nobel NV	Chemicals	A3/A-	585.59	**	-23.17 **	-0.92	0.21
Assa Abloy AB	Machinery/Engineering	NR/A-	106.27	**	-23.76 **	-1.20	-0.34
Atlas Copco AB	Machinery/Engineering	A3/A-	205.46	**	-23.76 **	0.01	0.32
British American Tobacco Plc	Tobacco	Baa1/BBB+	141.74	**	-23.58 **	-0.20	-0.65
BMW AG	Automotives	A1/NR	120.88	**	-23.84 **	-1.26	-0.27
Brisa-Auto Estradas de Portugal SA	Transport infrastructure	A3/A+	123.72	**	-23.93 **	-1.03	-0.38
Ciba Specialty Chemicals AG	Chemicals	A2/A	325.67	**	-23.85 **	0.38	-0.59
CIR - Compagnie Industriali Riunite SpA	Holding company	NR/BBB-	62.65	**	-23.13 **	-1.28	-0.23
DSM NA	Chemicals	A2/A-	279.20	**	-24.04 **	-0.68	0.19
Deutsche Telekom AG	Telecommunications	Baa1/A-	77.58	**	-23.95 **	-1.56	-0.51
Electrolux AB	Household appliance	Baa1/BBB+	182.43	**	-23.76 **	-0.47	0.60
Fiat SpA	Automotives	Ba3/BB-	137.45	**	-23.76 **	1.34	-1.16
Imperial Tobacco Group Plc	Tobacco	Baa3/BBB	94.97	**	-23.87 **	-1.26	-0.00
Royal KPN NV	Telecommunications	Baa1/A-	140.28	**	-24.04 **	-0.02	-1.20
Metro AG	General retail	Baa1/BBB	100.19	**	-29.93 **	-1.26	0.37
Metso Oyj	Machinery/Engineering	Ba1/BB	67.63	**	-23.76 **	-1.32	-0.01
Modern Times Group AB	Media	NR/BB-	47.43	**	-23.62 **	-1.17	-0.41
Norsk Hydro ASA	Oil and Gas	A2/A	5.49	**	-7.18 **	1.36	-1.55
Royal Philips Electronics NV	Electronics	Baa1/BBB+	85.81	**	-24.01 **	-1.26	-0.45
Portugal Telecom SGPS SA	Telecommunications	A3/A-	11.82	**	-11.74 **	-1.50	0.55
Repsol YPF SA	Oil and Gas	Baa2/BBB+	109.45	**	-23.74 **	-0.69	-0.44
Scania AB	Automotives	NR/A-	163.89	**	-23.70 **	-0.97	-0.33
Securitas AB	Services	Baa2/BBB+	55.49	**	-23.76 **	-1.40	-0.08
Siemens AG	Machinery/Engineering	Aa3/AA-	255.28	**	-21.88 **	2.13	-1.56
Solvay SA	Chemicals	A2/A	81.58	**	-24.01 **	-1.25	-0.29
Stora Enso Oyj	Paper and Packaging	Baa1/BBB+	133.86	**	-23.76 **	-0.81	-0.73
Suedzucker AG	Food producers	A3/NR	132.74	**	-23.95 **	-0.71	-0.69
Swedish Match AB	Tobacco	A3/A-	139.30	**	-23.76 **	-0.69	-0.90
TDC A/S	Telecommunications	A3/BBB+	87.89	**	-23.76 **	-1.29	-0.75
Telecom Italia SpA	Telecommunications	Baa2/BBB+	27.82	**	-21.48 **	-1.43	0.49
Telefonica SA	Telecommunications	A3/A	190.24	**	-23.63 **	67.50	-4.37
TeliaSonera AB	Telecommunications	Baa1/A-	73.54	**	-23.60 **	-1.29	-0.71
Unilever NV	Food producers	A1/A+	192.28	**	-24.04 **	-0.56	-0.13
UPM-Kymmene Oyj	Paper and Packaging	Baa1/BBB	123.32	**	-23.76 **	-1.11	-0.62
Wolters Kluwer NV	Media	Baa1/BBB+	145.46	**	-22.92 **	-1.03	0.01

Note: ** and * denote significance at $\alpha = 0.01$ and $\alpha = 0.05$ respectively; NR denotes the null hypothesis could not be rejected at either $\alpha = 0.01$ or $\alpha = 0.05$.

As can be seen from the table above, the null hypothesis was rejected for all bonds in the sample at both $\alpha = 0.01$ and $\alpha = 0.05$ significance levels using both the parametric and non-parametric tests. It is therefore possible to argue that bond-implied asset values are different to asset values calculated using equity prices and accounting values.

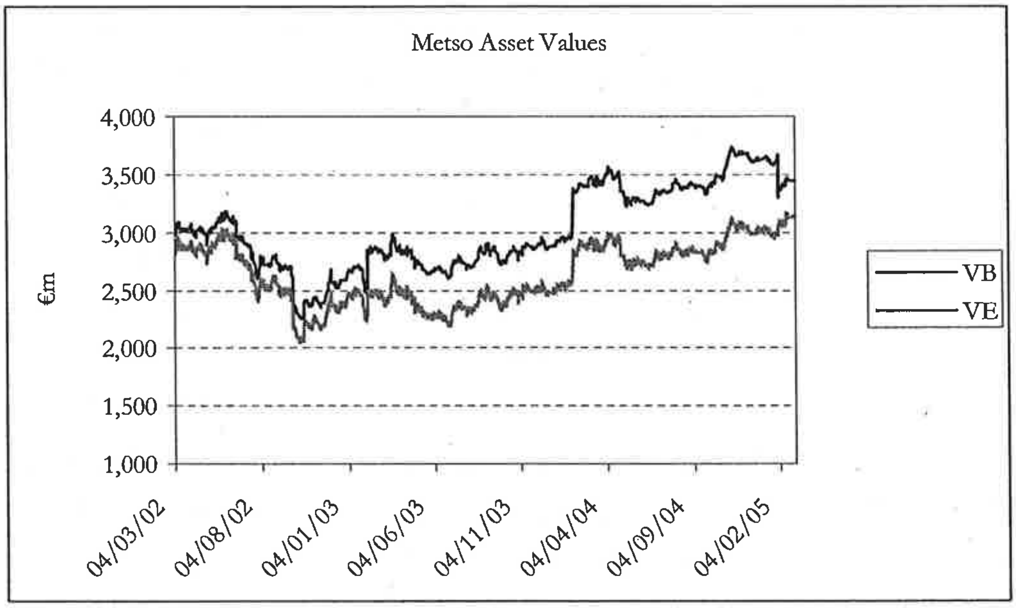
A further example of the differences is demonstrated in the two diagrams below, showing the development over time of bond implied asset values V_B and calculated values based on equity prices and accounting values, V_E. In order to demonstrate that that difference exist across the credit rating spectrum, Figure 6.1 shows values derived for Unilevler, an A1/A+ rated company and Figure 6.2 shows values for Metso, a Ba1/BB rated (i.e. sub-investment grade) company:

Figure 6.1: Unilever Asset Value



Source: Unilever Solution spreadsheet

Figure 6.2: Metso Asset Value



Source: Metso Solution spreadsheet

As can be seen from Figures 6.1 and 6.2 above, both bond-implied and equity-based asset values appear to follow similar general directions, but there is a clear difference between the values derived from bond prices using a structural model (denoted as V_B) and those calculated using equity values and accounting based inputs (denoted V_E).

As mentioned in section 5 above, the paired-difference test used in order to reject/not reject the null hypothesis assumes that the sampled populations are normally distributed and have equal variances. In order to test whether the data used for testing Hypothesis 1 is normally distributed, the Kurtosis and skewness of the pair-differences was tested. It appears that one company's data, Telefonica's, displays outlier characteristics when compared to the rest of the sample population. Telefonica's Kurtosis measure is 67.502 and its skewness measure is -4.370. The average for Kurtosis measure for the sample excluding the outlier is -0.693 and skewness is -0.349. The Kurtosis measure for Telefonica suggests that the distribution of the differences between V_B and V_E, denoted as d_V is very peaked compared with a normal distribution. The skewness measure for Telefonica suggests that distribution of the differences between V_B and V_E is skewed towards the tail containing the lower values. For the whole sample population (excluding the outlier data), the distribution of d_V appears to be slightly flatter than a perfectly normal distribution and slightly skewed towards the asymmetric tail containing the lower values of the distribution. Whilst these results indicate that the distribution of d_V is not perfectly normal, it is worthwhile noting that of the 36 data sets used for testing Hypothesis 1, 7 have positive Kurtosis measure (including Telefonica) and 9 display positive skew; in other words, not all data set distributions are flat or skewed towards the lower end of their values.

The evidence of slight departure from the normality assumptions raises the question of whether the *t*-test used is robust enough for hypothesis testing. McDougal and Rayner (2004) tested the effect of normal distribution on the strength of results using the *t*-test, Wilcoxon's signed rank test and sign-test for different sample sizes and concluded that all tests become more powerful as the sample size increases. Moore and McCabe (2002) and Weiss (1999) argue that the *t*-test is still appropriate when data are non-normal, as long as the sample size is large enough, suggesting sample sizes of between 15 and 40 as minimum, depending on the degree of non-normality. As the sample sizes used to test Hypothesis 1 for every bond ranged from 633 to 770 observations, thus exceeding the suggested large samples in the literature, it can be argued that using the *t*-test does not disadvantage the robustness of the results. As further evidence in support of the view that with large samples there is little difference to the results using parametric and non-parametric tests the results of the Wilcoxon's Signed Rank Test match those obtained using the *t*-test.

6.2. Hypothesis 2

Hypothesis 2 sought to test whether Bond-implied asset value volatility denoted as σ_{vB} is the same as equity-derived asset value volatility, denoted as σ_{vE}. Testing Hypothesis 2 also required both the paired difference t-test and the Wilcoxon's Signed Rank Test, for the same reasons applied to Hypothesis 1. Therefore, defining $\mu d_{\sigma v}$ as the mean of the differences between σ_{vB} and σ_{vE}, Hypothesis 2 was expressed as:

H_0: $\mu d_{\sigma v} = 0$

H_A: $\mu d_{\sigma v} \neq 0$

Hypothesis 2 was also tested using the same significance levels as Hypothesis 1, $\alpha = 0.01$ and $\alpha = 0.05$. The results of testing the null hypothesis of Hypothesis 2 using the paired difference t-test and the Wilcoxon's signed rank test, as well as the Kurtosis and skewness measures for each bond in the sample are shown in the table below:

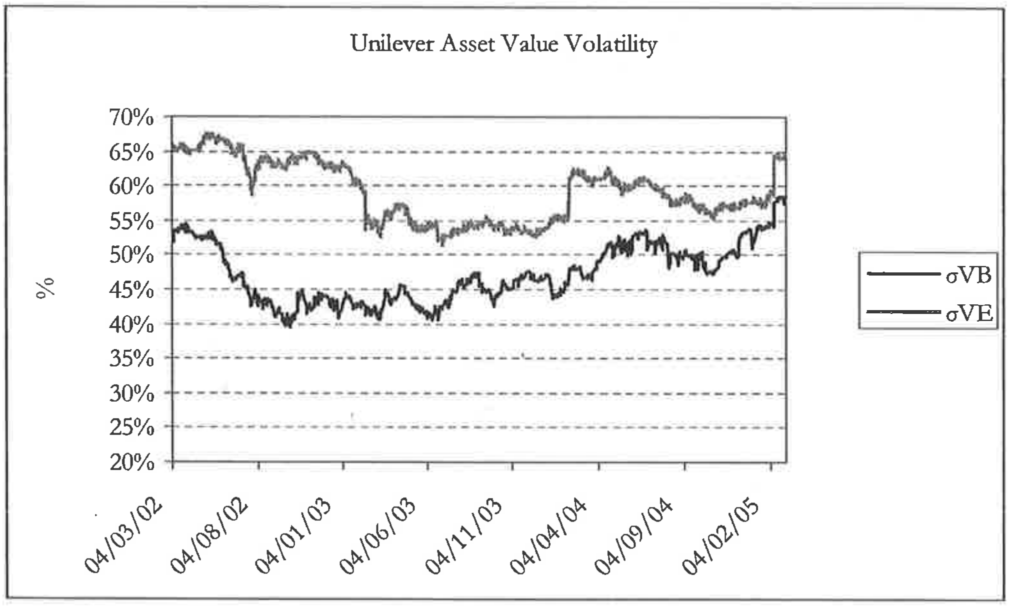
Company	Sector	Credit Rating	t-Test	Wilcoxon's Signed Rank Test	Kurtosis	Skewness
		Moodys/S&P	Calculated t	Calculated z		
Royal Ahold NV	Food retail	Ba2/ BB	-0.28 NR	-0.39 NR	1.42	0.76
Akzo Nobel NV	Chemicals	A3/A-	-9.36 **	-9.15 **	-0.81	0.39
Assa Abloy AB	Machinery/Engineering	NR/A-	-245.31 **	-23.76 **	-0.97	0.02
Atlas Copco AB	Machinery/Engineering	A3/A-	-179.28 **	-23.76 **	-0.97	0.03
British American Tobacco Plc	Tobacco	Baa1/BBB+	-312.55 **	-23.58 **	-0.67	-0.09
BMW AG	Automotives	A1/NR	-28.15 **	-18.99 **	-1.14	-0.30
Brisa-Auto Estradas de Portugal SA	Transport infrastructure	A3/A+	-263.44 **	-23.93 **	-1.26	0.02
Ciba Specialty Chemicals AG	Chemicals	A2/A	-88.83 **	-23.85 **	-1.27	0.33
CIR - Compagnie Industriali Riunite SpA	Holding company	NR/BBB-	7.16 **	-6.83 **	-0.77	-0.48
DSM NA	Chemicals	A2/A-	83.75 **	-24.04 **	-1.14	0.33
Deutsche Telekom AG	Telecommunications	Baa1/A-	-15.38 **	-13.46 **	-0.01	0.60
Electrolux AB	Household appliance	Baa1/BBB+	-249.44 **	-23.76 **	-0.28	-0.33
Fiat SpA	Automotives	Ba3/BB-	136.82 **	-23.76 **	0.33	0.91
Imperial Tobacco Group Plc	Tobacco	Baa3/BBB	-110.86 **	-23.87 **	-0.53	-0.23
Royal KPN NV	Telecommunications	Baa1/A-	37.15 **	-23.18 **	0.76	0.29
Metro AG	General retail	Baa1/BBB	-166.57 **	-29.93 **	0.27	1.08
Metso Oyj	Machinery/Engineering	Ba1/BB	-43.51 **	-23.75 **	-0.81	-0.59
Modern Times Group AB	Media	NR/BB-	9.45 **	-4.77 **	-0.89	0.55
Norsk Hydro ASA	Oil and Gas	A2/A	-83.85 **	-21.80 **	0.93	-1.28
Royal Philips Electronics NV	Electronics	Baa1/BBB+	-372.90 **	-24.01 **	0.40	-0.06
Portugal Telecom SGPS SA	Telecommunications	A3/A-	-62.05 **	-23.78 **	-0.29	0.92
Repsol YPF SA	Oil and Gas	Baa2/BBB+	-7.68 **	-7.76 **	-0.95	0.20
Scania AB	Automotives	NR/A-	-140.25 **	-23.70 **	0.02	0.69
Securitas AB	Services	Baa2/BBB+	-145.76 **	-23.76 **	-1.04	0.33
Siemens AG	Machinery/Engineering	Aa3/AA-	-213.52 **	-21.88 **	-0.27	-0.33
Solvay SA	Chemicals	A2/A	-228.91 **	-24.01 **	-0.58	0.03
Stora Enso Oyj	Paper and Packaging	Baa1/BBB+	-286.60 **	-23.76 **	0.98	-0.89
Suedzucker AG	Food producers	A3/NR	-243.60 **	-23.95 **	1.76	-0.28
Swedish Match AB	Tobacco	A3/A-	-116.12 **	-23.76 **	-0.95	0.08
TDC A/S	Telecommunications	A3/BBB+	-109.96 **	-23.76 **	-0.59	-0.07
Telecom Italia SpA	Telecommunications	Baa2/BBB+	-173.48 **	-23.76 **	-1.09	-0.00
Telefonica SA	Telecommunications	A3/A	-50.26 **	-23.23 **	-0.61	0.66
TeliaSonera AB	Telecommunications	Baa1/A-	-297.97 **	-23.60 **	1.74	-0.20
Unilever NV	Food producers	A1/A+	-63.91 **	-24.04 **	-0.70	-0.60
UPM-Kymmene Oyj	Paper and Packaging	Baa1/BBB	-108.98 **	-23.76 **	-1.25	-0.26
Wolters Kluwer NV	Media	Baa1/BBB+	-121.42 **	-22.92 **	-0.73	0.45

Note: ** and * denote significance at $\alpha = 0.01$ and $\alpha = 0.05$ respectively; NR denotes the null hypothesis could not be rejected at either $\alpha = 0.01$ or $\alpha = 0.05$.

From the table above it can be seen that the null hypothesis was rejected in 35 out of the 36 cases tested, i.e. 97.2% of the time. It can therefore be argued that bond implied asset value volatility σ_{vB} is different to equity-derived asset value volatility σ_{vE}. The one company in the sample where results appear to be different is Ahold, a supermarket chain that suffered from accounting irregularities and great bond and equity price volatility during the period used in this study.

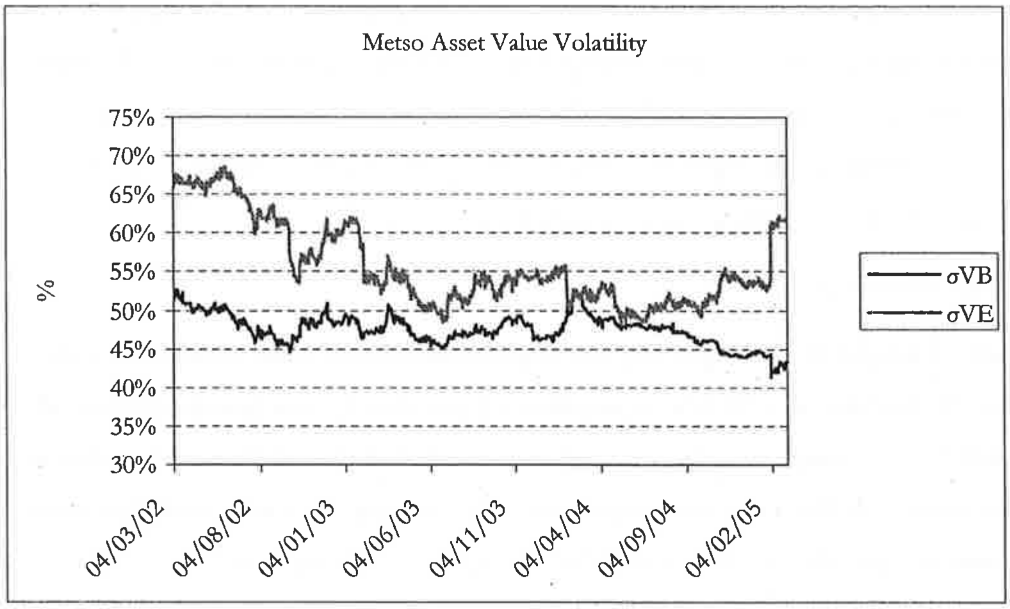
In order to provide anecdotal examples showing the differences between σ_{vB} and σ_{vE} , the diagrams below show the dynamics of asset value volatility for Unilever (Figure 6.3) and Metso (Figure 6.4):

Figure 6.3: Unilever asset value volatility



Source: Unilevler Solution Spreadsheet

Figure 6.4: Metso asset value volatility



Source: Metso Solution Spreadsheet

As can be seen from Figures 6.3 and 6.4 above, the paths followed by equity-derived asset value volatility and those derived from bond prices do not always follow the same direction, and there are sometimes significant differences in the absolute values of asset value volatility derived from the two methods.

As already mentioned in section 5 above, the paired-difference *t*-test used in order to reject/not reject the null hypothesis assumes that the sampled populations are normally distributed and have equal variances. In order to test whether the data used for testing Hypothesis 2 is normally distributed, the Kurtosis and skewness of the pair-differences was tested. Unlike in Hypothesis 1, there were no outliers in the Kurtosis and skenwness data for the sample used to test Hypothesis 2. The average for Kurtosis measure for the sample is -0.333 and skewness is -0.074. For the whole sample population, the distribution of $d_{\sigma v}$ appears to be slightly flatter than a perfectly normal distribution and slightly skewed towards the asymmetric tail containing the lower values of the distribution. Whilst these results indicate that the distribution of $d_{\sigma v}$ is not perfectly normal, it is worthwhile noting that of the 36 data sets used for testing Hypothesis 2, 10 have positive Kurtosis measure and 20 display positive skew; in other words, not all data set distributions are flat or skewed towards the lower end of their values.

Although the evidence of slight departure from the normality assumptions raises the question of whether the *t*-test used is robust enough for hypothesis testing, the sample sizes used to test Hypothesis 2 for every bond ranged from 633 to 770 observations, thus exceeding the suggested minimum large samples in the literature. It can therefore be argued that using the *t*-test does not disadvantage the robustness of the results. As is the case for Hypothesis 1, using the Wilcoxon's signed rank test provides similar results to those of the parametric test, supporting the view that with large samples both parametric and non-parametric tests lead to similar conclusions.

6.3. Hypothesis 3

Hypothesis 3 sought to determine whether bond-implied asset value volatility is stable over time. In the Literature Review section above it has already been mentioned that one of the underlying assumptions of options pricing models and thus of structural models is that asset value volatility is constant over time. By deriving asset value volatility from bond prices, it is possible to test whether this assumption holds in practice.

Hypothesis 3 was tested using single factor ANOVA. The null hypothesis of an ANOVA test is that each sample was drawn from the same underlying probability distribution; the alternative hypothesis is that the underlying probability distributions are not the same. The results of the ANOVA tests for every bond in the sample are shown in the table below. In order to determine the level of confidence with which the null hypothesis can be rejected, the table shows the *P*-value for each bond tested. If the *P*-value is less than 0.05, the null hypothesis can be rejected at a significance level $\alpha = 0.05$. Similarly, if the *P*-value is less than 0.01, the null hypothesis can be rejected at a significance level $\alpha = 0.01$:

Company	Sector	Credit Rating Moodys/S&P	Anova Test P-value	
Royal Ahold NV	Food retail	Ba2/ BB	5.78E-47	**
Akzo Nobel NV	Chemicals	A3/A-	4.06E-10	**
Assa Abloy AB	Machinery/Engineering	NR/A-	8.27E-20	**
Atlas Copco AB	Machinery/Engineering	A3/A-	1.01E-70	**
British American Tobacco Plc	Tobacco	Baa1/BBB+	1.27E-107	**
BMW AG	Automotives	A1/NR	1.53E-64	**
Brisa-Auto Estradas de Portugal SA	Transport infrastructure	A3/A+	2.95E-47	**
Ciba Specialty Chemicals AG	Chemicals	A2/A	2.06E-37	**
CIR - Compagnie Industriali Riunite SpA	Holding company	NR/BBB-	1.11E-81	**
DSM NA	Chemicals	A2/A-	3.42E-62	**
Deutsche Telekom AG	Telecommunications	Baa1/A-	6.02E-64	**
Electrolux AB	Household appliance	Baa1/BBB+	5.47E-88	**
Fiat SpA	Automotives	Ba3/BB-	1.29E-63	**
Imperial Tobacco Group Plc	Tobacco	Baa3/BBB	3.93E-86	**
Royal KPN NV	Telecommunications	Baa1/A-	2.73E-29	**
Metro AG	General retail	Baa1/BBB	5.49E-52	**
Metso Oyj	Machinery/Engineering	Ba1/BB	2.77E-36	**
Modern Times Group AB	Media	NR/BB-	1.89E-68	**
Norsk Hydro ASA	Oil and Gas	A2/A	5.78E-109	**
Royal Philips Electronics NV	Electronics	Baa1/BBB+	1.98E-42	**
Portugal Telecom SGPS SA	Telecommunications	A3/A-	7.92E-81	**
Repsol YPF SA	Oil and Gas	Baa2/BBB+	2.76E-13	**
Scania AB	Automotives	NR/A-	3.89E-42	**
Securitas AB	Services	Baa2/BBB+	4.58E-48	**
Siemens AG	Machinery/Engineering	Aa3/AA-	9.27E-52	**
Solvay SA	Chemicals	A2/A	7.00E-87	**
Stora Enso Oyj	Paper and Packaging	Baa1/BBB+	9.56E-32	**
Suedzucker AG	Food producers	A3/NR	1.87E-44	**
Swedish Match AB	Tobacco	A3/A-	4.44E-84	**
TDC A/S	Telecommunications	A3/BBB+	1.84E-60	**
Telecom Italia SpA	Telecommunications	Baa2/BBB+	3.29E-64	**
Telefonica SA	Telecommunications	A3/A	3.79E-46	**
TeliaSonera AB	Telecommunications	Baa1/A-	8.75E-18	**
Unilever NV	Food producers	A1/A+	2.36E-67	**
UPM-Kymmene Oyj	Paper and Packaging	Baa1/BBB	5.48E-50	**
Wolters Kluwer NV	Media	Baa1/BBB+	2.41E-56	**

Note: ** and * denote significance at $\alpha = 0.01$ and $\alpha = 0.05$ respectively; NR denotes the null hypothesis could not be rejected at either $\alpha = 0.01$ or $\alpha = 0.05$.

As can be seen from the table above, the *P*-values obtained for all the bonds in the sample are well below 0.01 and the null hypothesis can be rejected at significance levels $\alpha = 0.01$ and $\alpha = 0.05$. It appears that the structural models' assumption of constant asset value volatility does not hold in practice.

In order to test the null hypothesis for all the bonds in the sample together, a one-tail lower tailed statistical test was performed, treating all the *P*-values obtained in the ANOVA test as a single sample. The null hypothesis of the test was that $P_{avg} >= 0.05$ and that $P_{avg} >= 0.01$. The test was performed at $\alpha = 0.01$ and $\alpha = 0.05$ significance levels. The table below shows the results of the statistical test carried out:

H_0:	$P >= 0.05$	$P >= 0.01$
Average	1.13E-11	1.13E-11
Std Dev	6.77E-11	6.77E-11
No. of observations	36	36
z test statistic	-738,704,400 **	-147,740,880 **

Note: ** and * denote significance at α = 0.01 and α = 0.05 respectively; NR denotes the null hypothesis could not be rejected at either α = 0.01 or α = 0.05.

As can be seen from the table, the null hypothesis can be rejected at α = 0.01 and α = 0.05 significance levels, indicating that $P_{avg} < 0.05$ and that $P_{avg} < 0.01$.

6.4. Hypothesis 4

Hypothesis 4 sought to establish whether there is correlation between bond-implied asset value volatility and the bond's time to maturity. The test carried out for every bond in the sample was calculating the Pearson product moment coefficient of correlation, *r*, measuring the strength of a linear relationship between σ_{vB} and t. The closer *r* is to zero, the weaker the relationship between the two variables; an *r* value close to +1 (-1) indicates a strong positive (negative) correlation between the two variables. The hypothesis tested was:

$$H_0: r = 0$$

$$H_A: r \neq 0$$

The results of the test for each bond in the sample are shown in the table below:

Company	Sector	Credit Rating Moodys/S&P	Correlation σ_{VB} and time to maturity
Royal Ahold NV	Food retail	Ba2/ BB	77.7%
Akzo Nobel NV	Chemicals	A3/A-	82.4%
Assa Abloy AB	Machinery/Engineering	NR/A-	77.1%
Atlas Copco AB	Machinery/Engineering	A3/A-	-79.2%
British American Tobacco Plc	Tobacco	Baa1/BBB+	-85.2%
BMW AG	Automotives	A1/NR	3.6%
Brisa-Auto Estradas de Portugal SA	Transport infrastructure	A3/A+	87.7%
Ciba Specialty Chemicals AG	Chemicals	A2/A	42.1%
CIR - Compagnie Industriali Riunite SpA	Holding company	NR/BBB-	72.8%
DSM NA	Chemicals	A2/A-	-34.6%
Deutsche Telekom AG	Telecommunications	Baa1/A-	85.7%
Electrolux AB	Household appliance	Baa1/BBB+	-79.6%
Fiat SpA	Automotives	Ba3/BB-	82.0%
Imperial Tobacco Group Plc	Tobacco	Baa3/BBB	92.8%
Royal KPN NV	Telecommunications	Baa1/A-	82.1%
Metro AG	General retail	Baa1/BBB	-29.0%
Metso Oyj	Machinery/Engineering	Ba1/BB	53.6%
Modern Times Group AB	Media	NR/BB-	70.5%
Norsk Hydro ASA	Oil and Gas	A2/A	-89.2%
Royal Philips Electronics NV	Electronics	Baa1/BBB+	65.4%
Portugal Telecom SGPS SA	Telecommunications	A3/A-	-2.4%
Repsol-YPF SA	Oil and Gas	Baa2/BBB+	80.3%
Scania AB	Automotives	NR/A-	1.3%
Securitas AB	Services	Baa2/BBB+	-21.7%
Siemens AG	Machinery/Engineering	Aa3/AA-	47.7%
Solvay SA	Chemicals	A2/A	-47.9%
Stora Enso Oyj	Paper and Packaging	Baa1/BBB+	-22.1%
Suedzucker	Food producers	A3/NR	76.3%
Swedish Match AB	Tobacco	A3/A-	-87.7%
TDC A/S	Telecommunications	A3/BBB+	93.0%
Telecom Italia SpA	Telecommunications	Baa2/BBB+	72.5%
Telefonica SA	Telecommunications	A3/A	79.3%
TeliaSonera AB	Telecommunications	Baa1/A-	8.6%
Unilever NV	Food producers	A1/A+	-47.7%
UPM-Kymmene Oyj	Paper and Packaging	Baa1/BBB	79.9%
Wolters Kluwer NV	Media	Baa1/BBB+	86.3%

As can be seen from the table, the results do not lead to an obvious and clear cut conclusion regarding the relationship between σ_{vB} and t. The next stage in testing hypothesis 4 was to look at the correlation coefficients for the sample as a whole, testing the average correlation coefficient for all the bonds in the sample is analysed. The hypothesis tested was:

$$H_0 : \bar{r} = 0$$

$$H_A : \bar{r} \neq 0$$

Where $\bar{r}$ is the average correlation coefficient for all the bonds in the sample. In order to test Hypothesis 4 for the whole sample a two-tailed statistical test was performed enabling the rejection/non-rejection of the null hypothesis at a given confidence level.

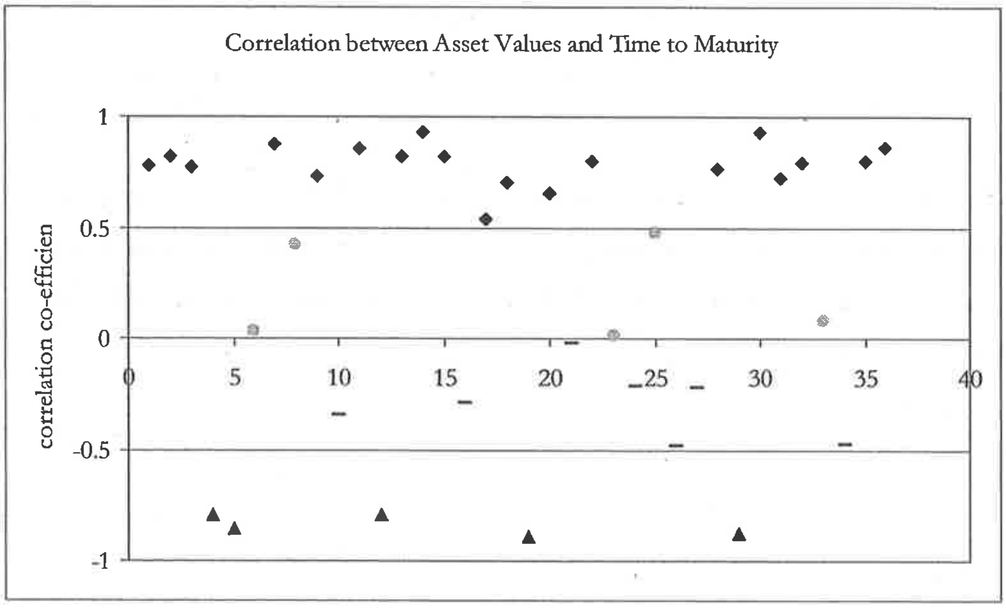
To reject a two-tailed hypothesis with a significance level of α = 0.05, the z value calculated for the sample of r values must be less than -1.96 or greater than +1.96. To reject the two-tail hypothesis with a significance level of α = 0.01, z must be less than -2.575 or greater than +2.575. The table below shows the test's results using the r values in the sample:

H_0:	r = 0
Average	0.271
Std Dev	0.634
No. of observations	36
z test statistic	0.427
	NR

Note: ** and * denote significance at α = 0.01 and α = 0.05 respectively; NR denotes the null hypothesis could not be rejected at either α = 0.01 or α = 0.05.

As can be seen from the two-tailed statistical test results above, it is not possible to reject the null hypothesis that there is no correlation between bond-implied asset value volatility and a bond's time to maturity at α = 0.01 or α = 0.05 significance levels. These results lead to further questions about the relationship between t and σ_{vB}. One method of seeking out a trend or relationship between two variables is through graphical representation. Figure 6.5 below plots the correlation coefficients for all the bonds in the sample against the firms in alphabetical order:

Figure 6.5: Distribution of Correlation between t and σ_{vB}.



If the null hypothesis could be rejected, most of the dots in the diagram would have been expected to be concentrated around the zero correlation coefficient level. However, as can be seen from the plot, most of the dots appear to be in the 0.5 – 1 region, indicating positive correlation between time to maturity and bond-implied asset value volatility. The table below provides further details of the breakdown of correlation coefficients in the sample:

Correlation Coefficient	Number of bonds	% of total bonds
1.0 – 0.5	19	52.8%
0.5 – 0	5	13.9%
0 – -0.5	7	19.4%
-0.5 – -1.0	5	13.9%

From the table above it appears that only 33.3% of the bonds in the sample displayed a low degree of correlation between asset value volatility and time to maturity, in the region of -0.5 - +0.5; just over 50% of the bonds in the sample displayed strong positive correlation between time to maturity and asset value volatility, and a further 13.9% displayed strong negative correlation. In order to establish whether certain characteristics are associated with high or low correlation between t and σ_{vB}, the table below provides the names, time to maturity and credit ratings of the companies used in the sample, divided according to where in the range of correlation coefficients they fall:

Name	Maturity	Rating	Time to Maturity	
1 > r >= 0.5				
Royal Ahold NV	May-08	Ba2/ BB	3.16	
Akzo Nobel NV	Nov-08	A3/A-	3.67	
Assa Abloy AB	Dec-06	NR/A-	1.75	
Brisa-Auto Estradas de Portugal SA	Dec-06	A3/A+	1.75	
CIR - Compagnie Industriali Riunite SpA	Mar-09	NR/BBB-	3.99	
Deutsche Telekom AG	May-08	Baa1/A-	3.16	
Fiat SpA	Dec-06	Ba3/BB-	1.75	
Imperial Tobacco Group Plc	Apr-09	Baa3/BBB	4.08	
Royal KPN NV	Nov-08	Baa1/A-	3.67	
Metso Oyj	Dec-06	Ba1/BB	1.75	
Modern Times Group AB	Jun-06	NR/BB-	1.24	
Royal Philips Electronics NV	May-11	Baa1/BBB+	6.20	
Repsol YPF SA	Dec-06	Baa2/BBB+	1.75	
Suedzucker AG	Feb-12	A3/NR	6.92	
TDC A/S	Apr-06	A3/BBB+	1.08	
Telecom Italia SpA	Apr-06	Baa2/BBB+	1.08	
Telefonica SA	Apr-09	A3/A	4.08	
UPM-Kymmene Oyj	Jan-12	Baa1/BBB	6.83	
Wolters Kluwer NV	Apr-08	Baa1/BBB+	3.08	
Average time to maturity				3.21
0.5 > r >= 0.0				
BMW AG	Sep-06	A1/NR	1.50	
Ciba Specialty Chemicals AG	Apr-09	A2/A	4.08	
Scania AB	Dec-08	NR/A-	3.75	
Siemens AG	Jul-11	Aa3/AA-	6.33	
TeliaSonera AB	Apr-09	Baa1/A-	4.08	
Average time to maturity				3.95
0.0 > r >= -0.5				
DSM NA	May-09	A2/A-	4.16	
Metro AG	May-07	Baa1/BBB	2.16	
Portugal Telecom SGPS SA	Feb-06	A3/A-	0.92	
Securitas AB	Mar-08	Baa2/BBB+	2.99	
Solvay SA	Jul-06	A2/A	1.33	
Stora Enso Oyj	Jun-07	Baa1/BBB+	2.24	
Unilever NV	Jun-06	A1/A+	1.24	
Average time to maturity				2.15
0.5 > r >= -1.0				
Atlas Copco AB	Sep-09	A3/A-	4.50	
British American Tobacco Plc	May-08	Baa1/BBB+	3.16	
Electrolux AB	Mar-08	Baa1/BBB+	2.99	
Norsk Hydro ASA	Jan-10	A2/A	4.83	
Swedish Match AB	Oct-06	A3/A-	1.58	
				3.41

As can be seen from the table above, the average time to maturity does not appear to indicate whether a company's σ_{vB} will be positively or negatively correlated with time to maturity, nor does the credit rating appear to provide a good indication.

The results of testing hypothesis 4 suggest that whilst it is not possible to reject the null hypothesis that there is no correlation between time to maturity and asset value volatility, the relationships observed do not lend themselves to a generalised conclusion that there is correlation between the two variables tested because some bonds displayed positive- and others displayed negative correlation. There may be other factors that affect the relationship between time to maturity and asset volatility (e.g. bond liquidity, firm size, capital structure, etc.) and it could be that the relationship between time to maturity and asset value volatility is not linear, and therefore calculating the Pearson product moment coefficient of correlation, *r*, is not the appropriate measure for it.

7. Conclusions

This study provides the first empirical research of bond implied asset values and volatilities. In order to achieve the study's aims, an extensive database of bond- and issuer information was assembled, as well as extensive time-series data of daily bond prices. The length of the time series used (three years of daily prices for each bond) and the empirical tests focusing on the characteristics of bond implied asset values and volatility set this study apart from other empirical studies in the field of structural credit modeling, which tend to focus on the pricing prediction accuracy of such models.

The study sought to identify the characteristics of two unobservable variables that are crucial inputs in every structural credit model developed thus far. These variables are the firm's asset value and the firm's asset value volatility, denoted as V and σ_V respectively. As highlighted in section 2.1.3 above, several methods are used by practitioners in order to calculate these unobservable variables, by using observed equity prices, their volatility, and the firm's leverage. Furthermore, the underlying assumption in most structural models is that asset value volatility is constant through time. This study set out to determine whether calculations of V and σ_V using equity and leverage are different to the values of these variables implied by observed bond prices, whether the assumption of constant volatility holds in practice, and whether bond implied asset volatility is correlated with the bond's time to maturity. In order to analyze these issues, the following hypotheses were tested:

Hypothesis 1: Bond-implied asset values are the same as equity-implied asset values.

Hypothesis 2: Bond-implied asset value volatility is the same as equity-derived asset value volatility.

Hypothesis 3: Bond-implied asset value volatility is stable over time.

Hypothesis 4: The bond-implied asset value volatility is not correlated with the bond's time to maturity.

The sample of bonds used in the study comprised 36 bonds issued by 36 different firms. All of the firms were publicly listed companies based in Western European countries. For every bond in the sample there was a time series of three years' daily prices.

The structural model used in order to derive bond implied asset value and asset value volatility was the one developed by Leland and Toft (1996), as it provides closed form solutions for the value of the firm's equity, its bonds and its total debt, and the default barrier is determined endogenously using the model's inputs. By solving a system of two equations and two unknowns the bond-implied values for every firm's asset value and asset value volatility were derived. The two equations used were for the firm's equity value and a single bond's value. An alternative method for estimating the values of the firm's assets and volatility was also used, based on methodology proposed by Jones, *et al.* (1984) which relies on the firm's equity volatility and leverage. This procedure resulted in two data sets for each bond in the sample:

1. Bond implied asset values and volatility

2. Equity-derived asset values and volatility

The two sets of data were then used to test the study's hypotheses.

The null hypothesis in Hypothesis 1 was that there is no statistically significant difference between bond-implied and equity derived asset values. In order to test the null hypothesis, two tests were used: a paired-difference *t*-test and, due to evidence of slight skewness and Kurtosis in the data, Wilcoxon's signed rank test. Using both tests, the null hypothesis was rejected at both $\alpha = 0.01$ and $\alpha = 0.05$ significance levels for all 36 bonds used, leading to the conclusion that there are statically significant differences between bond-implied and equity derived asset values.

Hypothesis 2 was tested in the same way as Hypothesis 1. The null hypothesis was that there is no statistically significant difference between bond-implied and equity derived asset value volatility. Using both the paired-difference *t*-test and the Wilcoxon's signed rank test the null hypothesis was rejected in 35 of the 36 bonds used at both $\alpha = 0.01$ and $\alpha = 0.05$ significance levels.

Hypothesis 3 sought to determine whether asset value volatility is constant over time. The null hypothesis for every bond in the sample was that volatility is constant over time and the test used was a single factor ANOVA using three samples for each bond. The resulting *P*-values were well below 0.01 for all bonds in the sample, suggesting that the null hypothesis can be rejected at $\alpha = 0.01$ and $\alpha = 0.05$ significance levels. In order to test the results for the sample as a whole, a one-tail lower tailed statistical test was carried out on the *P*-values obtained using the ANOVA. The null hypotheses used were that $P_{avg} >= 0.05$ and that $P_{avg} >= 0.01$. These hypotheses were both rejected at $\alpha = 0.01$ and $\alpha = 0.05$ significance levels, leading to the conclusion that asset value volatility is not constant over time.

Hypothesis 4 aimed to determine whether there is correlation between bond-implied asset value volatility and the bond's time to maturity. The hypothesis was tested using a Pearson product moment coefficient of correlation, *r*, measuring the strength of a linear relationship between σ_{vB} and t. The results of the test for the sample were inconclusive. In addition to looking at each bond individually, the average correlation coefficient for all the bonds in the sample was tested, where the null hypothesis is that the average correlation coefficient is zero, i.e. there is no correlation between bond-implied asset value volatility and the bond's time to maturity. The hypothesis was tested using a two-tailed statistical test and it was not possible to reject it at either significance level $\alpha = 0.01$ or $\alpha = 0.05$.

A graphical representation of the results for the sample indicated that for approximately half of the bonds in the sample the correlation between asset volatility and time to maturity lies between 0.5 and 1, but 33.3% of the bonds in the sample demonstrated a low degree of correlation. Time to maturity and credit rating also do not appear to influence the correlation between bond-implied asset value volatility and time to maturity. The results of testing hypothesis 4 suggest that it is not possible to reject the null hypothesis, but the relationships observed do not lend themselves to a generalised conclusion that there is correlation between the two variables tested, as some correlation coefficients were positive, others negative or close to zero.

This study analysed the properties of asset value and volatility empirically and resulted in the rejection of three out of the four null hypotheses set out. It proved that there are statistically significant differences between bond implied asset values and volatilities and those values calculated using equity prices. The differences identified have significant implications for the implementation of structural credit models – clearly, using equity values to calculate the unobservable variables required for structural credit models will result in pricing errors, as these variables are different to those implied by bond prices.

The study also proved that bond implied asset value volatility is not constant over time. Demonstrating that asset volatility is itself volatile over time could also explain the pricing prediction errors of structural models identified by many empirical studies, as shown in section 2.1.3 above. Fouque, Sircar and Solna (2004) proposed a structural model for pricing zero coupon bonds incorporating mean-reverting volatility and showed that the spreads it generates can match observed spreads more closely than spreads predicted by a constant-volatility structural model.

8. Contribution to Knowledge

The contribution of this study to the body of knowledge in the field of finance spans empirical, practical and theoretical aspects. Empirically, this study is the first attempt to derive bond-implied asset values and volatility and obtain a time series of these values. At a practical level, the study shows that using the Merton relationship between debt and equity in order to derive asset value volatility results in values that are different to those implied by bond prices, and may thus contribute to price prediction errors. At the theoretical level, some key assumptions that underlie structural credit models are tested. Structural models for pricing credit form an integral part of the theory of capital structure optimisation and are also widely used by practitioners who aim to value defaultable securities as well as measure and manage credit risk. Understanding the inputs required by such models can lead to better understanding of credit risk, more efficient pricing of risky debt and equity, and ultimately to optimising corporate capital structure.

Previous empirical studies in the field of structural credit models focused on the models' ability to predict bond prices and credit spreads. Jones, *et al.* (1984) tested the predictive power of a Merton type model in pricing corporate bonds. Ogden (1987) also tested the predictive power of a structural model, whilst later studies such as Wei and Guo (1997), Eom, *et al.* (2004), Houwelig and Vorst (2005), Gunduz and Uhrig-Homburg (2005) and others sought to compare the performance of different types of structural models in accurately predicting bond prices and credit spreads. No empirical study has thus far derived and analysed the characteristics of asset value volatility from observed bond prices. This study used a set of 36 Western-European corporate bonds with daily price history over three years in order to derive asset values and asset value volatility from observed bond prices. The data set of asset values and volatility that was obtained is the first of its kind in the field of structural credit models.

Most practical applications of structural models use the relationship between equity volatility and leverage as shown in the Merton (1974) model in order to derive the instantaneous asset value volatility, which is a critical input in any structural model. This study compared the time series of bond implied asset value volatility with the time series of asset value volatility derived from equity volatility and leverage and showed that there are statically significant differences between the two sets of data across 97.2% of the bonds in the sample. This finding suggests that using the Merton (1974) model to derive asset volatility and then using it in another, more sophisticated structural model may be inappropriate as the relationship between equity and debt are slightly different across the range of models that has been developed. Furthermore, using bond prices to derive asset value volatility may result in an inaccurate volatility figure because bond prices may contain other factors such as liquidity and lack of actual trading which may distort the results. Additionally, bonds of firms that are not close to default represent options that are out of the money, and using out-of-the-money options to derive implied volatility tends to lead to inaccurate results, as the option price less sensitive to volatility in such cases.

The theoretical underpinnings of structural models assume that asset value volatility is constant over time. This study derived extensive time series of bond-implied asset value volatility and showed that volatility is not constant over time. Although equity volatility has been shown to vary over time, this is the first study to use bond prices in order to show that asset value volatility is not constant over time. The implications of this finding to structural models are that when assuming constant volatility, the predictive power of the model may be greater for shorter time periods into the future. A possible enhancement to structural models would be to use a forecast of future volatility, based on methods such as GARCH.

Efficient pricing of risk is a key component of capital structure optimisation, as argued by Leland (1994) and Leland and Toft (1996). At the heart of structural models from Merton (1974) onwards, risk is represented by asset value volatility. This study contributes to the body of knowledge in the field of finance by deriving asset value volatility from observed bond prices, and analysing it. Better understanding of risk is the key to more efficient pricing of risky investments such as corporate bonds and equity. More efficient pricing of risky investments leads to more efficient allocation of capital.

9. Limitations of the Research

The aim of this study was to derive implied asset values and asset value volatility from bond prices and to compare them to calculated asset values and asset value volatility based on equity price movement. The limitations of the research can be divided into two broad categories: limitations of the research methodology and limitations of the implementation method. This section describes the limitations of the research, classified into these two broad categories.

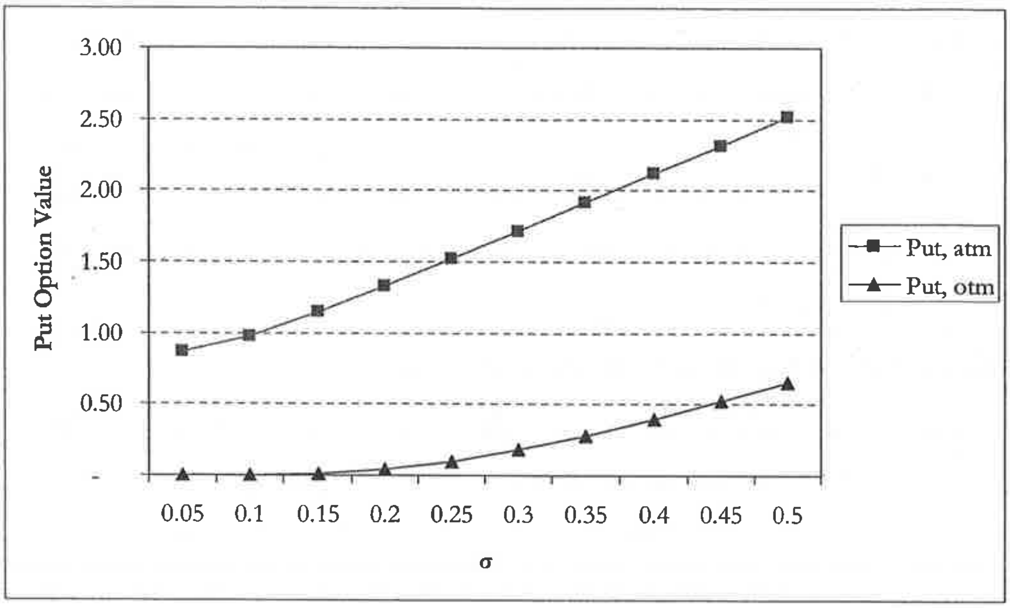
9.1. Limitations of the Research Methodology

The literature on bond-implied asset value volatility is not yet developed, but the literature on the differences between share price volatility implied by option prices, historical volatility and realised volatility is well developed. As structural models view bonds as, essentially, options written on the firm's assets, the observations made about the differences between implied and realised volatility in equity derivatives could be applicable to bond implied asset value volatility. There are several weaknesses in the process of estimating implied stock volatility that have been identified in the literature and may also be applicable to estimating bond implied asset value volatility.

Hentschel (2003) argued that whilst the Black and Scholes (1973) model assumes frictionless markets, when using traded options in order to derive implied volatility, the calculation is subject to errors due to the precision of the price quoted, the bid-ask spreads, and non-synchronous observations, all of which can lead to implied volatility calculation becoming an estimation.

Hentschel (2003) also argued that the estimation error of implied stock volatility increases the more out-of-the-money the option is. This is because the more out-of-the-money an option is, the less sensitive it is to changes in volatility, as can be seen in Figure 8.1 below:

Figure 8.1: Put options, moniness and volatility



As can be seen from Figure 8.1 above, the value of an out-of-the-money ('otm') put option remains close to zero for a range of volatility values, and its value slopes less steeply than that of an at-the-money ('atm') put option, *ceteris paribus*. Thus there is a higher degree of implied volatility estimation error when using out-of-the-money options than there is when using at-the-money-options.

All of these observations are valid when estimating bond-implied asset value volatility:

- Bond prices observed in the markets are subject to significant bid-offer spreads due to the relatively illiquid nature of the bond market.
- Some of the prices might be stale as not all bonds trade at similar volumes and some trade on an enquiry basis, but dealers might still post prices.
- In structural models, the further the company is from default, the more out-of-the-money the put option that shareholders have on the firm's assets. The put option becomes valuable only when the firm is close to bankruptcy; thus estimates of implied asset value volatility for high credit quality companies are akin to estimating implied equity volatility from out-of-the-money options – they could be subject to significant estimation errors which will not have a noticeable effect on the price of the option or the bond.

Another limitation of the research methodology is the use of a structural model to derive bond-implied asset value volatility. Structural models tend to be univeriate models, where one factor – asset value volatility – determines the fair price of a bond. Although some models contain other causes of uncertainty such as interest rate volatility or recovery rate volatility, structural models do not normally include more than two key factors in order to predict fair bond pricing and default probability. There is some evidence that suggests that credit spreads/bond prices and default probabilities are functions of more than just asset value volatility and post-default recovery rates. Stein (2005) argued that a simple multi-factor model can outperform a single-factor Merton-type model when predicting default and that enhancement of the Merton model by introducing additional variables can improve its performance. Studies looking at factors affecting credit spreads suggest that factors other than default probability play significant roles in determining the level of credit spreads demanded by the market. Studies by Elton, *et al.* (2001) and Collin-Dufresne, *et al.* (2001) found that default probability and recovery value in default explain a relatively small portion of market-observed credit spreads. Manning (2004) found that, for a sample of investment grade bonds, factors relating to liquidity appear to have greater importance than default probability for determining credit spreads. Churm and Panigirtzoglou (2005) found further evidence that non-credit factors such as overall risk in the financial system (due to macroeconomic factors) and liquidity risk account for a large proportion of credit spread changes. These findings support the view expressed by Eom, *et al.* (2004) that bond-implied estimates of asset value volatility may incorporate not only asset value volatility but also a range of other, unquantifiable factors such as liquidity premium, systematic risk in the financial system as a whole, and macro-economic factors.

As can be seen from the discussion above, attempting to derive implied volatility from bond prices suffers from the same shortcomings as attempting to derive implied volatility from equity derivatives. Additionally, using a univariate or a bivariate model in order to derive market implied asset value volatility may result in the unintended inclusion of effects of other factors that are not explicitly specified in the model.

9.2. Limitations of the Implementation Method

The results of this study are affected not only by the choice of methodology but also by the implementation of the study itself.

In order to compare bond-implied asset value and volatility with equity-derived values for the same variables, a method proposed by Jones, *et al.* (1984) was used, linking equity price volatility with the firm's leverage in order to obtain asset value volatility estimates.

This technique is also used, with certain variations, by KMV and CreditGrades™. Recent empirical studies by Ericsson and Reneby (2004) and Eom, *et al.* (2004) used maximum likelihood techniques proposed by Duan (1994) deriving asset value volatility and other variables from equity pricing formulae. The use of a different method for estimating equity-derived asset value volatility may yield different results to those obtained in this study. It is, however, worthwhile noting that Duan, Gauthier and Simonato (2004) found that estimates using KMV's method are identical to those obtained using maximum likelihood estimates techniques, although maximum likelihood estimates provide more information about the distributional properties of the variables being estimated.

The bond prices used in this sample were classified by Bloomberg™ as 'BGN' (an average of several dealers' quotes during the day) or sourced from a pricing contributor, usually a dealer whose prices are fed into Bloomberg™ . As noted by Ogden (1987) and Sarig and Warga (1989), liquidity in the secondary market for bonds can be very limited, and some bonds do not trade for months, leaving dealers with the need to estimate what would be a fair price for the bond at any given day. Therefore, it is possible that within the sample of bonds used, some of the prices are 'stale' (i.e. they did not change for a period of time) or may be inaccurate as they are a dealer's estimate of fair price rather than a market clearing price.

The sample used in this study contained only publicly traded companies with significant price data history for both their bonds and their equity. As a result, the sample may be biased towards firms that are large and survive, as it did not include defaulted firms. Given the low probability of default for many of the firms in the sample, the dynamics of equity and debt implied asset volatility could be different for them than for, say, firms that are close to default.

The limitations of this study that are due to the implementation method and the sample of data used raise the prospect for further research into these and related topics. Some of these areas are discussed in the following section.

10. Areas for Further Research

This study compared equity-derived asset value and asset value volatility with bond-implied values for the same variables over a period of three years – a comparison that has not been done before. The results suggest that a key assumption in structural models – that volatility is constant over time - does not hold in practice; furthermore, the study highlighted differences between calculations using equity and derivation using bond prices. These findings support the need for further research and development of structural models and their inputs. Some of the areas for further research are discussed in this section.

Given the significant and persistent differences between bond-implied and equity-derived asset value volatility found in this study, an area for further research is to explain what information bond-implied asset volatility actually contains. It is possible, as has been suggested by Eom, *et al.* (2004), that bond-implied volatility incorporates the effects of other factors that are not related to asset value volatility at all, such as the bond's liquidity premium, macro-economic factors and other determinants of bond prices that are not specifically built into structural models in their current form.

According to Christensen and Hansen (2002) it is widely believed that volatility implied by an option's price represents the options markets' forecast of future volatility over the remaining life of the option. This view is based on the rational expectations assumption, which states that the market uses all available information to form its expectations of future volatility. In efficient markets, the implied volatility is the best forecast of future volatility given all information currently available. Supporters of this view include Latané and Rendleman (1976), Chiras and Manaster (1978), Jorion (1995), Christensen and Prahala (1998), and Christensen and Hansen (2002) – all found evidence that supports the view that implied volatility is a more accurate forecast of future volatility than historical volatility. Other studies such as those by Day and Lewis (1992), Lamoureux and Lastrapes (1993), and Canina and Figlewski (1993) found that historical stock price volatility contains information that is not reflected in implied volatilities and that implied volatility provides no superior forecast of future volatility. An area for further research stemming out of this study is whether bond-implied asset values and volatility provide a better forecast of future asset value and volatility than those calculated using equity volatility.

Whilst this study focused on deriving implied asset value and asset value volatility from bond prices, it treated the expected recovery rate in default as constant for all bonds, based on historical studies. Turnbull (2005) described recovery rate information as critical input for the pricing of most credit-sensitive instruments and highlighted a range of factors that might be included when modelling recovery rates. Altman, Resti and Sironi (2004) found that recovery rates depend on seniority and collateral, vary over the credit cycle and across industries, and tend to have a binomial distribution. Cooper and Davydenko (2004) used observed bond spreads to derive market expected losses on default using a calibrated Merton (1974) model. They found that their approach generated forward looking expected default loss estimates that were similar to historical losses for similar credit rating categories. A structural model that incorporates the uncertainty of expected loss on default can be used in order to derive bond implied asset value volatility and determine whether these are statistically different to those estimates obtained when loss given default is assumed to be constant.

This study used a sample comprising Western European based companies. A similar study can be undertaken with different data sets (when available) in order to determine whether the differences between equity derived volatility and bond-implied volatility persist across different markets. Such research may help generalise the results obtained in this study.

The majority of firms in the sample (32 out of 36) were rated investment grade and were thus at a lower risk of default compared with highly leveraged firms. Using the structural model approach, the put options embedded in such firms' bonds are out-of-the-money – the probability that they will be exercised is small. This could result in higher estimation error of implied asset value volatility compared with sub-investment grade firms, where the embedded put options are closer to being at-the-money. Further studies focusing on highly-leveraged firms could advance our understanding of the dynamics of implied asset value volatilities.

As can be seen from the areas for further research highlighted above, research into the usefulness of structural models and the inputs that are needed in order to implement such models still appears to be at its infancy, despite more than thirty years of academic research and development. The field offers a range of opportunities for academic and practical contributions.

11. Glossary of Terms

Absorbing (bankruptcy) Barrier: an option whose payoff is conditional on the value of the underlying asset reaching a certain threshold (barrier). An absorbing barrier is one that once crossed, the path of the underlying asset cannot cross it back. In structural models, modelling bankruptcy barrier as an absorbing barrier option indicates that once a firm's value crossed the bankruptcy threshold, it cannot recover back to a higher value and bankruptcy is the only possible outcome.

Bivariate Normal Distribution: a distribution function of two normal random variables that are correlated. A bivariate normal distribution is calculated using the formulae below:

$$P(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{\left[-\frac{Z}{2(1-\rho^2)}\right]}$$

where: $Z = \frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2}$

$$\rho = cor(x_1, x_2)$$

Brownian Motion (Standard): a Markov process where the change in the value of a variable over a short time interval is normally distributed with a zero mean, a standard deviation of $\sqrt{\Delta t}$ and variance of Δt, where Δt denotes a small time interval. Over longer periods of, say, N time-intervals, the change in the value of the variable will be normally distributed with a mean equal to zero, variance of $N\Delta t = T$ and standard deviation equal to $\sqrt{T}$.

Cumulative Normal Density Function: the area under a curve denoting a normal density function, representing the probability that a variable is greater than, less than or between two specified values. A Cumulative Standard Normal Density Function is the area under the curve denoting a standard normal density function. The total area under the curve equals 1, or 100% probability that all outcomes are possible. The function for finding the area under the curve between two values of the variable z (in this case $-\infty \le z \le x$) is given by:

$$\Phi(z) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}z^2 dz}$$

Diffusion Process: a model where the value of a variable changes continuously and smoothly, without sudden jumps.

Jump Process: a model where the path of a variable through time is represented by sudden jumps that are independent of one another.

Jump-Diffusion Process: a combination of a diffusion process and a jump process, where a variable's path over time follows a diffusion process with jumps that are superimposed on it.

Log-normal Distribution: a continuous distribution in which the logarithm of a variable has a normal distribution.

Markov Process: a stochastic process where only the present value of a variable is relevant for estimating its future value. The future value of the variable is uncertain and is expressed in terms of probability distributions. However, how the variable arrived at its present value is irrelevant to its future value.

Monte-Carlo Simulation: a repeated random generation of values for uncertain variables in order to simulate a model.

Normal Distribution: the probability distribution of a normal random variable. A normal distribution can be represented by a perfectly symmetric bell-shaped curve with a mean μ at its centre and its spread determined by the value of its standard deviation σ. A standard normal distribution is one where $\mu = 0$ and $\sigma = 1$.

Normal Density Function: also known as the normal probability distribution is given by:

$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$ where x denotes a normal random variable with mean μ and a standard deviation σ.

Option: the right, but not the obligation to buy (call) or sell (put) an asset. A European option is one that can only be exercised at its pre-determined maturity date. An American option is one that can be exercised at any time during a pre-determined time period.

Poisson Distribution: a probability distribution that describes the number of events that will occur in a specific time period. In financial theory, the events are referred to as 'jumps' and the jump times are assumed to be independent of one another, and each jump is assumed to be of the same magnitude. The probability distribution, mean and variance of a Poisson random variable are represented by:

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \;;\; \mu = \lambda \;;\; \sigma^2 = \lambda$$

In a Poisson distribution, λ is the mean number of events during a given unit of time; it is also referred to as the *intensity* of the Poisson process.

Standard Normal Density Function: also known as standard normal probability distribution is given by:

$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$ where z denotes a standard normal random variable with a mean equal to 0 and a variance equal to 1.

Stochastic Process: a mathematical description of the behaviour of a stochastic variable. A stochastic variable is one whose future value is not known with certainty.

Wiener Process: see Brownian motion.

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13. Appendix 1

13.1. Explanation of Tables

Appendix 1 contains data relating to each of the bonds tested in this study. In order to facilitate understanding of the data contained here, a brief explanation of each table used in the appendix is provided below, followed by actual data obtained for each bond in the sample.

Company Name:	
Country of Incorporation:	

The table above contains the name of the company that issued the bond and its country of incorporation. Where bonds have been issued by a special purpose vehicle, the company name shown in the table refers to the parent company, which is also the entity whose shares are listed on the stock exchange.

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P

The table above provides information on the bond that was used in the study. The credit ratings shown are those that were assigned to the bond at the time the initial data search was done (4th March 2005).

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B						
V_E						
σ_B						
σ_E						

The table above contains summary data about the values for V_B, V_E, σ_B, and σ_E. For every variable the table shows the mean average, the number of observations, the standard deviation and standard error of the means. For every pair (i.e. V_B and V_E, σ_B, and σ_E), the table contains the correlation between the two variables and a measure of the significance applicable to the correlation calculated.

	Kurtosis	Skewness
V_B-V_E		
σ_B - σ_E		

The table above contains test of the data to indicate whether it is normally distributed or not.

Kurtosis is a measure of the degree to which data points are clustered around a central point. For a normal distribution, the kurtosis statistic should be zero. Positive kurtosis indicates that the data points cluster more and have longer tails than those of a normal distribution; Negative kurtosis indicates the data is less clustered and has shorter tails than those of a normal distribution.

Skewness is a measure of asymmetry of distribution. If data is perfectly normally distributed it has a skewness value of zero. If the skewness measure is significantly positive, it indicates that the data has a long right (positive) tail; negative skewness indicates a long left (negative) tail.

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$A = 0.05$						
V_B-V_E							
σ_B - σ_E							
	$A = 0.01$						
V_B-V_E							
σ_B - σ_E							

The table above contains the data obtained using the Paired-Sample T-Test. For every pair of variables used in the study, the table provides the mean of the differences, the standard deviation of the differences and the lower and upper boundaries between which the true mean difference is likely to be found with significance of $\alpha = 0.05$ and $\alpha = 0.01$. The t-statistic is obtained by dividing the mean difference by its standard error and if its absolute value is greater than the critical t required for rejecting the null hypothesis at a given significance level, then the null-hypothesis can be rejected. df denotes the degrees of freedom with which the calculation of the t-statistic has been performed. The Sig. (2-tailed) column shows the probability of obtaining a t-statistic whose absolute value is greater than or equal to the obtained t-statistic. If the obtained significance level is less than 0.05 or less than 0.01, it is possible to conclude that the mean difference observed is not due to chance. The 2-tailed significance level is also referred to as the *p*-value.

Wilcoxon Signed Rank Test

		Ranks			Test Statistics	
		N	Mean Rank	Sum of Ranks	Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks					
	Positive Ranks					
	Ties					
	Total					
σ_E - σ_B	Negative Ranks					
	Positive Ranks					
	Ties					
	Total					

The table above contains data obtained when performing a Wilcoxon Rank-Signed Test – a test that can be used to compare paired differences when the assumption of normal distribution of data does not hold. A Wilcoxon Rank-Signed Test tests the null hypothesis that two related medians are the same. Ranks are based on absolute value of the differences between the two test variables. The sign of the differences is used to classify cases into negative, positive and tied ranks. Tied cases (where the difference is zero) are ignored. In the table above, column N shows the number of ranks falling into each category. The Sum of the ranks shows the total of all ranks in the category. Z is the standardised measure of the distance between the rank sum of the negative group and the expected value. The two-tailed asymptotic significance provides an estimate of the probability of obtaining a Z statistic that is as or more extreme in absolute value than the Z statistic calculated for the sample. If the asymptotic significance is less than 0.05 or less than 0.01 then the null hypothesis can be rejected with a significance level of α = 0.05 and α = 0.01 respectively.

Correlation

	σ_B	t
σ_B	1	
t		1

The table above contains the Pearson product moment coefficient of correlation between time to maturity t and bond implied asset value volatility σ_B; the correlation coefficient is also known as r. The range of r is between -1 and +1, where r close to or equal to -1 indicates negative correlation between two variables and the closer r gets to +1, the more positive the correlation becomes; r close to or equal to 0 indicates no correlation between the two variables.

Anova

Group	Count	Sum	Average	Variance
σ_B 1				
σ_B 2				
σ_B 3				

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups						
Within Groups						
Total						
			α = 0.01			
Between Groups						
Within Groups						
Total						

The table above provides summary data for single-factor Analysis of Variance (ANOVA) test performed in order to determine whether the volatility of asset values is stable over time. The null hypothesis tested using ANOVA is that asset value volatility is stable over time.

The upper part of the table contains descriptive statistics of the samples used in the ANOVA test. The Count column contains the number of observations in each data set; the Sum column contains the sum of all observations in the data set, and the Average and Variance columns contain the average and variance values for each data set.

The lower part of the table contains the sources of variance between the sub-sets:

- The SS column contains the sum of the squares for variance between the groups and for variance within the group. The variance within the group is attributed to sampling error, and should thus be lower than the variance between groups, if the null hypothesis is to be rejected.

- The df column contains the degrees of freedom for each grouping – as there are 3 groups in each test, the degrees of freedom for variance between the groups is 2.
- The MS column contains the mean square measure, which is the sum of squares (SS) divided by the degrees of freedom (df).
- The F column contains the F statistic fo the ANOVA test, which is the MS between the groups divided by the MS within the groups.
- The P-value column contains the observed significance of the F-test. A P-value lower than 0.05 and 0.01 indicates that the null hypothesis can be rejected with a significance level of $\alpha = 0.05$ and $\alpha = 0.01$ respectively.
- The F-Critical column shows the minimum value of the F statistic needed in order to reject the null hypothesis.

Company Name:	**Royal Ahold N.V.**
Country of Incorporation:	The Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	1,500m	5.875%	May 2001	May 2008	Ba2	BB

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	31,835	761	7,887	285.913	0.978	0.000
V_E	29,133	761	7,276	263.784		
σ_B	46.618	761	8.878	0.322	0.489	6.556
σ_E	46.751	761	14.684	0.532		

	Kurtosis	Skewness
V_B-V_E	0.151	0.031
σ_B - σ_E	1.422	0.762

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	2,702	1,712	2,580	2,823	43.531	760	1.27e-208
σ_B - σ_E	0.000	12.925	0.000	0.787	-0.283	760	0.777
	$\alpha = 0.01$						
V_B-V_E	2,702	1,712	2,542	2,862	43.531	760	1.27e-208
σ_B - σ_E	0.000	12.925	0.000	1.077	-0.283	760	0.777

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	715	400.77	286,551		
	Positive Ranks	46	73.70	3,390		
	Ties	0				
	Total	761			-23.339(a)	1.767e-120
σ_E - σ_B	Negative Ranks	449	317.64	142,622		
	Positive Ranks	312	472.18	147,319		
	Ties	0				
	Total	761			-0.387(b)	0.699

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.777
t	0.777	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	15.034	0.342	1.42e-05
σ_B 2	44	17.936	0.408	2.95e-05
σ_B 3	43	22.365	0.520	0.004

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.707	2	0.353	273.791	5.78e-47	3.067
Within Groups	0.165	128	0.001			
Total	**0.872**	**130**				
			α = 0.01			
Between Groups	0.707	2	0.353	273.791	5.78e-47	4.775
Within Groups	0.165	128	0.001			
Total	**0.872**	**130**				

Company Name:	**Akzo Nobel N.V.**
Country of Incorporation:	The Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
DM	1,000m	5.375%	Nov 1998	Nov 2008	A3	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	18,198	715	2,330	87.126	0.999	0.000
V_E	15,407	715	2,361	88.312		
σ_B	54.748	715	3.038	0.115	0.139	0.000
σ_E	56.670	715	4.907	0.184		

	Kurtosis	Skewness
V_B-V_E	-0.916	0.205
σ_B - σ_E	-0.814	0.392

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	2,791	127	2,782	2,800	585.589	714	0.000
σ_B - σ_E	-1.921	5.420	-2.319	-1.524	-9.480	714	3.66e-020
	$\alpha = 0.01$						
V_B-V_E	2,791	127	2,779	2,803	585.589	714	0.000
σ_B - σ_E	-1.921	5.420	-2.445	-1.398	-9.480	714	3.66e-020

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	715	358.00	255,970		
	Positive Ranks	0				
	Ties	0				
	Total	715			-23.165(a)	1.02e-118
σ_E - σ_B	Negative Ranks	297	260.80	77,459		
	Positive Ranks	418	427.06	178,511		
	Ties	0				
	Total	715			--9.145(b)	5.95e-020

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.824
t	0.824	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	22.900	0.520	0.001
σ_B 2	44	24.171	0.549	0.000
σ_B 3	43	23.553	0.548	8.56e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.023	2	0.012	25.726	4.06e-10	3.067
Within Groups	0.058	128	0.000			
Total	**0.081**	**130**				
			α = 0.01			
Between Groups	0.023	2	0.012	25.726	4.06e-10	4.775
Within Groups	0.058	128	0.000			
Total	**0.081**	**130**				

Company Name:	**Assa Abloy AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	600m	5.125%	Dec 2001	Dec 2006	NR	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	5,617	752	584	21.305	0.986	0.000
V_E	5,189	752	630	22.693		
σ_B	0.552	752	0.042	0.002	0.930	0.000
σ_E	0.742	752	0.054	0.002		

	Kurtosis	Skewness
V_B-V_E	-1.200	-0.339
σ_B - σ_E	-0.975	0.023

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	428	110	420	436	106.271	751	0.000
σ_B - σ_E	-0.189	0.021	-0.191	-0.188	-245.315	751	0.000
	$\alpha = 0.01$						
V_B-V_E	428	110	418	438	106.271	751	0.000
σ_B - σ_E	-0.189	0.021	-0.191	-0.188	-245.315	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.37e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	752			-23.757(b)	9.37e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.771
t	0.771	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	22.953	0.534	0.000
σ_B 2	43	23.201	0.540	2.98e-05
σ_B 3	42	23.710	0.565	0.000

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.022	2	0.011	63.740	8.27e-20	3.069
Within Groups	0.022	125	0.000			
Total	**0.045**	**127**				
			α = 0.01			
Between Groups	0.022	2	0.011	63.740	8.27e-20	4.779
Within Groups	0.022	125	0.000			
Total	**0.045**	**127**				

Company Name:	**Atlas Copco AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
US$	250m	7.750%	Sep 1999	Sep 2009	A3	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	72,694	752	8,190	299	0.996	0.000
V_E	67,070	752	8,300	303		
σ_B	55.072	752	5.860	0.214	0.864	3.18e-225
σ_E	74.734	752	5.621	0.205		

	Kurtosis	Skewness
V_B-V_E	0.008	0.320
σ_B - σ_E	-0.971	0.029

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	5,614	749	5,561	5,668	205.459	751	0.000
σ_B - σ_E	-19.662	3.008	-19.878	-19.447	-179.276	751	0.000
	$\alpha = 0.01$						
V_B-V_E	5,614	749	5,544	5,685	205.459	751	0.000
σ_B - σ_E	-19.662	3.008	-19.945	-19.379	-179.276	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.575(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	752			-23.575(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.792
t	-0.792	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	28.446	0.662	0.000
σ_B 2	43	23.802	0.554	0.000
σ_B 3	42	21.383	0.509	0.000

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.524	2	0.262	761.330	1.01e-70	3.069
Within Groups	0.043	125	0.000			
Total	**0.567**	**127**				
			α = 0.01			
Between Groups	0.524	2	0.262	761.330	1.01e-70	4.780
Within Groups	0.043	125	0.000			
Total	**0.567**	**127**				

Company Name:	**British American Tobacco Plc**
Country of Incorporation:	United Kingdom

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
US$	330m	6.875%	May 1998	May 2008	Baa1	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	23,254	741	2,278	83	0.994	0.000
V_E	21,868	741	2,370	87		
σ_B	0.501	741	0.034	0.001	0.835	4.46e-194
σ_E	0.717	741	0.244	0.009		

	Kurtosis	Skewness
V_B-V_E	-0.202	-0.651
σ_B - σ_E	-0.672	-0.090

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,386	266	1,367	1,405	141.743	740	0.000
σ_B - σ_E	-0.216	-.019	-0.217	-0.214	-312.510	740	0.000
	$\alpha = 0.01$						
V_B-V_E	1,386	266	1,361	1,41	141.743	740	0.000
σ_B - σ_E	-0.216	-.019	-0.217	-0.214	-312.510	740	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	741	371.00	274,911		
	Positive Ranks	0				
	Ties	0				
	Total	741			-23.582(a)	5.86e-123
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	741	371.00	274,911		
	Ties	0				
	Total	741			-23.582(b)	5.86e-123

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.852
t	-0.852	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	23.622	0.549	2.60e-05
σ_B 2	44	22.870	0.520	4.16e-05
σ_B 3	43	19.254	0.448	4.99e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
$\alpha = 0.05$						
Between Groups	0.235	2	0.117	3,000	1.30e-107	3.068
Within Groups	0.005	127	3.91e-05			
Total	**0.240**	**129**				
$\alpha = 0.01$						
Between Groups	0.235	2	0.117	3,000	1.30e-107	4.776
Within Groups	0.005	127	3.91e-05			
Total	**0.240**	**129**				

Company Name:	**BMW AG**
Country of Incorporation:	Germany

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	750m	5.250%	Feb 2001	Sep 2006	A1	NR

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	56,489	757	5,477	199	0.941	0.000
V_E	47,214	757	4,139	150		
σ_B	0.454	757	0.013	0.000	0.396	6.94e-030
σ_E	0.503	757	0.051	0.001		

	Kurtosis	Skewness
V_B-V_E	-1.260	-0.271
σ_B - σ_E	-1.136	-0.300

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	9,275	2,111	9,124	9,426	120.882	756	0.000
σ_B - σ_E	-0.048	0.047	-0.052	-0.045	-28.151	756	7.97e-120
	$\alpha = 0.01$						
V_B-V_E	9,275	2,111	9,077	9,473	120.882	756	0.000
σ_B - σ_E	-0.048	0.047	-0.053	-0.044	-28.151	756	7.97e-120

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	757	379.00	386,903		
	Positive Ranks	0				
	Ties	0				
	Total	757			-23.835(a)	1.44e-125
σ_E - σ_B	Negative Ranks	210	138.85	29,158		
	Positive Ranks	547	471.20	247,745		
	Ties	0				
	Total	757			-18.991(b)	2.04e-080

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.036
t	0.036	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	19.826	0.451	2.25e-06
σ_B 2	44	19.966	0.454	1.15e-05
σ_B 3	43	18.569	0.432	1.84e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
α = 0.05						
Between Groups	0.012	2	0.006	571.741	1.53e-64	3.067
Within Groups	0.001	128	1.07e-05			
Total	**0.014**	**130**				
α = 0.01						
Between Groups	0.012	2	0.006	571.741	1.53e-64	4.774
Within Groups	0.001	128	1.07e-05			
Total	**0.014**	**130**				

Company Name:	**Brisa – Auto Estrades de Portugal SA**
Country of Incorporation:	Portugal

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	650m	4.875%	Dec 2001	Dec 2006	A3	A+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	5,448	763	760	27	0.996	0.000
V_E	4,916	763	662	24		
σ_B	0.466	763	0.033	0.001	0.866	5.79e-231
σ_E	0.676	763	0.043	0.002		

	Kurtosis	Skewness
V_B-V_E	-1.032	-0.383
σ_B - σ_E	-1.264	0.018

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	532	118	524	541	142.212	762	0.000
σ_B - σ_E	-0.211	0.022	-0.212	-0.209	-264.486	762	0.000
	$\alpha = 0.01$						
V_B-V_E	532	118	521	542	142.212	762	0.000
σ_B - σ_E	-0.211	0.022	-0.213	-0.208	-264.486	762	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	763	382.00	291,4663		
	Positive Ranks	0				
	Ties	0				
	Total	763			-23.930(a)	1.51e-126
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	763	382.00	291,4663		
	Ties	0				
	Total	763			-23.930(b)	1.51e-126

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.877
t	0.877	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	19.139	0.435	7.68e-05
σ_B 2	44	20.912	0.475	1.52e-05
σ_B 3	43	20.756	0.483	0.000

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.058	2	0.029	277.359	2.95e-47	3.067
Within Groups	0.013	128	0.000			
Total	**0.071**	**130**				
			$\alpha = 0.01$			
Between Groups	0.058	2	0.029	277.359	2.95e-47	4.775
Within Groups	0.013	128	0.000			
Total	**0.071**	**130**				

Company Name:	**Ciba Specialty Chemicals AG**
Country of Incorporation:	Switzerland

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
CHF	300m	3.250%	Apr 1999	Apr 2009	A2	A

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	12,103	758	1,106	40	0.992	0.000
V_E	10,324	758	1,150	42		
σ_B	0.467	758	0.040	0.001	0.054	0.135
σ_E	0.605	758	0.018	0.007		

	Kurtosis	Skewness
V_B-V_E	0.380	-0.589
σ_B - σ_E	-1.269	0.333

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,778	150	1,767	1,789	325.672	757	0.000
σ_B - σ_E	-13.870	0.004	-14.176	-13.563	-88.835	757	0.000
	$\alpha = 0.01$						
V_B-V_E	1,778	150	1,764	1,792	325.672	757	0.000
σ_B - σ_E	-13.870	0.004	-14.273	-13.467	-88.835	757	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	758	379.50	287,661		
	Positive Ranks	0				
	Ties	0				
	Total	758			-23.851(a)	9.86e-126
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	758	379.50	287,661		
	Ties	0				
	Total	758			-23.851(b)	9.86e-126

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.421
t	0.421	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	17.766	0.404	2.03e-05
σ_B 2	43	20.594	0.479	0.001
σ_B 3	42	20.036	0.477	0.001

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.160	2	0.080	177.805	2.06e-37	3.068
Within Groups	0.057	126	0.000			
Total	**0.216**	**128**				
			α = 0.01			
Between Groups	0.160	2	0.080	177.805	2.06e-37	4.778
Within Groups	0.057	126	0.000			
Total	**0.216**	**128**				

Company Name:	**CIR – Comagnie Industriali Riunite SpA**
Country of Incorporation:	Italy

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	400m	5.250%	Mar 1999	Mar 2009	NR	BBB-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	2,726	713	402	15	0.987	0.000
V_E	2,477	713	311	12		
σ_B	0.444	713	0.039	0.001	-0.694	1.45e-103
σ_E	0.418	713	0.066	0.002		

	Kurtosis	Skewness
V_B-V_E	-1.279	-0.225
σ_B - σ_E	-0.774	-0.476

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	249	106.252	241	257	62.652	712	6.36e-292
σ_B - σ_E	0.026	0.097	0.019	0.033	7.161	712	2.00e-012
	$\alpha = 0.01$						
V_B-V_E	249	106.252	239	260	62.652	712	6.36e-292
σ_B - σ_E	0.026	0.097	0.017	0.035	7.161	712	2.00e-012

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	713	357.00	254,541		
	Positive Ranks	0				
	Ties	0				
	Total	713			-23.133(a)	2.17e-118
σ_E - σ_B	Negative Ranks	468	352.27	164,864		
	Positive Ranks	245	366.03	89,677		
	Ties	0				
	Total	713			-6.833(a)	8.31e-012

(a) Based on positive ranks.

Correlation

	σ_B	t
σ_B	1	0.728
t	0.728	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	16.666	0.379	1.60e-05
σ_B 2	33	16.554	0.502	4.12e-05
σ_B 3	39	18.611	0.477	0.000

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.339	2	0.170	1,474.242	1.11e-81	3.077
Within Groups	0.013	113	0.000			
Total	**0.352**	**115**				
			α = 0.01			
Between Groups	0.339	2	0.170	1,474.242	1.11e-81	4.800
Within Groups	0.013	113	0.000			
Total	**0.352**	**115**				

Company Name:	DSM NA
Country of Incorporation:	Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
US$	250m	6.750%	May 1999	May 2009	A2	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	6,577	770	366	13	0.986	0.000
V_E	5,880	770	392	14		
σ_B	52.327	770	3.500	0.126	-0.452	4.10e-040
σ_E	24.452	770	7.109	0.256		

	Kurtosis	Skewness
V_B-V_E	-0.677	0.191
σ_B - σ_E	-1.143	0.330

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	696	69	691	701	281.400	769	0.000
σ_B - σ_E	27.875	9.236	27.222	28.528	83.748	769	0.000
	$\alpha = 0.01$						
V_B-V_E	696	69	690	702	281.400	769	0.000
σ_B - σ_E	27.875	9.236	27.015	28.734	83.748	769	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	770	385.00	296,835		
	Positive Ranks	0				
	Ties	0				
	Total	770			-24.039(a)	1.09e-127
σ_E - σ_B	Negative Ranks	770	385.00	296,835		
	Positive Ranks	0				
	Ties	0				
	Total	770			-24.039(a)	1.09e-127

(a) Based on positive ranks.

Correlation

	σ_B	t
σ_B	1	-0.346
t	-0.346	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	25.334	0.576	0.000
σ_B 2	44	22.355	0.508	0.000
σ_B 3	43	21.171	0.492	0.000

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.172	2	0.086	520.235	3.42e-62	3.067
Within Groups	0.021	128	0.000			
Total	**0.193**	**130**				
			$\alpha = 0.01$			
Between Groups	0.172	2	0.086	520.235	3.42e-62	4.774
Within Groups	0.021	128	0.000			
Total	**0.193**	**130**				

Company Name:	**Deutsche Telekom AG**
Country of Incorporation:	Germany

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	2,000m	5.250%	May 1998	May 2008	Baa1	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	137,672	764	16,681	603	0.960	0.000
V_E	117,021	764	10,337	374		
σ_B	46.663	764	3.600	0.130	0.035	0.335
σ_E	49.191	764	2.901	0.105		

	Kurtosis	Skewness
V_B-V_E	-1.557	-0.515
σ_B - σ_E	-0.008	0.595

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	20,650	7,357	20,128	21,173	77.582	763	0.000
σ_B - σ_E	-2.529	4.544	-2.851	-2.206	-15.381	763	1.05e-046
	$\alpha = 0.01$						
V_B-V_E	20,650	7,357	19,964	21,338	77.582	763	0.000
σ_B - σ_E	-2.529	4.544	-2.953	-2.104	-15.381	763	1.05e-046

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	764	382.50	292,230		
	Positive Ranks	0				
	Ties	0				
	Total	764			-23.945(a)	1.04e-126
σ_E - σ_B	Negative Ranks	192	333.24	63,983		
	Positive Ranks	572	399.03	228,247		
	Ties	0				
	Total	764			-13.460(b)	2.70e-041

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.857
t	0.857	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	18.914	0.430	5.89e-06
σ_B 2	44	19.629	0.446	1.36e-05
σ_B 3	43	20.711	0.482	0.000

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
α = 0.05						
Between Groups	0.061	2	0.030	558.291	6.02e-64	3.067
Within Groups	0.007	128	5.45e-05			
Total	**0.068**	**130**				
α = 0.01						
Between Groups	0.061	2	0.030	558.291	6.02e-64	4.775
Within Groups	0.007	128	5.45e-05			
Total	**0.068**	**130**				

Company Name:	**Electrolux AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	300m	6.000%	Mar 2001	Mar 2008	Baa1	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	64,570	752	6,508	237	0.995	0.000
V_E	59,980	752	6,321	231		
σ_B	45.509	752	5.860	0.214	0.892	2.99e-260
σ_E	75.585	752	3.252	0.119		

	Kurtosis	Skewness
V_B-V_E	-0.467	-0.598
σ_B - σ_E	-0.281	-0.333

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	4,590	690	4,540	4,639	182.427	751	0.000
σ_B - σ_E	-30.076	3.306	-30.312	-29.839	-249.441	751	0.000
	$\alpha = 0.01$						
V_B-V_E	4,590	690	4,525	4,655	182.427	751	0.000
σ_B - σ_E	-30.076	3.306	-30.387	-29.764	-249.441	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	752			-23.757(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.796
t	-0.796	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	23.306	0.542	2.52e-04
σ_B 2	43	19.880	0.462	7.10e-10
σ_B 3	42	14.465	0.368	3.25e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
$\alpha = 0.05$						
Between Groups	0.643	2	0.321	1,497.736	5.47e-88	3.069
Within Groups	0.027	125	0.000			
Total	**0.670**	**127**				
$\alpha = 0.01$						
Between Groups	0.643	2	0.321	1,497.736	5.47e-88	4.779
Within Groups	0.027	125	0.000			
Total	**0.670**	**127**				

Company Name:	**Fiat SpA**
Country of Incorporation:	Italy

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	500m	5.500%	Dec 1999	Dec 2006	Ba3	BB-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	43,114	752	3,319	121	0.933	0.000
V_E	37,057	752	2,909	106		
σ_B	50.784	752	6.768	0.247	0.528	2.79e-055
σ_E	21.325	752	4.931	0.180		

	Kurtosis	Skewness
V_B-V_E	1.337	-1.162
σ_B - σ_E	0.327	0.912

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	6,057	1,209	5,971	6,144	137.451	751	0.000
σ_B - σ_E	29.459	5.905	29.037	29.882	136.819	751	0.000
	$\alpha = 0.01$						
V_B-V_E	6,057	1,209	5,944	6,171	137.451	751	0.000
σ_B - σ_E	29.459	5.905	28.903	30.015	136.819	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.40e-125
σ_E - σ_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.40e-125

(a) Based on positive ranks.

Correlation

	σ_B	t
σ_B	1	0.820
t	0.820	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	19.138	0.435	1.76e-05
σ_B 2	44	22.194	0.504	2.90e-04
σ_B 3	43	25.018	0.582	9.84e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.469	2	0.235	550.952	1.29e-63	3.067
Within Groups	0.055	128	0.000			
Total	**0.542**	**130**				
			α = 0.01			
Between Groups	0.469	2	0.235	550.952	1.29e-63	4.775
Within Groups	0.055	128	0.000			
Total	**0.542**	**130**				

Company Name:	**Imperial Tobacco Group Plc**
Country of Incorporation:	United Kingdom

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
US$	600m	7.125%	Apr 1999	Apr 2009	Baa3	BBB

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	11,674	759	1,837	67	0.998	0.000
V_E	10,781	759	1,602	58		
σ_B	61.471	759	5.525	0.201	0.857	5.47e-220
σ_E	74.977	759	2.959	0.107		

	Kurtosis	Skewness
V_B-V_E	-1.257	-0.000
σ_B - σ_E	-0.526	-0.226

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
			$\alpha = 0.05$				
V_B-V_E	892	259	874	911	94.972	758	0.000
σ_B - σ_E	-13.506	3.357	-13.745	-13.267	-110.855	758	0.000
			$\alpha = 0.01$				
V_B-V_E	892	259	868	917	94.972	758	0.000
σ_B - σ_E	-13.506	3.357	-13.821	-13.191	-110.855	758	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	759	380.00	288,420		
	Positive Ranks	0				
	Ties	0				
	Total	759			-23.867(a)	6.78e-126
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	759	380.00	288,420		
	Ties	0				
	Total	759			-23.867(b)	6.78e-126

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.928
t	0.928	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	22.739	0.529	1.86e-05
σ_B 2	43	25.436	0.578	4.15e-05
σ_B 3	44	27.020	0.628	1.79e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.213	2	0.107	1,342	3.93e-86	3.068
Within Groups	0.010	127	7.94e-05			
Total	**0.223**	**129**				
			α = 0.01			
Between Groups	0.213	2	0.107	1,342	3.93e-86	4.776
Within Groups	0.010	127	7.94e-05			
Total	**0.223**	**129**				

Company Name:	**Royal KPN N.V.**
Country of Incorporation:	The Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	1,500m	4.750%	Nov 1998	Nov 2008	Baa1	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	39,654	770	3,260	117	0.958	0.000
V_E	34,069	770	2,527	91		
σ_B	51.511	770	4.105	0.148	-0.498	2.15e-049
σ_E	42.466	770	3.700	0.013		

	Kurtosis	Skewness
V_B-V_E	-0.017	-1.201
σ_B - σ_E	0.765	0.290

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	5,568	1,105	5,507	5,664	140.284	769	0.000
σ_B - σ_E	9.045	6.756	8.567	9.523	37.149	769	8.82e-174
	$\alpha = 0.01$						
V_B-V_E	5,568	1,105	5,507	5,664	140.284	769	0.000
σ_B - σ_E	9.045	6.756	8.416	9.671	37.149	769	8.82e-174

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	770	385.50	296,835		
	Positive Ranks	0				
	Ties	0				
	Total	770			-24.039(a)	1.09e-127
σ_E - σ_B	Negative Ranks	710	410.60	291,523		
	Positive Ranks	60	88.53	5,312		
	Ties	0				
	Total	770			-23.179(a)	7.48e-119

(a) Based on positive ranks.

Correlation

	σ_B	t
σ_B	1	0.821
t	0.821	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	20.576	0.468	6.02e-04
σ_B 2	44	21.702	0.493	1.44e-05
σ_B 3	43	22.208	0.516	5.71e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.052	2	0.026	114.855	2.73e-29	3.067
Within Groups	0.029	128	0.000			
Total	**0.081**	**130**				
			$\alpha = 0.01$			
Between Groups	0.052	2	0.026	114.855	2.73e-29	4.775
Within Groups	0.029	128	0.000			
Total	**0.081**	**130**				

Company Name:	**Metro AG**
Country of Incorporation:	Germany

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	100m	5.900%	May 2001	May 2007	Baa1	BBB

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	18,158	763	2,759	100	0.989	0.000
V_E	16,389	763	2,545	90		
σ_B	45.237	763	2.577	0.093	0.858	9.00e-223
σ_E	63.535	763	4.943	0.179		

	Kurtosis	Skewness
V_B-V_E	-1.262	0.375
σ_B - σ_E	0.267	1.080

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
				$\alpha = 0.05$			
V_B-V_E	1,768	488	1,734	1,803	100.189	762	0.000
σ_B - σ_E	-18.298	3.034	-18.513	-18.082	-166.569	762	0.000
				$\alpha = 0.01$			
V_B-V_E	1,768	488	1,723	1,814	100.189	762	0.000
σ_B - σ_E	-18.298	3.034	-18.581	-18.014	-166.569	762	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	763	382.00	291,466		
	Positive Ranks	0				
	Ties	0				
	Total	759			-29.930(a)	1.51e-126
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	763	382.00	291,466		
	Ties	0				
	Total	759			-29.930(b)	1.51e-126

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.290
t	-0.290	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	20.424	0.464	2.64e-05
σ_B 2	44	20.301	0.461	2.19e-05
σ_B 3	43	18.019	0.419	1.98e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
α = 0.05						
Between Groups	0.055	2	0.028	340.690	5.49e-52	3.067
Within Groups	0.010	128	8.13e-05			
Total	**0.066**	**130**				
α = 0.01						
Between Groups	0.055	2	0.028	340.690	5.49e-52	4.775
Within Groups	0.010	128	8.13e-05			
Total	**0.066**	**130**				

Company Name:	**Metso Oyj**
Country of Incorporation:	Finland

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	156m	6.250%	Dec 2001	Dec 2006	Ba1	BB

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	3,023	752	362	13	0.920	1.938e-306
V_E	2,635	752	267	10		
σ_B	47.471	752	1.976	0.720	0.301	2.968e-017
σ_E	55.603	752	5.362	0.200		

	Kurtosis	Skewness
V_B-V_E	-1.321	-0.008
σ_B - σ_E	-0.811	-0.586

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	387	157	376	399	67.629	751	0.000
σ_B - σ_E	8.132	5.125	-8.499	-7.765	-43.510	751	1.871e-207
	$\alpha = 0.01$						
V_B-V_E	387	157	373	402	67.629	751	0.000
σ_B - σ_E	8.132	5.125	-8.615	-7.649	-43.510	751	1.871e-207

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.397e-125
σ_E - σ_B	Negative Ranks	3	4.67	14		
	Positive Ranks	749	377.99	283,114		
	Ties	0				
	Total	752			-23.754(b)	9.937e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.536
t	0.536	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	18.687	0.435	9.85e-05
σ_B 2	43	21.090	0.490	4.69e-04
σ_B 3	42	20.102	0.479	8.94e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.075	2	0.037	169.128	2.77e-36	3.069
Within Groups	0.028	125	0.000			
Total	**0.102**	**127**				
			α = 0.01			
Between Groups	0.075	2	0.037	169.128	2.77e-36	4.780
Within Groups	0.028	125	0.000			
Total	**0.102**	**127**				

Company Name:	**Modern Times Group AP**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	120m	5.500%	Jun 2001	Jun 2006	NR	BB-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	14,910	752	3,619	132	0.991	0.000
V_E	13,819	752	3,815	116		
σ_B	68.741	752	8.914	0.325	-0.695	2.00e-109
σ_E	63.732	752	6.843	0.250		

	Kurtosis	Skewness
V_B-V_E	-1.175	-0.414
σ_B - σ_E	-0.887	0.546

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,091	631	1,046	1,136	47.426	751	4.57e-228
σ_B - σ_E	5.008	14.528	3.968	6.048	9.454	751	4.09e-020
	$\alpha = 0.01$						
V_B-V_E	1,091	631	1,032	1,150	47.426	751	4.57e-228
σ_B - σ_E	5.008	14.528	3.640	6.376	9.454	751	4.09e-020

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	724	389.90	282,284		
	Positive Ranks	28	30.14	844		
	Ties	0				
	Total	752			-23.615(a)	2.707e-123
σ_E - σ_B	Negative Ranks	356	477.53	170,002		
	Positive Ranks	396	285.67	113,126		
	Ties	0				
	Total	752			-4.772(a)	1.821e-006

(a) Based on positive ranks.

Correlation

	σ_B	t
σ_B	1	0.705
t	0.705	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	26.259	0.611	5.24e-04
σ_B 2	43	27.470	0.639	1.88e-04
σ_B 3	42	32.414	0.772	6.49e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.627	2	0.313	695.106	1.89e-68	3.069
Within Groups	0.056	125	0.000			
Total	**0.683**	**127**				
			α = 0.01			
Between Groups	0.627	2	0.313	695.106	1.89e-68	4.779
Within Groups	0.056	125	0.000			
Total	**0.683**	**127**				

Company Name:	**Norsk Hydro ASA**
Country of Incorporation:	Norway

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	300m	6.250%	Oct 1999	Jan 2010	A2	A

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	140,788	633	21,267	845	0.971	0.000
V_E	139,516	633	17,841	709		
σ_B	44.126	633	11.987	0.477	0.882	9.65e-209
σ_E	67.134	633	6.597	0.262		

	Kurtosis	Skewness
V_B-V_E	1.365	-1.552
σ_B - σ_E	0.925	-1.279

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,272	5,832	817	1,727	5.487	632	5.94e-008
σ_B - σ_E	-23.008	6.904	-23.547	-22.649	-83.850	632	0.000
	$\alpha = 0.01$						
V_B-V_E	1,272	5,832	673	1,871	5.487	632	5.94e-008
σ_B - σ_E	-23.008	6.904	-23.717	-22.299	-83.850	632	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	460	289.91	133,357		
	Positive Ranks	173	389.04	67,304		
	Ties	0				
	Total	633			-7.175(a)	7.22e-013
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	633	317.00	200,661		
	Ties	0				
	Total	633			-21.797(b)	2.46e-105

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.892
t	-0.892	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	24.782	0.563	4.55e-04
σ_B 2	44	22.135	0.503	4.29e-04
σ_B 3	43	11.748	0.273	1.01e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	2.031	2	1.016	3,079.447	5.8e-109	3.067
Within Groups	0.042	128	0.000			
Total	**2.074**	**130**				
			α = 0.01			
Between Groups	2.031	2	1.016	3,079.447	5.8e-109	4.775
Within Groups	0.042	128	0.000			
Total	**2.074**	**130**				

Company Name:	**Royal Philips Electronics N.V.**
Country of Incorporation:	The Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	750m	6.125%	May 2001	May 2011	Baa1	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	35,436	768	5,972	216	0.994	0.000
V_E	33,413	768	6,094	220		
σ_B	57.048	768	3.103	0.112	0.866	6.66e-233
σ_E	81.407	768	3.621	0.131		

	Kurtosis	Skewness
V_B-V_E	-1.265	-0.449
σ_B - σ_E	0.403	-0.059

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	2,022	653	1,976	2,068	85.808	767	0.000
σ_B - σ_E	-24.360	1.810	-24.488	-24.231	-372.905	767	0.000
	$\alpha = 0.01$						
V_B-V_E	2,022	653	1,961	2,083	85.808	767	0.000
σ_B - σ_E	-24.360	1.810	-24.528	-24.191	-372.905	767	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	768	384.50	295,296		
	Positive Ranks	0				
	Ties	0				
	Total	768			-24.008(a)	2.30e-127
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	768	384.50	295,296		
	Ties	0				
	Total	768			-24.008(b)	2.30e-127

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.654
t	0.654	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	23.290	0.529	2.85e-04
σ_B 2	44	25.888	0.588	3.70e-05
σ_B 3	43	24.047	0.559	1.94e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
α = 0.05						
Between Groups	0.077	2	0.038	222.935	1.98e-42	3.067
Within Groups	0.022	128	0.000			
Total	**0.099**	**130**				
α = 0.01						
Between Groups	0.077	2	0.038	222.935	1.98e-42	4.775
Within Groups	0.022	128	0.000			
Total	**0.099**	**130**				

Company Name:	**Portugal Telecom SGPS SA**
Country of Incorporation:	Portugal

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	1,000m	5.750%	Feb 2001	Feb 2006	A3	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	13,562	758	3,591	130	0.927	0.000
V_E	14,388	758	1,955	71		
σ_B	52,082	758	4.026	0.146	0.333	4.24e-021
σ_E	61,441	758	3.026	0.110		

	Kurtosis	Skewness
V_B-V_E	-1.504	0.551
σ_B - σ_E	-0.292	0.917

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	-826	1,924	-963	-689	-11.825	757	1.00e-029
σ_B - σ_E	-9.360	4.153	-9.656	-9.063	-62.050	757	4.23e-299
	$\alpha = 0.01$						
V_B-V_E	-826	1,924	-1,006	-646	-11.825	757	1.00e-029
σ_B - σ_E	-9.360	4.153	-9.749	-8.970	-62.050	757	4.23e-299

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	255	286.35	73,019		
	Positive Ranks	503	426.76	214,642		
	Ties	0				
	Total	758			-11.743(a)	7.716e-032
σ_E - σ_B	Negative Ranks	24	17.54	421		
	Positive Ranks	734	391.34	287,240		
	Ties	0				
	Total	758			-23.781(a)	5.217e-125

(a) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.024
t	-0.024	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	42	22.577	0.538	9.56e-05
σ_B 2	43	27.356	0.636	2.60e-04
σ_B 3	43	21.523	0.505	2.02e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
α = 0.05						
Between Groups	0.442	2	0.211	1,132.874	7.92e-81	3.069
Within Groups	0.023	125	0.000			
Total	**0.446**	**127**				
α = 0.01						
Between Groups	0.442	2	0.211	1,132.874	7.92e-81	4.779
Within Groups	0.023	125	0.000			
Total	**0.446**	**127**				

Company Name:	**Repsol YPF S.A.**
Country of Incorporation:	Spain

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	750m	5.750%	Dec 2001	Dec 2006	Baa2	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	43,590	751	2,181	80		9.84e-107
V_E	37,220	751	1,293	47		
σ_B	47.395	751	3.850	0.141		4.84e-101
σ_E	50.078	751	6.549	0.239		

	Kurtosis	Skewness
V_B-V_E	-0.686	-0.443
σ_B - σ_E	-0.954	0.195

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	6,370	1,595	6,256	6,484	109.447	750	0.000
σ_B - σ_E	-2.683	9.579	-3.370	-1.997	-7.677	750	5.08e-014
	$\alpha = 0.01$						
V_B-V_E	6,370	1,595	6,220	6,520	109.447	750	0.000
σ_B - σ_E	-2.683	9.579	-3.586	-1.781	-7.677	750	5.08e-014

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	751	376.00	282,376		
	Positive Ranks	0				
	Ties	0				
	Total	751			-23.741(a)	1.37e-124
σ_E - σ_B	Negative Ranks	271	350.76	95,056		
	Positive Ranks	480	390.25	187,320		
	Ties	0				
	Total	751			-7.757(b)	8.69e-015

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.803
t	0.803	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	18.850	0.438	2.79e-04
σ_B 2	43	19.635	0.457	2.77e-06
σ_B 3	42	19.207	0.457	1.16e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.010	2	0.005	36.774	2.76e-13	3.069
Within Groups	0.017	125	0.000			
Total	**0.027**	**127**				
			$\alpha = 0.01$			
Between Groups	0.010	2	0.005	36.774	2.76e-13	4.779
Within Groups	0.017	125	0.000			
Total	**0.027**	**127**				

Company Name:	**Scania AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	500m	6.000%	Dec 2001	Dec 2008	NR	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	81,177	749	9,579	350	0.994	0.000
V_E	71,371	749	8,254	302		
σ_B	43.342	749	4.434	0.126	0.734	1.37e-127
σ_E	58.394	749	4.305	0.157		

	Kurtosis	Skewness
V_B-V_E	-0.967	-0.333
σ_B - σ_E	0.025	0.689

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	9,806	1,638	9,689	9,924	163.885	748	0.000
σ_B - σ_E	-15.052	2.937	-15.263	-14.841	-140.249	748	0.000
	$\alpha = 0.01$						
V_B-V_E	9,806	1,638	9,652	9,961	163.885	748	0.000
σ_B - σ_E	-15.052	2.937	-15.329	-14.775	-140.249	748	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	749	375.00	280,875		
	Positive Ranks	0				
	Ties	0				
	Total	749			-23.701(a)	2.90e-124
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	749	375.00	280,875		
	Ties	0				
	Total	749			-23.701(b)	2.90e-124

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.013
t	0.013	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	20.034	0.466	1.12e-04
σ_B 2	43	18.484	0.430	4.22e-05
σ_B 3	42	15.963	0.380	9.11e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
α = 0.05						
Between Groups	0.158	2	0.079	222.868	3.89e-42	3.069
Within Groups	0.044	125	0.000			
Total	**0.202**	**127**				
α = 0.01						
Between Groups	0.158	2	0.079	222.868	3.89e-42	4.780
Within Groups	0.044	125	0.000			
Total	**0.202**	**127**				

Company Name:	**Securitas AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	500m	6.125%	Mar 2001	Mar 2008	Baa2	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	58,294	752	10,924	398	0.994	0.000
V_E	55,163	752	11,822	431		
σ_B	49.706	752	4.920	0.179	0.479	2.25e-044
σ_E	75.569	752	4.597	0.168		

	Kurtosis	Skewness
V_B-V_E	-1.401	-0.079
σ_B - σ_E	-1.036	0.327

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	3,131	1,547	3,020	3,242	55.488	751	6.73e-268
σ_B - σ_E	-25.863	4.866	-26.212	-25.515	-145.762	751	0.000
	$\alpha = 0.01$						
V_B-V_E	3,131	1,547	2,985	3,277	55.488	751	6.73e-268
σ_B - σ_E	-25.863	4.866	-26.321	-25.405	-145.762	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	752			-23.757(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.217
t	-0.217	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	23.277	0.541	3.14e-05
σ_B 2	43	21.088	0.490	3.06e-04
σ_B 3	42	18.045	0.430	1.03e-03

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.265	2	0.133	295.023	4.58e-48	3.069
Within Groups	0.056	125	0.000			
Total	**0.322**	**127**				
			$\alpha = 0.01$			
Between Groups	0.265	2	0.133	295.023	4.58e-48	4.779
Within Groups	0.056	125	0.000			
Total	**0.322**	**127**				

Company Name:	**Siemens AG**
Country of Incorporation:	Germany

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	2,000m	5.750%	Jun 2001	Jul 2008	Aa3	AA-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	63,314	638	9,158	363	0.999	0.000
V_E	59,207	638	9,060	359		
σ_B	54.564	638	2.383	0.094	0.178	5.92e-006
σ_E	79.848	638	2.281	0.090		

	Kurtosis	Skewness
V_B-V_E	2.127	-1.559
σ_B - σ_E	-0.270	-0.325

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	4,107	406	4,076	4,139	255.275	637	0.000
σ_B - σ_E	-25.284	2.991	-25.517	-25.052	-213.516	637	0.000
	$\alpha = 0.01$						
V_B-V_E	4,107	406	4,066	4,149	255.275	637	0.000
σ_B - σ_E	-25.284	2.991	-25.590	-24.978	-213.516	637	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	638	319.50	203,841		
	Positive Ranks	0				
	Ties	0				
	Total	638			-21,883(a)	3.75e-106
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	638	319.50	203,841		
	Ties	0				
	Total	638			-21,883(b)	3.75e-106

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.477
t	0.477	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	22.264	0.506	2.67e-05
σ_B 2	44	24.694	0.561	1.31e-05
σ_B 3	43	23.608	0.549	2.93e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.074	2	0.037	373.391	9.27e-52	3.067
Within Groups	0.014	128	0.000			
Total	**0.088**	**130**				
			α = 0.01			
Between Groups	0.074	2	0.037	373.391	9.27e-52	4.775
Within Groups	0.014	128	0.000			
Total	**0.088**	**130**				

Company Name:	**Solvay SA**
Country of Incorporation:	Belgium

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	700m	5.250%	Jul 2001	Jul 2006	A2	A

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	7,756	768	732	26	0.976	0.000
V_E	7,281	768	734	26		
σ_B	54.451	768	2.983	0.108	0.424	7.31e-035
σ_E	77.252	768	1.832	0.066		

	Kurtosis	Skewness
V_B-V_E	-1.249	-0.286
σ_B - σ_E	-0.576	0.031

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	475	161	460	490	81.581	767	0.000
σ_B - σ_E	-22.800	2.760	-22.996	-22.605	-228.913	767	0.000
	$\alpha = 0.01$						
V_B-V_E	475	161	460	490	81.581	767	0.000
σ_B - σ_E	-22.800	2.760	-23.057	-22.5423	-228.913	767	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	768	384.50	295,296		
	Positive Ranks	0				
	Ties	0				
	Total	768			-24,008(a)	2.30e-127
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	768	384.50	295,296		
	Ties	0				
	Total	768			-24,008(b)	2.30e-127

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.479
t	-0.479	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	26.820	0.610	1.29e-04
σ_B 2	44	24.356	0.554	3.03e-05
σ_B 3	43	21.177	0.492	1.72e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.298	2	0.149	1,356.207	7.00e-87	3.067
Within Groups	0.014	128	0.000			
Total	**0.312**	**130**				
			$\alpha = 0.01$			
Between Groups	0.298	2	0.149	1,356.207	7.00e-87	3.067
Within Groups	0.014	128	0.000			
Total	**0.312**	**130**				

Company Name:	**Stora Enso Oyj**
Country of Incorporation:	Finland

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	375m	6.375%	Jun 2000	Jun 2007	Baa1	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	17,055	752	1,077	39	0.970	0.000
V_E	15,203	752	1,318	48		
σ_B	42.754	752	2.113	0.077	0.775	8.00e-152
σ_E	60.929	752	2.780	0.103		

	Kurtosis	Skewness
V_B-V_E	-0.806	-0.729
σ_B - σ_E	0.977	-0.893

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,852	379	1,825	1,880	133.864	751	0.000
σ_B - σ_E	-18.354	1.756	-18.480	-18.228	-286.605	751	0.000
	$\alpha = 0.01$						
V_B-V_E	1,852	379	1,817	1,888	133.864	751	0.000
σ_B - σ_E	-18.354	1.756	-18.519	-18.189	-286.605	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23,757(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	752			-23,757(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.221
t	-0.221	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	18.224	0.424	2.81e-05
σ_B 2	43	18.646	0.434	3.77e-04
σ_B 3	42	16.491	0.393	2.76e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.039	2	0.019	133.470	9.57e-32	3.069
Within Groups	0.018	125	0.000			
Total	**0.057**	**127**				
			$\alpha = 0.01$			
Between Groups	0.039	2	0.019	133.470	9.57e-32	4.779
Within Groups	0.018	125	0.000			
Total	**0.057**	**127**				

Company Name:	**Sudzuecker AG**
Country of Incorporation:	Germany

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	500m	5.750%	Feb 2002	Feb 2012	A3	NR

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	4,379	764	198	7	0.943	0.000
V_E	4,062	764	190	7		
σ_B	48.664	764	2.345	0.085	0.615	1.27e-080
σ_E	66.738	764	2.328	0.084		

	Kurtosis	Skewness
V_B-V_E	-0.711	-0.691
σ_B - σ_E	1.765	-0.277

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	318	66	313	322	132.740	763	0.000
σ_B - σ_E	-18.075	2.051	-18.220	-17.929	-243.600	763	0.000
	$\alpha = 0.01$						
V_B-V_E	318	66	311	324	132.740	763	0.000
σ_B - σ_E	-18.075	2.051	-18.266	-17.883	-243.600	763	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	764	382.50	292,230		
	Positive Ranks	0				
	Ties	0				
	Total	764			-23,945(a)	1.04e-126
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	764	382.50	292,230		
	Ties	0				
	Total	764			-23,945(b)	1.04e-126

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.763
t	0.763	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	18.958	0.431	7.17e-04
σ_B 2	44	21.698	0.493	8.71e-06
σ_B 3	43	21.480	0.499	3.85e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.125	2	0.063	244.630	1.87e-44	3.067
Within Groups	0.033	128	0.000			
Total	**0.158**	**130**				
			$\alpha = 0.01$			
Between Groups	0.125	2	0.063	244.630	1.87e-44	4.775
Within Groups	0.033	128	0.000			
Total	**0.158**	**130**				

Company Name:	**Swedish Match AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	180m	6.125%	Oct 1999	Oct 2006	A3	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	30,371	752	2,329	85	0.987	0.000
V_E	28,428	752	2,376	87		
σ_B	50.013	752	8.355	0.305	0.744	1.22e-133
σ_E	79.704	752	1.972	0.072		

	Kurtosis	Skewness
V_B-V_E	-0.695	-0.904
σ_B - σ_E	-0.951	0.079

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,942	383	1,916	1,970	139.296	751	0.000
σ_B - σ_E	-29.692	7.012	-30.194	-29.190	-116.120	751	0.000
	$\alpha = 0.01$						
V_B-V_E	1,942	383	1,907	1,979	139.296	751	0.000
σ_B - σ_E	-29.692	7.012	-30.352	-29.031	-116.120	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23,755(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	764			-23,755(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.879
t	-0.879	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	28.348	0.659	1.26e-03
σ_B 2	43	22.490	0.523	1.55e-04
σ_B 3	42	16.043	0.382	4.87e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	1.634	2	0.817	1,284.957	4.44e-84	3.069
Within Groups	0.079	125	0.001			
Total	**1.713**	**127**				
			α = 0.01			
Between Groups	1.634	2	0.817	1,284.957	4.44e-84	4.779
Within Groups	0.079	125	0.001			
Total	**1.713**	**127**				

Company Name:	**TDC A/S**
Country of Incorporation:	Denmark

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	685m	5.875%	Apr 2001	Apr 2006	A3	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	93,713	752	10,953	399	0.958	0.000
V_E	81,371	752	8,257	301		
σ_B	39.307	752	4.686	0.171	0.808	7.55e-175
σ_E	50.390	752	3.605	0.131		

	Kurtosis	Skewness
V_B-V_E	-0.592	-0.071
σ_B - σ_E	-1.294	-0.749

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	12,341	3,851	12,066	12,617	87.885	751	0.000
σ_B - σ_E	-11.083	2.764	-11.281	-10.885	-109.960	751	0.000
	$\alpha = 0.01$						
V_B-V_E	12,341	3,851	11,979	12,704	87.885	751	0.000
σ_B - σ_E	-11.083	2.764	-11.343	-10.823	-109.960	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23,757(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	764			-23,757(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.930
t	0.930	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	15.109	0.343	5.27e-05
σ_B 2	44	17.519	0.398	2.59e-04
σ_B 3	43	18.510	0.430	2.12e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.169	2	0.084	484.957	1.84e-60	3.067
Within Groups	0.022	128	0.000			
Total	**0.191**	**130**				
			α = 0.01			
Between Groups	0.169	2	0.084	484.957	1.84e-60	4.775
Within Groups	0.022	128	0.000			
Total	**0.191**	**130**				

Company Name:	**Telécom Italia SpA**
Country of Incorporation:	Italy

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating Moody's	Credit Rating S&P
€	3,000m	6.125%	Apr 2000	Apr 2006	Baa2	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	71,653	752	15,416	562	0.988	0.000
V_E	65,776	752	9,953	563		
σ_B	46.758	752	6.196	0.226	0.940	0.000
σ_E	60.434	752	6.312	0.230		

	Kurtosis	Skewness
V_B-V_E	-1.434	0.495
σ_B - σ_E	-1.093	-0.001

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	5,877	5,794	5,462	6,292	27.815	751	1.35e-117
σ_B - σ_E	-13.682	2.167	-13.837	-13.527	-173.478	751	0.000
	$\alpha = 0.01$						
V_B-V_E	5,877	5,794	5,331	6,423	27.815	751	1.35e-117
σ_B - σ_E	-13.682	2.167	-13.886	-13.478	-173.478	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	730	427.89	269,570		
	Positive Ranks	122	111.13	13,558		
	Ties	0				
	Total	752			-21.481(a)	2.33e-102
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	764			-23,757(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.725
t	0.725	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	20.043	0.456	7.13e-06
σ_B 2	44	18.944	0.431	2.44e-05
σ_B 3	43	20.846	0.485	1.41e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.064	2	0.032	564.193	3.29e-64	3.067
Within Groups	0.007	128	0.000			
Total	**0.071**	**130**				
			α = 0.01			
Between Groups	0.064	2	0.032	564.193	3.29e-64	4.775
Within Groups	0.007	128	0.000			
Total	**0.071**	**130**				

Company Name:	**Telefonica SA**
Country of Incorporation:	Spain

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	500m	4.500%	Apr 1999	Apr 2009	A3	A

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	88,373	751	10,220	373	0.993	0.000
V_E	78,999	751	9,416	343		
σ_B	53.614	751	5.017	0.183	-0.289	6.44e-016
σ_E	65.583	751	2.969	0.108		

	Kurtosis	Skewness
V_B-V_E	67.502	-4.337
σ_B - σ_E	-0.612	0.656

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	9,737	1,403	9,637	9,838	190.238	750	0.000
σ_B - σ_E	-11.969	6.527	-12.436	-11.501	-50.256	750	2.70e-242
	$\alpha = 0.01$						
V_B-V_E	9,737	1,403	9,603	9,870	190.238	750	0.000
σ_B - σ_E	-11.969	6.527	-12.584	-11.354	-50.256	750	2.70e-242

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	750	375.61	281,709		
	Positive Ranks	1	667.00	667		
	Ties	0				
	Total	751			-23.629(a)	1.96e-123
σ_E - σ_B	Negative Ranks	59	51.83	3,058		
	Positive Ranks	692	403.64	279,318		
	Ties	0				
	Total	751			-23,227(b)	2.45e-119

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.793
t	0.793	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	20.953	0.487	9.72e-06
σ_B 2	43	22.197	0.516	3.33e-05
σ_B 3	42	23.534	0.560	6.03e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.115	2	0.057	270.643	3.79e-46	3.069
Within Groups	0.027	125	0.000			
Total	**0.141**	**127**				
			α = 0.01			
Between Groups	0.115	2	0.057	270.643	3.79e-46	4.779
Within Groups	0.027	125	0.000			
Total	**0.141**	**127**				

Company Name:	**TeliaSonera AB**
Country of Incorporation:	Sweden

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	203m	4.625%	Apr 1999	Apr 2009	Baa1	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	195,937	742	24,973	910	0.995	0.000
V_E	186,884	742	22,473	825		
σ_B	57.421	742	9.929	0.108	0.622	1.56e-080
σ_E	82.660	742	2.063	0.076		

	Kurtosis	Skewness
V_B-V_E	-1.288	-0.710
σ_B - σ_E	1.738	-0.200

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	6,054	3,354	8,812	9,295	73.539	741	0.000
σ_B - σ_E	-25.239	2.307	-25.405	-25.072	-297.971	741	0.000
	$\alpha = 0.01$						
V_B-V_E	6,054	3,354	8,736	9,372	73.539	741	0.000
σ_B - σ_E	-25.239	2.307	-25.457	-25.020	-297.971	741	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	742	371.50	275,653		
	Positive Ranks	0				
	Ties	0				
	Total	742			-23.598(a)	4.02e-123
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	742	371.50	275,653		
	Ties	0				
	Total	742			-23.598(b)	4.02e-123

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.086
t	0.086	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	25.454	0.592	4.56e-04
σ_B 2	43	24.982	0.581	2.20e-04
σ_B 3	32	17.492	0.547	3.89e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
$\alpha = 0.05$						
Between Groups	0.040	2	0.020	56.348	8.75e-18	3.075
Within Groups	0.040	115	0.000			
Total	**0.080**	**117**				
$\alpha = 0.01$						
Between Groups	0.040	2	0.020	56.348	8.75e-18	4.795
Within Groups	0.040	115	0.000			
Total	**0.080**	**117**				

Company Name:	**Unilever NV**
Country of Incorporation:	The Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	1,000m	5.125%	Jun 2001	Jun 2006	A1	A-

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	61,751	770	3,770	136	0.946	0.000
V_E	53,257	770	3,435	124		
σ_B	47.238	770	4.277	0.154	0.287	5.03e-016
σ_E	59.025	770	4.292	0.155		

	Kurtosis	Skewness
V_B-V_E	-0.565	-0.134
σ_B - σ_E	-0.695	-0.597

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	8,494	1,226	8,408	8,581	192.279	769	0.000
σ_B - σ_E	-11.787	5.118	-12.149	-11.425	-63.910	769	0.000
	$\alpha = 0.01$						
V_B-V_E	8,494	1,226	8,380	8,609	192.279	769	0.000
σ_B - σ_E	-11.787	5.118	-12.263	-11.311	-63.910	769	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	770	385.50	296,835		
	Positive Ranks	0				
	Ties	0				
	Total	770			-24.039(a)	1.09e-127
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	770	385.50	296,835		
	Ties	0				
	Total	770			-24.039(b)	1.09e-127

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	-0.477
t	-0.477	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	24.331	0.553	5.29e-04
σ_B 2	44	20.336	0.462	2.59e-04
σ_B 3	43	18.380	0.427	7.64e-05

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.367	2	0.183	639.454	2.36e-67	3.067
Within Groups	0.037	128	0.000			
Total	**0.404**	**130**				
			α = 0.01			
Between Groups	0.367	2	0.183	639.454	2.36e-67	4.775
Within Groups	0.037	128	0.000			
Total	**0.404**	**130**				

Company Name:	**UPM-Kymmene Oyj**
Country of Incorporation:	Finland

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
€	600m	6.125%	Jan 2002	Jan 2012	Baa1	BBB

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	16,049	752	1,001	37	0.941	0.000
V_E	14,523	752	930	34		
σ_B	42.662	752	3.435	0.129	0.747	3.29e-135
σ_E	57.309	752	5.483	0.200		

	Kurtosis	Skewness
V_B-V_E	-1.112	-0.623
σ_B - σ_E	-1.253	-0.264

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	1,526	339	1,502	1,551	123.317	751	0.000
σ_B - σ_E	-14.647	3.686	-14.911	-14.383	-108.978	751	0.000
	$\alpha = 0.01$						
V_B-V_E	1,526	339	1,494	1,558	123.317	751	0.000
σ_B - σ_E	-14.647	3.686	-14.994	-14.300	-108.978	751	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	752	376.50	283,128		
	Positive Ranks	0				
	Ties	0				
	Total	752			-23.757(a)	9.40e-125
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	752	376.50	283,128		
	Ties	0				
	Total	752			-23.757(b)	9.40e-125

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.799
t	0.799	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	43	15.588	0.636	5.27e-05
σ_B 2	43	18.274	0.425	4.00e-05
σ_B 3	42	17.373	0.414	3.56e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			$\alpha = 0.05$			
Between Groups	0.095	2	0.048	321.262	5.48e-50	3.069
Within Groups	0.018	125	0.000			
Total	**0.114**	**127**				
			$\alpha = 0.01$			
Between Groups	0.095	2	0.048	321.262	5.48e-50	4.779
Within Groups	0.018	125	0.000			
Total	**0.114**	**127**				

Company Name:	**Wolters Kluwer NV**
Country of Incorporation:	The Netherlands

Bond Data

Currency	Principal	Coupon	Issue Date	Maturity Date	Credit Rating	
					Moody's	S&P
NLG	500m	5.250%	Apr 1998	Apr 2008	Baa1	BBB+

Value and Value Volatility Data

	Mean	No.	Std. Dev.	Std. Error of Means	Correlation	Significance
V_B	7,852	700	884	33	0.997	0.000
V_E	7,307	700	955	36		
σ_B	45.176	700	4.191	0.158	0.760	8.50e-133
σ_E	60.316	700	5.047	0.191		

	Kurtosis	Skewness
V_B-V_E	-1.026	0.005
σ_B - σ_E	-0.735	0.454

Paired Samples T Test

	Mean	Std. Dev.	Lower Limit	Upper Limit	t	df	Sig. (2-tailed)
	$\alpha = 0.05$						
V_B-V_E	546	99	538	552	145.460	699	0.000
σ_B - σ_E	-15.140	3.300	-15.385	-14.895	-121.423	699	0.000
	$\alpha = 0.01$						
V_B-V_E	546	99	535	555	145.460	699	0.000
σ_B - σ_E	-15.140	3.300	-15.462	-14.818	-121.423	699	0.000

Wilcoxon Signed Rank Test

		Ranks N	Mean Rank	Sum of Ranks	Test Statistics Z	Asymp. Sig. (2-tailed)
V_E - V_B	Negative Ranks	700	350.50	245,350		
	Positive Ranks	0				
	Ties	0				
	Total	700			-22.921 (a)	2.87e-116
σ_E - σ_B	Negative Ranks	0				
	Positive Ranks	700	350.50	245,350		
	Ties	0				
	Total	700			-22.921 (b)	2.87e-116

(a) Based on positive ranks.
(b) Based on negative ranks.

Correlation

	σ_B	t
σ_B	1	0.863
t	0.863	1

Anova

Group	Count	Sum	Average	Variance
σ_B 1	44	17.256	0.392	1.02e-04
σ_B 2	44	20.112	0.457	4.54e-05
σ_B 3	43	19.323	0.449	2.60e-04

Source of Variance	SS	df	MS	F	*P*-Value	*F*-Critical
			α = 0.05			
Between Groups	0.110	2	0.055	409.393	2.14e-56	3.067
Within Groups	0.017	128	0.000			
Total	**0.128**	**130**				
			α = 0.01			
Between Groups	0.110	2	0.055	409.393	2.14e-56	4.775
Within Groups	0.017	128	0.000			
Total	**0.128**	**130**				